Generic Issues for Dynamic (Macro)Economic Models
(using the Solow gowth model handout as an example)

1. Exogenous and endogenous variables

Those variables determined by the model, and those independent of it. In most simple models there is an equation associated with each endogenous variable, so there are as many equations as 'unknowns' (endogenous variables), although often this association may be arbitrary (e.g. quantity and price with a supply and demand curve).

Solow: exogenous = (s,n) endogenous = (y,c,k,r,w)

2. Dynamic and static equations

A model with 'x' dynamic equations is sometimes described as a 'xth' order dynamic system.

Solow: capital accumulation is the only dynamic equation, so it is s first order dynamic system

3. Functional forms

Equations may be given a specific functional form, or may be kept general. Analysis often attempts to keep functions as general as possible, but some specific form may be required to get results or keep the algebra simple

Solow: a general production function is y=f(k), a particular production function would be $y=Ak^a$., where 'A' and 'a' are parameters (Cobb-Douglas)

4. Shocks

A 'shock' is anything that moves a variable away from its associated equation. A shock may have a particular dynamic structure, from purely transitory (one period only) to permanent (often described as a 'shift' in an equation).

Solow: a shock to the production function could be represented as some non-zero value of u_t in $y_t=Ak_t^a+ u_t$ *or a permanent change in A.*

5. Equations or diagrams

Dualism. Diagrams may be more intuitive, but they are normally only two-dimensional and may contain implicit assumptions about functional forms (e.g. a straight line implies a linear relationship.)

6. Equilibrium

Variables settle to a steady state. (Can be a static or steady growth equilibrium. Equilibrium may not exist or it may not be unique).

Solow: when dk/dt=0, we have a unique equilibrium given certain assumptions about the production function (diminishing marginal product)

7. Recursive or simultaneous structure

Normally every endogenous variable is influenced, directly or indirectly, by every other (the model is completely simultaneous). However this need not always be true. A recursive structure can help us understand how a model works.

Solow: if we know f(.), then n and s give us k and y, independently of c or r. So if we add government spending to the GDP identity, for example, this will not influence k.

8. Comparative Statics

The impact of a change in an exogenous variable on endogenous variables in equilibrium.

An example from the Solow model: The impact of a change in the savings rate on steady state capital (the value of dk/ds)

9. Stability

If the economy starts away from equiulibrium, will its dynamics move it towards the equilibrium? Without stability, comparative statics is rather pointless. A rigorous analysis of stability requires computing the roots (eigenvalues) of the system (or its linear approximation), but a diagrammatic analysis of a first or second order system is possible (see the handout on the neoclassical growth model).

Solow: the model is stable as long as f''(k)<0.

10. Backward of Forward Looking Dynamics

In a dynamic relationship, is the variable today influenced by past values (backward looking), future values (forward looking) or both. If the variable is forward looking, then it is sometimes called a 'jump' variable. The presence of forward looking variables makes a big difference to any analysis of stability – see the handout on the neoclassical growth model.

11. Determinacy

A model involving forward looking variables is sometimes described as 'determinate' or 'indeterminate', rather than stable or unstable. This is because at least one unstable root is required to give the forward looking elements a unique (determinate) starting point. These models are sometimes called 'saddle path stable'.

12. Impulse response

The complete dynamic response of the system to a change in an exogenous variable

13. Welfare

If our model contains some measure of agent's utility (or we are able to add such a measure), then we can attempt to ask whether an equilibrium or dynamic path is pareto efficient.

Solow: individual consumption is maximised when r=n. Any other equilibria is not pareto efficient.