

W.M. GORMAN (1923-2003)

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1. Introduction

William Moore Gorman, known to all as Terence, died in Oxford on 12 January 2003. The greatest Irish economist since Edgeworth, he was, like Edgeworth, totally unknown to the general public, both in his native country and in Britain where he made his career. He was the purest of pure theorists, whose life was devoted to scholarship and teaching, and whose work of forbidding technical difficulty was incomprehensible to most of his contemporaries. Yet, paradoxically, he was always concerned with applied issues, and the tools and theorems he developed have had a lasting influence on empirical work.

2. The Life

Gorman was born in Kesh, County Fermanagh, on 17 June 1923. His father, a veterinary surgeon, having died when he was young, he was raised by his mother and spent part of his childhood in what was then Rhodesia. He liked to recount that it was his African nanny who rejected William as a not very Irish name and rechristened him Terence, by which he was thereafter universally known. Back in Ireland he attended Mount Temple College in Dublin and Foyle College in Derry before going up to Trinity College Dublin in 1941. He served as a Rating and then Petty Officer in the Royal Navy from 1943 to 1946, and then returned to Trinity where he graduated in Economics in 1948 and in Mathematics in 1949.¹

¹ Gorman always spoke fondly of the then Whately Professor in Trinity, George Duncan. In a late paper on the Le Chatelier Principle, which appeared in a *festschrift* for Ivor Pearce, Gorman wrote "I would like to praise George Duncan ... who introduced me to economics as an engine of thought, and who, in particular, taught me to expect the result that I will attempt to prove, and that in one of the first lectures of the first term of my first year in Trinity College Dublin." Gorman continued, with his characteristic bluntness, that Duncan was "A man in many ways like Ivor [Pearce] who might have become just as distinguished had he known more mathematics. He could not make head nor tail of the accelerator: but taught us about what have come to be known as Arrow-Debreu goods in one of his first lectures." ([B.27], pp. 1 and 16.)

After Trinity, Gorman moved to Britain where he held a succession of posts at leading economics departments. From 1949 to 1962 he taught in the University of Birmingham, which was a leading centre for theoretical research in the 1950's, with Frank Hahn and Maurice McManus among his colleagues. In 1962 he was appointed to a chair in Economics at Oxford and in 1967 he moved to a chair at the London School of Economics, where he played a central role in the development of a taught master's programme in econometrics and mathematical economics, then a rarity outside the U.S. He returned to Oxford in 1979 as an Official Fellow of Nuffield College, becoming Senior Research Fellow in 1984 and Emeritus Fellow in 1990. He also spent periods as Visiting Professor at several U.S. universities, including Iowa, Johns Hopkins, North Carolina and Stanford. Meanwhile honours and awards were piling up, most notably Presidency of the Econometric Society in 1972, as well as Fellowship of the British Academy, membership of Academia Europaea, honorary foreign membership of the American Academy of Arts and Sciences and of the American Economic Association, and honorary doctorates from University College London and the Universities of Birmingham and Southampton. In Ireland too his achievements were recognised, with an honorary doctorate from the National University of Ireland in 1986 and an honorary fellowship from Trinity College Dublin some years later. After retirement, he continued to live in Oxford, also spending summers in County Cork with his wife Dorinda, whom he had met in Trinity, until in his last years illness impaired his mobility.

3. The Work

It was sometimes said that Gorman published relatively little, and it is true that many of his papers circulated for years in mimeo form, some of them to be rescued by the editors of his Collected Works ([A.1]). However, as the bibliography at the end of this paper shows,

even his published output was formidable, and would have satisfied the most demanding research assessment exercise.

Gorman's own summary of his principal contributions is worth quoting in full:²

"James Davidson at Foyle College, Derry, and George Duncan at Trinity College Dublin, taught me to think of mathematics and economics as styles of thought, not collections of theorems, and Birmingham taught me to think of the social sciences as a unity with history as one way of holding them together. My research has accordingly been devoted to the end of flexible modelling, that is, to allow economists to immerse themselves in their data and in the opinions of other social scientists, and then to choose forms which seem capable of handling this information. This has been even more true of my teaching, largely through workshops for students beginning research."

A reader unfamiliar with Gorman's works might interpret this as the manifesto of a woolly inter-disciplinarian. But the key phrase is "flexible modelling". Gorman was younger than Hicks, who in *Value and Capital* relegated his mathematics to appendices, and Samuelson, who in his *Foundations* proselytised for the value of a mathematical approach to economics. To Gorman, technical difficulty was taken for granted, though not as an end in itself. Most of his research pursued the goal of using whatever tools were appropriate (and frequently developing new ones) in order to throw light on a central issue in economic theory: the links between individual preferences and market behaviour. Here we comment on some of the main topics which he illuminated.

² See Blaug (1986), p. 328.

3.1 Aggregation

In his first published paper [B.1], Gorman provided the definitive answer to a key question in economics: when does a society of utility-maximising individuals behave as if it was a single individual? In other words, when does a community indifference map exist? He showed that a necessary and sufficient condition is that, assuming all individuals face the same prices, their income-consumption or Engel curves should be parallel straight lines. Thus for individual (or household) h , the Hicksian demand function for good i should take the following form:³

$$x_i^h(\mathbf{p}, u) = f_i^h(\mathbf{p}) + u^h g_i(\mathbf{p}) \quad (1)$$

The location of the h superscripts on the right-hand side is crucial. Individuals can differ greatly in their responses to price changes as far as the f_i^h functions are concerned. However, their differences must be independent of income (or utility): all individuals must have the same g_i function, so that at the margin they have identical responses to changes in u . Hence aggregate demands have the same form as (1):

$$X_i(\mathbf{p}, u) = F_i(\mathbf{p}) + U g_i(\mathbf{p}) \quad (2)$$

where X_i , F_i and U are the sums over all individuals of the corresponding micro terms.

In [B.12] Gorman returned to this question, now using the much more powerful tools of duality which he and others had developed in the interim. This short paper is bed-time reading by contrast with the 1953 paper, yet it contains what is probably his best-known contribution. Here Gorman derived an explicit expression for the form of preferences which give rise to linear Engel curves. He showed that individual h 's expenditure function must

³ This result was independently obtained by Antonelli (1886) and Nataf (1953). However, taken together, Gorman's two papers, [B.1] and [B.12], provide an explicit characterisation of the preferences which are consistent with exact aggregation.

take the simple form:

$$e^h(\mathbf{p}, u^h) = f^h(\mathbf{p}) + u^h g(\mathbf{p}) \quad (3)$$

where the functions $f^h(p)$ and $g(p)$ are homogeneous of degree one in prices (so ensuring that this property is exhibited by the expenditure function itself), and their derivatives equal the coefficients in (1). They have nice interpretations: $f^h(p)$ is the expenditure needed to reach a reference utility level of zero, while $g(p)$ is the price index which deflates the excess money income $e^h(p, u^h) - f^h(p)$ needed to attain a level of utility or real income u^h . Inverting (3) gives utility as a function of prices and expenditure:

$$v^h(\mathbf{p}, I^h) = \frac{I^h - f^h(\mathbf{p})}{g(\mathbf{p})} \quad (4)$$

which Gorman called "the polar form of the underlying utility function". With this unconventional term Gorman was drawing attention to the fact that using what we would now call the indirect utility function amounts to switching from Cartesian to a form of polar coordinates in describing the indifference surface. Specifically, expenditure I may be taken as analogous to the radius and the vector of prices p to the angle in solid geometry. In any case, the term "Gorman polar form" has come to be universally applied to the functional form in (4).⁴

By construction the Gorman polar form plays a central role in consumer theory, and it has also been hugely important in empirical work. On the one hand, special cases with particular functional forms for $f^h(p)$ and $g(p)$ proved amenable to estimation, even before the advent of high-speed computers. Gorman himself showed that, if the marginal propensities to consume (which equal $p_i g_i / g$) are constant, then the function $g(p)$ can be written as a

⁴ Blackorby, Boyce and Russell (1978) appear to have been the first to refer to it as such.

geometric mean of prices:

$$g(p) = \prod p_i^{\rho_i}, \quad \rho_i = p_i g_i / g, \quad \sum \rho_i = 1 \quad (5)$$

The linear expenditure system, developed by R.C. Geary among others, is a further special case, corresponding to the combination of (5) with a linear form for $f(p)$.⁵ On the other hand, Gorman's results did not prove a barrier to extending the theory to more general demand systems which avoid the implausible restrictions on income effects of (3). Muellbauer (1975) showed that a richer family of demand systems could be generated if the traditional requirement, used by Gorman, that aggregate demands behave like the sum of individual demands, was replaced by the weaker requirement that they generate only the same budget *shares*. This in turn has spawned a huge empirical literature applying members of Muellbauer's family and its extensions, such as the "Almost Ideal" demand system of Deaton and Muellbauer (1980).

Gorman also explored in [B.21] the conditions that must be satisfied for the existence of an aggregate stock of a fixed factor such as capital. The necessary and sufficient condition turns out to be formally very similar to that for aggregation of demands over individual consumers. Each firm must have a restricted profit function similar in form to (3), where utility is replaced by a function of the amount of capital used by the firm. In his own words, this result "certainly does not help justify the practice of fitting aggregate production functions." This contribution of Gorman's to the capital theory controversies of the 1960s lacked the fireworks of those that emanated from the two Cambridges (England and Massachusetts), but it is probably of more lasting importance.

⁵ See Neary (1997) for further discussion and references.

3.2 Separability

“Suppose you were interested in the demand for tomatoes in Ireland.” Thus begins Gorman's 1987 article on separability in the *Palgrave Dictionary of Economics* ([B.30]), recalling his own early applied work, characterized by his widow Dorinda as involving “careering around Dublin on a bike, looking in greengrocers' windows” (private communication). For him, separability assumptions were what allowed the researcher to abstract from the mass of institutional detail accumulated on such trips: detail that could conceivably be relevant, but was certainly going to make analysis impossibly complex. “Separability”, he wrote, “is about the structure we are to impose on our model: what to investigate in detail, what can be sketched in with broad strokes without violence to the facts.”

As for the researcher, so also for the household or the enterprise. Practical decision-making often calls for short-cuts relative to full intertemporal optimization of a preference function. Gorman was confident that in reality most households engage in two-stage budgeting, in which the family budget is first allocated between broad classes of spending (clothing, food, etc.) and then choices are made within each class.

But just how good is this as a way of making decisions? Each of the two stages is problematic. Can the first-stage allocation safely be made just on the basis of some price aggregates for each class of goods, and without looking at the relative prices of all goods? Even if the first-stage allocations are correct, can the choice of goods within each class safely be made without reference to the prices on offer or quantities chosen of goods in other classes? It turns out that the validity of such a procedure for achieving the optimum requires that the household's utility function satisfy some fairly drastic separability restrictions – more stringent than had been recognized in the literature.

In particular, Strotz (1957) had argued that a sufficient condition for two-stage budgeting is that the household's utility function be separable, i.e. expressible in the form:

$$u = F[v_1(x_1), v_2(x_2), \dots, v_n(x_n)] \quad (6)$$

where x_r denotes the vector of consumptions in class r . Gorman showed that, while necessary, separability is not sufficient.⁶ In addition, it is required that the sub-utility functions, which Gorman called "specific satisfaction functions", v_r , enter utility either additively or through an intermediate function which is homogeneous of degree one in its components.⁷

That these constraints were severe was for Gorman "in a sense a good thing"; since (he knew) households did adopt two-stage budgeting, it must be that their preferences were so restricted. Knowledge of this fact would ease the task of applied researchers wishing to estimate the relevant parameters.

What motivated Gorman here was the tension between two goals of economic modelling. On the one hand, the conceptual need for a coherent and psychologically or organizationally credible theoretical representation of decision-making; on the other hand, the operational need to have a workable algebraic representation of this behaviour. The basic assumptions of utility theory are too weak to yield specific functional forms or to make many

⁶ See [B.9]. Gorman had refereed Strotz's paper but (according to the account he gave to Blackorby and Shorrocks, [A.1], p. 31) his report, handwritten and covered with strawberry jam, was disregarded by the editor, Robert Solow!

⁷ More precisely, the condition is that the utility function must be expressible in one of the following three forms, where f is homogeneous of degree one:

- (i) $u = F(v_1, v_2)$ (the case where there are only two classes)
- (ii) $u = F[v_1, f(v_2, \dots, v_n)]$ (all but one class can be grouped into a homogeneous function)
- (iii) $u = F[v_1 + \dots + v_d + f(v_{d+1}, \dots, v_n)]$ (classes 1 to d and a homogeneous function grouping the remaining classes all enter additively)

predictions about individual or aggregate behaviour. Further assumptions are needed if real progress is to be made in applied economics, but these assumptions must be more-or-less reasonable. Looking from the other side, it is evident that simple algebraic representations of behaviour are needed for applied econometrics. Simplicity is also needed if the theory is to be mathematically manipulated to yield further predictions. But all such uses are empty if the algebraic specification implies incoherent decision-making. In practice, most of the algebraic representations with which demand and production theory deal are linear functions of prices or quantities, or are simple transformations of linear functions. Here questions of separability become central.

An interesting example of how specific separability assumptions could help in underpinning a linear representation of behaviour is provided by Gorman's 1980 paper "Facing an Uncertain Future" ([C.7]). In this paper, Gorman's goal is to show that the assumptions required to justify a linear representation of the intertemporal objective function are much weaker and more credible than had hitherto been recognized in the literature.

For a static environment, Allais, Samuelson, Von Neumann and Morgenstern and others had presented the conditions under which decision-making under conditions of uncertainty could be represented as the maximization of a linear function – a weighted average – of the various alternative possibilities.⁸ The key assumption in this expected utility hypothesis, Samuelson's weak independence axiom (or "sure thing principle"), is one of separability.

If we widen the focus to intertemporal decisions (still under uncertainty), can we get as simple an objective function with equally weak assumptions? The objective function that

⁸ The weights are usually interpreted as subjective probabilities, an interpretation which Gorman found unhelpful: "Frequently they seem to me to obscure, rather than enlighten" ([C.7], p. 215).

is commonly – indeed almost universally used – is a double sum:

$$\sum_s \sum_t f^{st}(y_{st}) \tag{7}$$

where y_{st} is the vector of flows which occur in period t if state s occurs.⁹ Can we derive such a simple form from assumptions that are as mild and acceptable as those underlying expected utility? If we are prepared to assume an extended version of the sure thing principle, so that it applies over time as well as between uncertain states of the world, we will get this double summation form of the objective function. But Gorman points out that extending weak independence in this way is logically problematic.

Before doing so, he notes that such an extension to a second dimension is permissible in the case of a social welfare function under uncertainty, where households rather than time are the extra dimension. Thus, if social welfare is increasing in every household's utility, if each household is “self-regarding”,¹⁰ and if Samuelson's weak independence axiom holds, then, drawing on a powerful theorem from an earlier paper of his on the structure of utility functions [B.20],¹¹ Gorman shows that the social welfare function can be expressed in the same double summation form as (7), except with y_{sh} as the consumption of household h in

⁹ We often make the further simplification $f^{st}(y_{st}) = \delta_s r_t y_{st}$, where δ is a probability and r a discount factor, but Gorman does not force such an interpretation.

¹⁰ i.e. considers only its own consumption; this is what gives separability or independence in the additional dimension.

¹¹ The theorem states that if two separable sets overlap in their membership, then their intersection and differences are also separable. In the social welfare case the overlapping arises because the separable (self-regarding) individuals are all involved in states of the world to which Samuelson's weak independence axiom applies; in the intertemporal choice case the separability is induced by the partial myopia mentioned below.

state of the world s instead of y_{st} . These simple and acceptable¹² assumptions are thus all that is needed to produce “Bentham and Bernoulli at a stroke”.

But to assume that households or firms are not only able to calculate their utility over all possible future states of the world but assert independence of each set of states of the world and time periods is a step too far for Gorman. Such an argument “assumes from the outset that we are all very bright, and especially so at computation”. Instead, he proposes the contrary idea, that “we are all pretty limited beings, only able to hold a few things in our minds at a time ... and that organizations are collectively quite as limited as their members”. Specifically, he assumes that “we look ahead two periods in detail, summarizing the impact of our choices on more distant prospects in a single figure”. He then proceeds to show that this, partially myopic but more realistic, vision of decision-making, embodying a very weak (undemanding) form of intertemporal separability, is enough to generate the double summation form of the objective function.

Here, Gorman has armed applied econometricians with a justification for doing what they had always intended — use a linear functional form. The behavioral assumptions are somewhat restrictive, but also characteristically down-home: the firm is planning for now and next year, and for a general sense of what it will bequeath later years. If that is not how firms and households behave exactly, yet it seems not too unrealistic.

3.3 Characteristics

Separability may be justified between goods that satisfy widely different needs; but for other goods it is their close similarities that attract attention. It is not on the basis of their

¹² He was of course fully aware of the continuing controversy over the weak independence axiom for choice under uncertainty and the fact that it has been rejected by many empirical experiments.

essence that a consumer will choose between goods, but on the basis of the satisfaction they will produce. Even for closely related goods, this in turn may depend on more than one characteristic.

Switching from tomatoes to eggs in deference to his original audience for the topic – an agricultural economics seminar in the Iowa State College of Agriculture and the Mechanic Arts in 1956 – Gorman entitled his first paper on the topic of characteristics “A Possible Procedure for Analysing Quality Differentials in the Egg Market”. (The paper finally appeared in 1980: see [B.25].) Ever concerned with the interests of the applied economist, he saw the paper as a response to the need of Iowan farmers to understand what drove price differentials for eggs of different qualities.

The basic idea is simple: consumers buy different varieties of eggs solely for certain measurable characteristics (for example, he suggests, their vitamin content). If only two characteristics are relevant, then, given arbitrary prices, we may expect that at most two varieties of eggs will be bought by any given consumer; if three characteristics are relevant, at most three varieties.¹³ Only in the “degenerate case” where the relative prices happened to be just right would the consumer be indifferent between three or more varieties. But – and here is where things get interesting – as soon as we consider market equilibrium, the prices will not be arbitrary: the degenerate case will prove to be the normal one, as it is “*the only case in which every type of egg could find a sale*” (emphasis in the original). This degenerate case can be characterized by a shadow price q_j of characteristic j such that, if purchased variety i delivered quantity a_{ij} of characteristic j , the price p_{it} of each variety i at each time t should always equal the value of the sum of its characteristics measured at the shadow

¹³ Determined by the tangent of the consumer's indifference curve with the convex hull of the affordable combinations of the two characteristics.

prices:

$$p_{it} = \sum_j a_{ij} q_{jt} \quad (8)$$

Building on this insight, essentially an argument from the assumption that market prices should not embody arbitrage opportunities, Gorman proposed an empirical research agenda. The specific quantity of each characteristic delivered by each variety, though measurable for the consumer, is unknown to the researcher, as are the shadow prices. But a sufficiently long time series on prices of different varieties could allow both to be identified, even if the prices were also somewhat influenced by other, less important, elements. If the number of varieties is I and the number of characteristics J , then price data for T time periods yields IT data points to estimate $I+J$ parameters. Statistical techniques such as factor analysis are available for such analysis. Gorman sensed that many of these ideas were already known,¹⁴ but the arbitrage argument seems to be original to him.

For all his warmth towards the challenges faced by applied econometricians, Gorman had little real interest in pursuing applied empirical work. His attempts to operationalize the characteristics model on an ambitiously large scale using quarterly regional data on the consumption of over a hundred categories of food for 1956-71 proved somewhat inconclusive. (See [B.23]).

Yet the characteristics model has assumed an empirical life of its own: far from egg or tomato markets, this insight now underpins the most widely used asset-pricing models in

¹⁴ The idea of hedonic indexes can indeed be traced back to Waugh (1928). Griliches (1961) advocated their use for the US CPI, and this suggestion was acted on from 1984. A recognition of more rapid and systematic quality change has led to increasingly widespread use of such price indices.

modern finance theory.¹⁵ After all, most financial assets are closely substitutable, and investors' choices between them are largely driven by their potential to deliver a relatively small number of yield characteristics. Whereas Markowitz (1959) asserted that investors were seeking to balance portfolio risk and return, measured by mean and variance, modern theories allow the goals of investors to be unmeasured characteristics of the stream of future returns. Market-clearing prices of the various assets must, in these theories, be adapted to the shadow prices of these characteristics in the market just as Gorman saw. Thus, such price processes are estimated by factor-analysis type methods (Campbell et al., 1997). Even the famous option pricing model of Black and Scholes (1972) and Merton (1973) appeals to precisely the same arbitrage logic so lucidly presented by Gorman more than fifteen years earlier.

3.4 Duality

Over and above his substantive contributions, a recurring theme in Gorman's writings was the need to select the appropriate technical tools for the problem at hand. Typically, this meant using "dual" tools, functions defined over prices rather than quantities. Because households and firms typically take prices as given, it is much easier to understand their behaviour in terms of expenditure, cost and profit functions than in terms of the primal utility and production functions. The latter only take account of tastes and technology, the former add optimising behaviour. Gorman was not alone in advocating this approach, but he was one of its most ardent proponents. The great virtue of duality is that it avoids matrix inversion, which he called "the only technically difficult operation in general equilibrium theory." Even a cursory comparison between modern textbooks and Hicks's *Value and*

¹⁵ It has also been applied in a great variety of other fields, notably through the later work of Lancaster (1966).

Capital or Samuelson's *Foundations* shows how much more powerful are dual methods.

A nice example of the value of the dual approach was Gorman's contribution to the issue of household equivalence scales. Such scales, which attempt to correct consumption patterns for differences in household composition, had been used for years in applied budget studies, though without any theoretical foundation. Barten (1964) pioneered the exploration of such scales in the context of utility theory. But Barten used the primal approach, expressing utility as a function of consumption per "equivalent adult", where the scale which determines equivalence varies between commodities. Gorman in [B.23], argued that the insight of an "otherwise obtuse" schoolmaster he once had put it better: "When you have a wife and baby, a penny bun costs threepence." Leaving aside the banality (and, to a modern ear, the sexism) of the aphorism, Gorman noted that it gets to the heart of the issue: differences in household composition are better thought of as altering the effective prices which must be paid, rather than the effective number of consumers. This approach, implemented using the expenditure function, led to a substantial simplification and extension of Barten's results.¹⁶

Expenditure and profit functions are usually the appropriate tools. However, in some problems quantities may be the exogenous variables. In such cases the appropriate technical tool is the distance function, defined implicitly as the scalar by which an arbitrary consumption bundle must be deflated to yield a target level of utility: $u[q/d(q,u^0)] = u^0$. This can be viewed as the natural inverse of the direct utility function. But it also turns out to bear a dual relationship to the expenditure function. Just as (by Shephard's Lemma) the price derivatives of the expenditure function equal the optimal quantities, so the quantity derivatives of the distance function equal the optimal shadow prices. Gorman developed this concept in

¹⁶ Muellbauer (1974) independently rederived Barten's results using the dual approach.

full, independently of others. In [C.3] he gave what appears to be the first statement of the duality between cost and distance functions, while in [C.5] he examined the properties of the distance function in detail. These papers however remained unpublished, so modern treatments typically give precedence to Debreu (1951) and Malmquist (1953) and pass over Gorman's pioneering explorations.

Gorman's emphasis in all this was on the need for careful thought about which theoretical tools were appropriate for a particular problem. As he wrote in notes for a 1986 seminar in University College Dublin, doing economic theory "is like eating an apple pie. If you know there is one in the fridge, and where the light switches are, there is nothing to it. Look around when you next visit a strange house, in case you should feel hungry in the night."

4. Conclusion

Gorman wrote in his first paper [B.1]: "In writing this article I have been torn between a desire for rigour and a desire for simplicity, and each has had to be sacrificed in part to the other." Even to today's technically trained economists, his writings seem characterised more by rigour than by simplicity. Yet his legacy, carried on in part by generations of students to whom he devoted so much of his time and attention, is a set of results and of tools which make it immeasurably easier for future economists "to tailor models for particular problems and particular data".

Publications of W.M. Gorman

A. Books

1. *Separability and Aggregation: Collected Works of W.M. Gorman, Volume 1*, edited by C. Blackorby and A.F. Shorrocks, Oxford: Clarendon Press, 1995.

B. Articles¹⁷

- 1.* "Community preference fields," *Econometrica*, 21, June 1953, 63-80.
2. "The intransitivity of certain criteria used in welfare economics," *Oxford Economic Papers*, February 1955, 23-55.
3. "Convex indifference curves and diminishing marginal utility," *Journal of Political Economy*, 65, February 1957, 40-50.
4. "How surprising is a chain of coincidences?," *Metroeconomica*, 9, April 1957, 112-115.
5. "A note on 'A revised theory of expectations'," *Economic Journal*, 67, September 1957, 549-551.
6. "Some comments on Professor Hicks' *Revision of Demand Theory*," *Metroeconomica*, 9, December 1957, 167-180.
7. "Tariffs, retaliation and the elasticity of demand for imports," *Review of Economic Studies*, 25, June 1958, 133-162; reprinted in M.J. Farrell (ed.): *Readings in Welfare Economics*, London: Macmillan, 1973.
8. "The effect of tariffs on the level and terms of trade," *Journal of Political Economy*, 67, June 1959, 246-265.
- 9.* "Separable utility and aggregation," *Econometrica*, 27, July 1959, 469-481; reprinted in K.J. Arrow (ed.): *Selected Readings in Economic Theory from Econometrica*, Cambridge, Mass.: MIT Press, 1971.
10. "Are social indifference curves convex?," *Quarterly Journal of Economics*, 73, August 1959, 485-496.
11. "Tariffs and trade in a two good world," *International Economic Review*, September 1960.

¹⁷ Articles denoted with an asterisk are included in A.1, *Separability and Aggregation: Collected Works of W.M. Gorman, Volume 1*.

- 12.* "On a class of preference fields," *Metroeconomica*, 13, August 1961, 53-56.
13. "Elasticity of demand for imports with close but imperfect substitutes," *International Economic Review*, September 1961.
14. "Additive logarithmic preferences, a further note," *Review of Economic Studies*, 30, February 1963.
15. "More scope for qualitative economics," *Review of Economic Studies*, 31, January 1964.
- 16.* "Professor Friedman's consumption function and the theory of choice," *Econometrica*, 32, January 1964, 189-197.
17. "Production functions in which the elasticities of substitution stand in fixed proportion to each other," *Review of Economic Studies*, 32, July 1967, 217-224.
18. "Tastes, habits and choices," *International Economic Review*, 8, July 1967, 218-222.
- 19.* "Conditions for additive separability," *Econometrica*, 36, 1968, 605-609.
- 20.* "The structure of utility functions," *Review of Economic Studies*, 35, October 1968, 367-390.
- 21.* "Measuring the quantities of fixed factors," in J.N. Wolfe (ed.): *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh: Edinburgh University Press, 1968, 141-172.
22. "Preference, revealed preference and indifference," in J.S. Chipman, L. Hurwicz, M.K. Richter and H.F. Sonnenschein (eds.): *Preferences, Utility and Demand: A Minnesota Symposium*, New York: Harcourt Brace Jovanovich, 1971.
23. "Tricks with utility functions," in M.J. Artis and A.R. Nobay (eds.): *Essays in Economic Analysis*, Cambridge: Cambridge University Press, 1976, 211-243 (invited address to the 1975 Association of University Teachers in Economics Conference).
24. (J.R. Boyle, W.M. Gorman, and S.E. Pudney) "The demand for related goods: A progress report," in M.D. Intriligator (ed.): *Frontiers of Quantitative Economics, Volume IIIA*, Amsterdam: North-Holland, 1976, 87-101 (invited paper at the Third World Congress of the Econometric Society).
25. "The demand for related goods: A possible procedure for analysing quality differentials in the egg market," *Review of Economic Studies*, 47, June 1980, 843-856; first circulated as Journal Paper No. 2319, Iowa Agricultural Experiment Station, November 1956.
- 26.* "Some Engel curves," in A. Deaton (ed.): *Essays on the Theory and Measurement of*

Demand in Honour of Sir Richard Stone, Cambridge: Cambridge University Press, 1981.

27. "Le Chatelier and general equilibrium," in A. Ingham and A. Ulph (eds.): *Demand, Equilibrium and Trade: Essays in Honour of Ivor F. Pearce*, London: Macmillan, 1984, 1-18.
- 28.* "Assembling efficient organizations," in W.P. Heller, R.M. Starr and D.A. Starrett (eds.): *Uncertainty, Information and Communication: Essays in Honor of Kenneth J. Arrow, Volume III*, Cambridge: Cambridge University Press, 1986, 213-228.
29. (W.M. Gorman and G.D. Myles) "Characteristics," in J. Eatwell, M. Milgate and P. Newman (eds.): *The New Palgrave: A Dictionary of Economics*, Volume 1, London: Macmillan Press, 1987, 403-406.
- 30.* "Separability," in J. Eatwell, M. Milgate and P. Newman (eds.): *The New Palgrave: A Dictionary of Economics*, Volume 4, London: Macmillan Press, 1987, 305-311.
- 31.* "More measures for fixed factors," in G.D. Myles (ed.): *Measurement and Modelling in Economics*, Amsterdam: North Holland, 1990.

C. Unpublished Papers¹⁸

1. "The demand for fish: An application of factor analysis," paper read to the Amsterdam meeting of the Econometric Society; Discussion paper A.6, Faculty of Commerce and Social Science, Birmingham University, September 1959.
2. "A note on Hotelling's method of extracting the latent roots and vectors of a matrix," Discussion Paper A.12, Faculty of Commerce and Social Science, Birmingham University, September 1960.
- 3.* "Consumer budgets and price indices," unpublished typescript, 1965.
- 4.* "Conditions for generalised additive separability," Discussion Paper No. 9, Department of Economics, London School of Economics, September 1970.
- 5.* "Quasi separable preferences, costs and technologies," paper read at Harvard University, November 1970.
6. "Sketch for the demand for related goods," Presidential Address to the Econometric Society, 1972.

¹⁸ As explained in the text, much of Gorman's work was never published, though many previously unpublished papers were collected in [A.1]. This brief selection includes only those papers which he included in his curriculum vitae of 1985 or which are discussed in the text.

- 7.* "Facing an uncertain future," Technical Report No. 359, Institute for Mathematical Studies in the Social Sciences, Stanford University, January 1982.

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