Contracts, Asynchronised Price Setting, Imperfect Competition and Nominal Inertia

Consider a monopolistic competitor facing a demand curve of the form

$$y_i = (\frac{P_i}{P})^{-\theta} M / P$$

where p is the aggregate price (which each firm treats as exogenous) and aggregate demand is represented by real money balances. The firm maximises *real* profits given by

$$y_i(\frac{P_i}{P}) - \frac{k}{\beta} y_i^{\beta}$$

Optimising and using the demand curve to substitute for costs we can obtain

$$P_i / P = [(\frac{k\theta}{\theta - 1})(\frac{M}{P})^{\beta-1}]^{1/[1+\theta(\beta-1)]}$$

(Romer Ch.6,B&F Ch.8) Taking logs, and using lower case to denote logs, we can write this as

$$p_i = p + a(m-p) + \text{constant} \qquad (1)$$

where a is a constant which is a function of θ and β.

Now consider an economy made up of only two groups of firms. Each group sets its price for two periods (*contracts*). Each group is identical except that one sets prices in even periods and the other in odd periods (*asynchronisation or staggering*). Each firm faces the demand curve (1), where demand depends on relative prices and aggregate demand *(imperfect competition*).

Let p_1 and p_2 represent the log of the price level of a firm in each group. Aggregate real money balances are given by $m - (p_1 + p_2)/2$. If each group could vary its price each period, then following (1) prices would be given by

$$p_{1t} = p_{2t} + 2a \{ E[m_t] - (p_{1t} + p_{2t})/2 \} \qquad (2)$$

If aggregate demand was unimportant (a=0), one group would follow the other. If m_t and p_{2t} doubled, so would p_{1t} (homogeneity). There would be a similar equation for p_{2t} .

Equation (2) can be rewritten as

$$p_{1t} = b\, p_{2t} + (1-b)\, E[m_t] \qquad (3)$$

also $$p_{1t+1} = b\, p_{2t+1} + (1-b)\, E[m_{t+1}] \qquad (3')$$

However firms have to set prices for two periods. Let x_t denote the contract price set by group 1 in period t for t and t+1. A natural choice for firms faced with this constraint[1] is to set

$$x_t = (p_{1t} + p_{1t+1})/2$$

$$= (bp_{2t} + (1-b)E[m_t|I_t])/2 + (bE[p_{2t+1}|I_t] + (1-b)E[m_{t+1}|I_t])/2$$

However, as each group is identical, $p_{2t} = x_{t-1}$, $p_{2t+1} = x_{t+1}$. We therefore have

$$x_t = b(x_{t-1} + E[x_{t+1}|I_t])/2 + (1-b)(E[m_t|I_t]+E[m_{t+1}|I_t])/2 \qquad (4)$$

This is a second order difference equation, which has a solution (see earlier handout)

$$x_t = \lambda x_{t-1} + \frac{(1-\lambda)^2}{2}\sum_{i=0}^{\infty}\lambda^i E[m_{t+i} + m_{t+i+1}|I_t] \qquad (5)$$

To check this subtract λx_{t+1} from x_t using (5). You will find

$$b = 2\lambda / (1+\lambda^2) \qquad 0 < \lambda < 1$$

The aggregate price level is $p_t = (x_t + x_{t-1})/2$.

A surprise, permanent increase in money

Suppose $m_t = 0$ until T, and $m_{T+i} = 1$ for all i>0. The increase in m at T is a surprise, but once it has happened it is expected to continue. We have

$$\begin{aligned} x_T &= \frac{(1-\lambda)^2}{2}\sum_{i=0}^{\infty}2(1+\lambda+\lambda^2+\lambda^3+..... \\ &= 1-\lambda \\ x_{T+1} &= \lambda(1-\lambda)+(1-\lambda) = (1-\lambda)(1+\lambda) \\ x_{T+2} &= (1-\lambda)(1+\lambda+\lambda^2) \\ etc \end{aligned}$$

If $y_t = m_t - p_t$, then $y_t = 0$ for t<T

$y_T = (1+\lambda)/2$

$y_{T+1} = \lambda(1+\lambda)/2$

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[1] Here we ignore discounting for simplicity

$y_{T+2} = \lambda^2(1+\lambda)/2$ etc

The influence of the increase in money on output is persistent. Note if $b=0$ (no imperfect competition), $\lambda=0$ and $y_{T+i} = 0$ for $i>0$, or in other words output increases for a period which is just less than the contract.

In this example m_T was part of I_T. If m_T ,and changes to all subsequent m, were unknown until T+1, then the initial increase in output would have been 1, followed by the above series led one quarter.

An anticipated, permanent increase in money

At t=0, $E[m_i, i \geqslant T]=1$, $E[m_i, I<=T]=0$. We can now plug this into (5). Noting that $x_{-1}=0$, we have

$$x_0 = \frac{(1-\lambda)^2}{2}[0+0+...+\lambda^{T-1}+2\lambda^T+2\lambda^{T+1}+...]$$

This can be simplified as follows

$$\begin{aligned}
x_0 &= \frac{(1-\lambda)^2}{2}\lambda^{T-1}[1+2\lambda+2\lambda^2+.. \\
&= \frac{(1-\lambda)^2}{2}[\lambda^{T-1}+(1-\lambda)^2\lambda^T(1+\lambda+\lambda^2+.. \\
&= \frac{(1-\lambda)^2}{2}\lambda^{T-1}+\lambda^T(1-\lambda) \quad = \frac{(1-\lambda^2)}{2}\lambda^{T-1} = \frac{(1-\lambda)}{2}(\lambda^{T-1}+\lambda^T)
\end{aligned}$$

Note that as $\lambda<1$, contract prices rise by a small amount, which is smaller the larger is T. This makes intuitive sense: the further into the future the increase in money is expected to be, the less prices rise. However x_t is always positive, so some price increase will occur.

When we come to x_1, we now need to take into account the non-zero lagged value of x, which is simply λ times the expression above. The forward term involving m is simply going to be the above divided by λ. Combining the two gives

$$\begin{aligned}
x_1 &= \frac{(1-\lambda^2)}{2}\lambda^T + \frac{(1-\lambda^2)}{2}\lambda^{T-2} \\
&= \frac{(1-\lambda^2)(1+\lambda^2)}{2}\lambda^{T-2} \\
&= \frac{(1-\lambda)}{2}(\lambda^{T-2}+\lambda^{T-1}+\lambda^T+\lambda^{T+1})
\end{aligned}$$

This term is clearly larger than the last, which again makes sense - firms are gradually raising their prices. Generally we have

$$x_t = \frac{(1-\lambda)}{2}(\lambda^{T-t-1} + \lambda^{T-t} + ... + \lambda^{T+t-1} + \lambda^{T+t})$$

We can use this to calculate the contract price at t=T-1. This is

$$x_{T-1} = \frac{(1-\lambda)}{2}(1+\lambda \ \ +..+\lambda^{2T-1})$$

This actually tells us something quite useful. Suppose T was very large. In this case the power series in λ almost cancels with $1-\lambda$, and we get an answer roughly equal to 1/2. The intuition for this is simple. The long run value of x is one. Firms want to adjust gradually towards that, but also want to minimise the difference between x and m. If they are given infinite notice of the change in m, they will time their adjustment so that they are half way there when m actually changes.

If x_{T-1} was zero, then $x_T = 1-\lambda$. But x_{T-1} is not zero, so in fact

$$x_T = (1-\lambda) + \lambda x_{T-1}$$

Using the same logic

$$x_{T+1} = (1-\lambda)(1+\lambda) + \lambda^2 x_{T-1}$$

As we move forward, the influence of the x_{T-1} term dies out, and we come closer to the path for x that would have occurred if the change in m had been a surprise.

We know that $y_t = m_t - p_t = m_t - (x_t + x_{t-1})/2$. Using this and our expressions above we can derive

$$y_0 = 0 - \frac{(1-\lambda^2)}{4}\lambda^{T-1}$$

$$y_1 = 0 - \frac{(1-\lambda^2)}{4}\lambda^{T-1} - \frac{(1-\lambda^2)(1+\lambda^2)}{4}\lambda^{T-2}$$

$$= -\frac{(1-\lambda^2)}{4}\lambda^{T-2}(1+\lambda+\lambda^2)$$

The key point here is that output falls before T, by an amount that gradually increases. Firms are raising prices in anticipation of higher money, but before money actually increases, so real balances fall. The initial impact of an anticipated expansionary monetary policy is therefore deflationary.
As the maximum prices could reach before T is 1/2, then once money increases, output will increase, but by less the more it is anticipated.