A Study of Sale Rates and Prices in Impressionist and Contemporary Art Auctions

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Abstract

This paper presents a comprehensive study of sale rates and prices in impressionist and contemporary art auctions. Our analysis is based on a detailed dataset of impressionist and modern art auctions and contemporary art auctions. We use this data to estimate and test a "search" model of seller behavior and reserve prices, similar to that developed in the labor economics literature. In our model, we find that that the reserve price should be a constant proportion of the estimated price. Furthermore, the reserve price and therefore sales rate, should depend only upon the variance in log-prices and the seller's discount rate. We estimate that the secret reserve price is set on average between 70% and 80% of the low estimate.

1. Introduction

Among frequent auction-goers, it is a well-known fact that not all items that are put up for sale are sold. Sellers of individual items will set a secret reserve price, and if the bidding does not reach this level, the items will go unsold. An item that has not been sold may be put up for sale at a later auction, sold elsewhere, or taken off the market. Sale rates vary tremendously across time. For example, between 1982 and 1994, the sale rate for contemporary art varied between 52% and 91% in different auctions. Between 1980 and 1990, in different auctions of 58 selected impressionist and modern artists at Christie's and Sotheby's in New York and London, sale rates varied between 31% and 100%. Sale rates also vary systematically over different types of auctions. During 1995 and 1996, 96% of items put up for sale in auctions of arms and armour were sold, 89% of wine at auction was sold, and 71% of impressionist and modern art items were sold.

This paper addresses the question of why sale rates differ so dramatically over time and across departments by looking at the determinants of a seller's reserve price in a model of sequential search similar to those applied to the labor market (see Devine and Kiefer, 1991, for a review of empirical search models). We explore in detail the idea that the highest observed price in a particular auction may be thought of as a "job offer" which will be accepted only if it exceeds the reserve price, as suggested in Ashenfelter (1989).

We begin our study by looking in detail at sale rates, prices and price shocks. We can conclude that while prices have trended upwards with a peak in 1989-1990, sale rates have shown no discernible trend or consistent correlations with price levels. We then show a strong relationship between sale rates and price shocks, as measured by the difference between realized prices and auctioneer's estimates of prices. This relationship is consistent with a model of search, where sellers search for buyers of paintings at different auctions, setting their reserve price as a function of the expected price, similar to the way in which workers set their reservation wages when searching for jobs.

Extending the standard search model, we find that under plausible assumptions about the price distribution, the optimal policy is to set a reserve price that is a constant proportion of the current expected price. The constant proportion depends only on the variance of log prices and the seller's discount rate, and not on overall price levels. Using this model of reserve prices, we then show that sale rates are explained by price shocks and a constant, or "natural sale rate," that can vary across types of items that are auctioned. We estimate that in art auctions, the reserve price is generally set to be between about 70 and 80% of the lower estimate published by the auctioneer before the sale.

This paper proceeds as follows. In section 2 we describe the data and look in more detail at how prices and sale rates have behaved across time and across different items, and we look at the relationship of price shocks to sale rates. In sections 3 and 4 we develop and estimate an empirical model of model of search behavior in auction markets and determine a relationship between sales rates and unexpected price changes. In section 5 we perform various tests on the model, and in section 6 we conclude the analysis.

2. Sale Rates and Prices in Art Auctions

2.1. The Data

Our data consist of objects sold in contemporary art auctions held at Christie's auction house located on King Street in London and objects sold in impressionist and modern art auctions held at Christie's and Sotheby's auction houses in London and New York. We also have aggregate data on sale rates and average values of items sold in different departments at Christie's.

The dataset on impressionist and modern art auctions was constructed by Orley Ashenfelter and Andrew Richardson and includes sales of 58 selected impressionist and modern artists that took place at Christie's and Sotheby's auction houses between 1980 and 1990. The artists in this sample were selected because their art is well represented at auction. The auction prices were collected from public price lists, and the estimated prices and various observable characteristics were collected from pre-sale catalogues.

The dataset on contemporary art was constructed by Kathryn Graddy and includes all sales of contemporary art at Christie's auction house on King Street in London between 1982 and 1994. The data were gathered from the archives of Christie's auction house, and for each item, the observable characteristics were hand-copied from the pre-sale catalogues. The information on whether or not a lot is sold and the final bid from 1988 onwards was taken primarily from Christie's internal property system. Before 1988, many of the lots were missing from the internal system. An assistant in the archives department said that after a certain period of time, some of the lots are deleted from the system, for no predictable reason. From December 1982 through December 1987, we had access to the auctioneer's books and were able to track the missing items in that manner.

The impressionist and modern art dataset includes over 150 auctions and 16,000 items for sale, and the contemporary art dataset includes 35 auctions and approximately 4500 items for sale. In the pre-sale catalogues, the auction houses publish a low- and a high-price estimate for each work of art. The auction house does not publish, and indeed is very secretive about, the seller's reserve price for the work of art. The auction houses observe an unwritten rule of setting the secret reserve price at or below the low estimate.

In addition to sale prices and the low and high price estimates for these paintings, the observable characteristics we have on each item are title of painting, artist, size of painting, medium, and painting date. For the impressionist and modern dataset, we also know whether or not the painting was signed, monogrammed, or stamped. For the contemporary art dataset, we know whether or not the item was subject to VAT and we know the highest bids for items that were not sold. Having information on the highest bids of items that were not sold is a primary benefit in the contemporary art dataset. For a further description of the contemporary art and impressionist and modern art dataset, please see Beggs and Graddy (1997) and Ashenfelter and Graddy (2003).

The data on sale rates and average values were obtained directly from Christie's. We have sale rate, value of items sold divided by total estimated value, and associated department for all sales that took place between January of 1994 and September of 1996. In addition, we have average sold lot value by department, for 1995 and 1996.

2.2. Summary Statistics

Table 1 presents summary statistics on prices and sale rates for contemporary and impressionist auctions. A simple comparison of the datasets is interesting. The average price for impressionist and modern art is approximately nine times the average price for contemporary art, and the standard deviation in the average price for impressionist art is approximately fifteen times the standard deviation in the contemporary art sample. However, these differences vary tremendously throughout the sample, with contemporary art appreciating relative to impressionist art over the time of the sample. There does appear to be an upward trend in prices, though prices appear to have peaked in 1989 or 1990, with a price decline after that. It should be noted that the last years of each sample, 1991 for impressionist art and 1994 for contemporary art, are not fully representative of prices and sale rates in that year, as we do not have data for the full year. Different auctions throughout the year can systematically differ as to the ”quality” of art that is sold; for example, some auctions are considered major sales where the most expensive items are auctioned. Sale rates clearly vary over the time of the sample, but they do not appear to vary systematically. The average sale rate for impressionist and modern art is 71% over the period of the sample, and the average sale rate for contemporary art is 77%.

Table 1
Summary Statistics of Prices and Sale Rates
(1990 dollars)

	Contemporary Art					Impressionist and Modern Art			
year	Prices (Sold Items)	Prices (Unsold Items)	Sale Rates	No. of Items	No. of Auctions	Prices (Sold Items)	Sale Rates	No. of Items	No. of Auctions
1980						$170,459 ($344,708)	0.75 (0.43)	647	8
1981						$139,463 ($215,730)	0.69 (0.46)	895	10
1982	$5,943 ($9,733)	$3,279 ($4,528)	0.67 (0.47)	233	2	$90,566 ($179,029)	0.66 (0.48)	761	11
1983	$9,144 ($15,565)	$5,390 ($11,771)	0.80 (0.40)	171	2	$86,549 ($145,232)	0.72 (0.45)	815	7
1984	$7,659 ($18,369)	$4,311 ($6,990)	0.72 (0.45)	305	2	$105,294 ($290,878)	0.69 (0.46)	1565	12
1985	$13,381 ($20,896)	$7,036 ($11,624)	0.83 (0.38)	235	2	$107,543 ($222,307)	0.67 (0.47)	1866	18
1986	$16,658 ($24,994)	$6,916 ($11,843)	0.79 (0.41)	246	2	$135,838 ($326,088)	0.80 (0.40)	1719	16
1987	$28,295 ($50,924)	$16,130 ($20,159)	0.73 (0.45)	315	2	$245,680 ($558,538)	0.74 (0.44)	2239	16
1988	$43,986 ($69,315)	$25,784 ($36,745)	0.84 (0.37)	332	3	$294,544 ($738,154)	0.75 (0.43)	1727	15
1989	$81,852 ($128,499)	$48,701 ($86,435)	0.85 (0.36)	453	3	$751,239 ($2,331,040)	0.74 (0.44)	1856	15
1990	$57,746 ($105,841)	$90,199 ($263,407)	0.68 (0.47)	501	4	$839,212 ($3,856,147)	0.60 (0.49)	1889	19
1991	$37,314 ($58,139)	$33,180 ($53,972)	0.66 (0.48)	325	4	$205,748 ($45,318)	0.72 (0.45)	247	4
1992	$42,668 ($65,584)	$28,939 ($42,045)	0.85 (0.36)	418	4				
1993	$26,834 ($59,757)	$17,928 ($19,280)	0.76 (0.43)	550	3				
1994	$36,004 ($60,953)	$71,505 ($306,441)	0.82 (0.39)	215	2				
Overall	$36,476 ($74,736)	$32,686 ($129,183)	0.77 (0.42)	4299	35	$304,585 ($1,526,606)	0.71 (0.45)	16226	151

What can we infer from looking at the above data? First, one can conclude that art prices have trended upwards over time, with a peak in 1989-1990. Secondly, sale rates, while moving around, appear to have no noticeable trend. Finally, it is difficult to conclude anything about the correlation of sale rates to prices. The correlation of the yearly average of sales rates to yearly average of prices is about .26 for contemporary art, and about -.28 for impressionist and modern art. So, the data for impressionist art indicate that price levels are positively correlated to sale rates, but the data for contemporary art indicate that price levels are negatively correlated to sale rates.[1]

Table 2
Average Sale Rates by Department

Department	Average Sold Lot Value*	No. of Auctions in Sample	Sale Rate (% of Lots Sold)		% Sold by Value	
			Mean	Std. dev	Mean	Std. dev.
Impressionist	£129,125	8	71%	(0.11)	80%	(0.10)
Old Masters Drawings	£39,940	4	77%	(0.09)	89%	(0.08)
Contemporary	£36,830	7	79%	(0.04)	87%	(0.06)
British Pictures	£26,635	7	78%	(0.14)	83%	(0.17)
Old Master Pictures	£17,870	11	73%	(0.15)	82%	(0.15)
Continental Pictures	£16,130	7	72%	(0.11)	79%	(0.10)
Clocks	£9,735	4	88%	(0.03)	89%	(0.07)
Jewellery	£9,470	8	86%	(0.05)	89%	(0.04)
Furniture	£9,945	25	85%	(0.09)	92%	(0.06)
Silver	£8,495	10	87%	(0.11)	92%	(0.07)
Sculpture	£8,705	5	78%	(0.21)	81%	(0.20)
Modern British Pictures	£8,765	9	70%	(0.05)	81%	(0.05)
Victorian Pictures	£8,930	6	66%	(0.13)	75%	(0.11)
British Drawings & Watercolours	£6,280	14	72%	(0.14)	87%	(0.10)
Rugs & Carpets	£6,430	8	80%	(0.17)	85%	(0.14)
Topographical Pictures	£8,325	2	68%	(0.13)	81%	(0.00)
Islamic	£6,810	5	68%	(0.22)	82%	(0.12)
Cars	£6,680	6	71%	(0.16)	65%	(0.22)
Chinese Works of Art	£6,020	8	70%	(0.19)	79%	(0.16)
Books & Manuscripts	£4,745	15	81%	(0.12)	86%	(0.09)
Russian Works of Art	£4,985	4	64%	(0.14)	69%	(0.15)
Japanese	£3,625	5	72%	(0.04)	76%	(0.05)
Musical Instruments	£4,035	5	77%	(0.05)	76%	(0.16)
Watches	£3,030	6	71%	(0.09)	81%	(0.11)
Prints-Old Modern and Contemporary	£4,040	8	81%	(0.12)	92%	(0.09)
Miniatures	£3,305	2	82%	(0.05)	92%	(0.07)
Antiquities	£3,450	3	57%	(0.08)	66%	(0.13)
Porcelain and Glass	£2,650	14	76%	(0.12)	85%	(0.10)
Tribal Art	£2,370	3	67%	(0.08)	75%	(0.19)
Photographica	£2,120	3	61%	(0.27)	79%	(0.08)
Modern Guns	£3,065	5	93%	(0.06)	94%	(0.04)
Garden Statuary	£1,830	4	91%	(0.10)	91%	(0.11)
Arms & Armour	£2,145	4	96%	(0.03)	99%	(0.01)
Frames	£2,030	4	81%	(0.15)	85%	(0.14)
Stamps	£740	22	78%	(0.13)	82%	(0.12)
Wine	£635	37	89%	(0.09)	91%	(0.08)

*Average of 1995 and 1996 average sold lot values.

[1] As items are typically unique, or nearly so, in art auctions, a better way of looking at price movements throughout time is by construction of a repeat sales or hedonic price index. These indices have been constructed for the Contemporary Art and the Impressionist Art dataset and are reported in Ashenfelter and Graddy (2003). The indices confirm the general movements that are suggested by the average prices in the table.

Table 2 looks at the differences in average sold lot value and the differences in sale rates across departments at Christie's. Christie's measures sale rates in two ways, by the percentage of lots that were sold in an auction and also by the percentage sold by value. This measure divides the sum of the prices of sold items in an auction by the sum of the high bids of all items in the auction. Therefore, if 100% of the lots were sold, then the percentage sold by value would be 1. The correlation between average sold lot value and the percentage of lots that were sold in an auction is -.11, and the correlation between average sold lot value and % sold by value is -.03.

Another way to look at the data on art auctions is to graph the relationship of the percentage of paintings that go unsold in a particular auction to unexpected changes in price. Figure 1 relates to impressionist and modern art, where the price shock is calculated only across sold items, and figure 2 relates to contemporary art in which all items are used. The price shock ps_{it} for item i at time t is calculated as $ps_{it} \equiv \ln p_{it} - \ln \hat{p}_{it}$, where p_{it} is the highest bid for the item (the sale price if the item is sold) and $\hat{p}_{it}$ is the midpoint of the high and low price estimates as listed in the pre-sale catalogue. These individual price shocks are then averaged over an auction. In both figures, there is a strong inverse relationship between price shocks and the percentage of paintings that go unsold. (The correlation of price shocks to the buy-in rate for contemporary art is -.86. For impressionist art, the correlation is -.50.) The above graphs are suggestive of a Phillips curve. Mortensen (1970) used a labor market sequential search model to explain the Phillips curve. An unexpected positive wage shock raises employment because more workers receive wage offers above their reservation wage. We pursue this analogy with the labour market below.

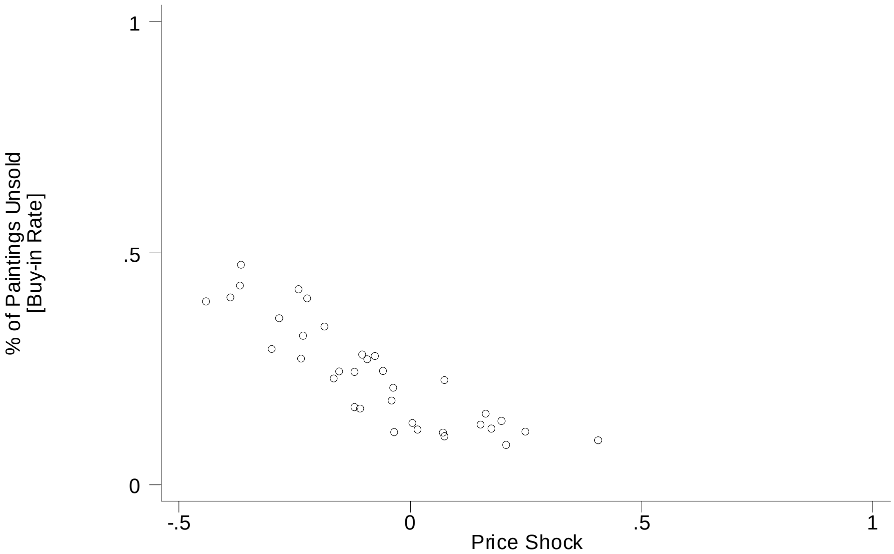

Figure 1: Buy-in Rates and Shocks for Contemporary Art

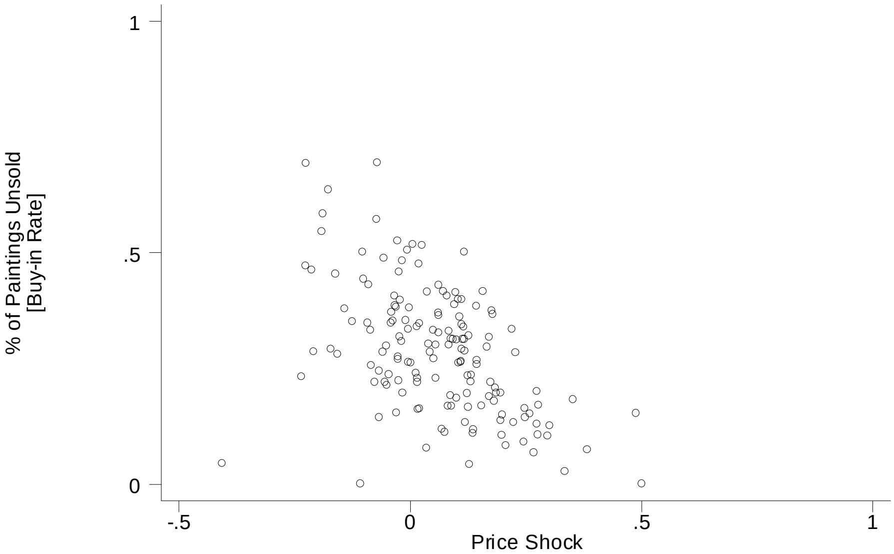

Figure 2: Buy-in Rates and Shocks for Impressionist Art

3. A Model of the Reserve Price

To determine the seller's reserve price, we follow an approach similar to the developed in the labor economics literature to model the behavior of a worker searching for a job, receiving wage offers sequentially from a known distribution of wages. As is well known (see for example, Lippman and McCall, 1976) the worker's optimal policy is to set a reservation wage, and to accept the first wage offer that exceeds it. The seller of a painting faces a problem similar to that of the unemployed worker: auctions occur sequentially; if he participates in an auction the highest bid for the painting can be regarded as a random draw from some price distribution; and when setting the reserve price he must decide at what price he would be indifferent between selling now and waiting for the next auction. However, for the application to art auctions it would not be appropriate to assume, as is usual for reservation wages, that the price distribution remains stationary[2]: as described in section 2, there were huge changes in price indices over a period of twelve years, which may have influenced the behavior of sellers. Instead, we will assume that the mean of log-prices may evolve over time, while the standard deviation of log-prices (the coefficient of variation) remains constant.

Suppose that the owner of a particular painting wishes to sell it. There is a sequence of auctions, one in each time period, and the best price offer for the painting at auction t is a random variable p_t, where:

$$\ln p_t = \mu_t + \varepsilon_t \qquad \text{and} \qquad \varepsilon_t \sim IID(0,\psi^2) \tag{1}$$

Here $\mu_t = E_t[\ln p_t]$, the expectation of the log-price immediately before the auction takes place, and ε_t is the price shock at auction t, assumed to be a white-noise process. While the painting remains unsold, the mean μ_t evolves according to a random walk:

$$\mu_{t+1} = \mu_t + \eta_t \tag{2}$$

where the random variables η_t are independent and identically distributed[3]. It is not necessary to assume that $E[\eta_t] = 0$: the mean may drift up or down, perhaps because of a general trend in art prices, or because the popularity of the artist is changing over

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[2] Lippman and McCall (1976) criticised attempts to explain the Phillips curve without incorporating a changing wage distribution. They extended the standard model to allow the distribution to evolve according to a Markov Chain. A reservation wage policy is still optimal, but the model does not provide clear predictions for the dynamics of unemployment.

[3] ε_t and η_t may not be independent: it would be plausible to assume that part of the price shock at auction t is permanent, affecting the mean at subsequent auctions.

time. A downward drift could also capture the possibility that an unsold painting is burned – that is, the future value of the painting falls if the painting is not sold because the market interprets the failure to sell as a negative signal of its value. Together, (1) and (2) imply that the expected price two auctions ahead is given by:

$$E_t[p_{t+1}] = \gamma E_t[p_t] \qquad \text{where} \quad \gamma = E_t[e^{\eta_t}] \tag{3}$$

Let $F_t(p)$ be the distribution function for the price of the painting at auction t, and δ be the seller's discount factor for the time between auctions. We assume that, having decided to sell the painting, the seller obtains no utility while it remains in his possession[4] – his payoff is simply the reserve price if and when it is sold. The optimal reserve price r_t for auction t is the price at which the seller is indifferent between selling now, and waiting until auction $t+1$ then using an optimal policy. Hence the sequence of optimal reserve prices satisfies:

$$r_t = \delta E_t\left[r_{t+1} F_{t+1}(r_{t+1}) + \int_{r_{t+1}} p\, dF_{t+1}(p) \right] \tag{4}$$

where the expression in square brackets is the payoff from using an optimal reserve price r_{t+1} at auction t+1. The following result is proved in the Appendix:

PROPOSITION 1: *Provided that* $\delta\gamma<1$, *the seller has an optimal reserve price* r_t, *which is a constant proportion* θ *of the expected price:*

$$r_t = \theta E_t[p_t] \tag{5}$$

where θ *increases with the discount factor* δ, *expected price growth* γ, *and the variance of the price shock,* ψ^2.

Thus the reserve at auction t is set equal to the seller's current expectation of the price, adjusted by a factor θ, which we will call the *reserve factor*. The

[4] We think that this is a reasonable assumption: due to transportation costs a painting that is not sold will often remain at the auction house for three months until the next auction. However our theoretical

comparative statics results are intuitive: a patient seller (higher δ) sets a higher reserve, and if prices are expected to be high at future auctions (high γ) there is more incentive to wait, so again the reserve should be set high. In fact if prices are expected to grow (γ>1) a patient seller will wait for ever; thus the condition $\delta\gamma<1$ is required for existence of an optimal reserve price. Finally, if prices are more variable (high ψ^2) the seller also benefits more from waiting, since a high price may be realized in future.

Since the reserve price moves in proportion to the mean, it follows that the probability of sale is the same at every auction:

PROPOSITION 2: *The painting is sold if and only if the price shock ε_t exceeds a constant threshold $\underline{\varepsilon}(\delta,\gamma,\psi^2)$, and the probability of sale decreases with δ, γ, and ψ^2.*

The proof is given in the Appendix. Note, in particular, that the probability of sale does not depend on the mean μ_t. The model is consistent with the evidence in section 2.3 that sale rates are related to unexpected price shocks, but not to price levels. In addition, the model predicts that paintings with a lot of price uncertainty have a lower sale probability[5].

Before we proceed to apply this model, it should be noted that in the case when the price distribution is stationary, our central result (5) appears to contradict the standard comparative statics result for labor market search (see Devine and Kiefer, 1991) that the derivative of the reservation wage with respect to the mean of the wage distribution lies strictly between 0 and 1. In contrast, θ may be greater than one. However, the standard result follows from the assumption that the *variance* of wages is held constant. Here, we are holding the *coefficient of variation* constant, as is

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results carry through if sellers do obtain some utility from an unsold painting, provided that this utility is proportional to the current expected price.

appropriate for our context: equation (5) implies that the ratio of the reserve price to the mean price is constant, under the assumption that for a painting with higher mean, the variability of prices is also higher, in proportion; it would not be sensible to assume the same variance in price *levels* for paintings whose expected prices differed by a factor of perhaps two or more.

4. An Empirical Application: Sale Rates and Unexpected Price Changes

4.1. Random Effects Probit

Now consider an art auction at which there are many sellers, each of whom sets a reserve price according to the model described above. Before an auction, the auction house publishes estimates of the value of each item for sale, but does not reveal the reserve price. By comparing the price estimate for item *i* at auction *t*, $\hat{p}_{it}$, with the realized price p_{it}, we can obtain a measure of the price shock ps_{it} for that item. Then, applying our theoretical model, we can use our observations of the price shocks to estimate the (secret) reserve factor θ used by sellers. We define:

$$ps_{it} \equiv \ln p_{it} - \ln \hat{p}_{it} \tag{6}$$

and let $y_{it} = 1$ if the item is sold, $y_{it} = 0$ otherwise. If we interpret $\hat{p}_{it}$ as the expected price $E_t[p_{it}]$, equation (5) for the reserve price implies:

$$y_{it} = 1 \iff ps_{it} > \ln \theta_{it} \tag{7}$$

where θ_{it} is the reserve factor of the seller of item *it*. We model the reserve factors for individual sellers as:

$$\ln \theta_{it} = \ln \overline{\theta} + u_t + \omega_{it} \tag{8}$$

where $\overline{\theta}$ is an "average" reserve factor for all sellers, $u_t \sim IN(0, \sigma_u^2)$ is an effect

[5] This result is similar to that of Balvers (1990), who found that the probability of sale decreases with a mean-preserving spread of the offer distribution.

common to the sellers at auction *t*, and $\omega_{it} \sim IN(0,\sigma_{\omega}^2)$ is an individual seller effect. Then (7) can be written:

$$y_{it} = 1 \iff ps_{it} > \ln\overline{\theta} + u_t + \omega_{it} \qquad (9)$$

Thus we have a Random Effects Probit specification, which we can use to estimate the average reserve factor $\overline{\theta}$ and the standard deviation σ_ω between sellers. In the special case of no auction-specific reserve factor effects (u_t=0) we have the standard Probit model for which:

$$\Pr[y_{it} = 1] = \Phi\left(\frac{ps_{it} - \ln\overline{\theta}}{\sigma_\omega}\right) \qquad (10)$$

where Φ is the standard normal distribution function.

4.2. Ordinary Least Squares

The Probit model can be used only when we observe the realized price p_{it} for both sold and unsold items, as we do for the Contemporary Art dataset. An alternative approach, applicable to both datasets, is to consider the relationship between the *sales rate* S_t and the *average price shock* ps_t at auction t:

$$S_t \equiv \frac{1}{n_t}\sum_i y_{it} \quad \text{and} \qquad ps_t \equiv \frac{1}{n_t}\sum_i ps_{it} \qquad (11)$$

(where n_t is the number of items for sale). We can write (9) as:

$$y_{it} = 1 \iff ps_t > \ln\overline{\theta} + u_t + \omega_{it} + v_{it} \qquad (12)$$

where $v_{it} = ps_t - ps_{it}$. Assuming that the item-specific deviations from the average price shock are normally-distributed: $v_{it} \sim IN(0,\sigma_v^2)$, the expected sale rate at auction *t*, given the average price-shock and the average reserve factor of the sellers at the auction, is given by:

$$E[S_t \mid ps_t, u_t] = \Pr[y_{it} = 1 \mid ps_t, u_t] = \Phi\left(\frac{ps_t - \ln\overline{\theta} - u_t}{\sigma_{\omega+v}}\right) \qquad (13)$$

This suggests estimating $\overline{\theta}$ in an OLS regression:

$$\Phi^{-1}(S_t) = \frac{1}{\sigma_{w+v}}(ps_t - \ln\overline{\theta}) + \pi_t \tag{14}$$

Here, the error term π_t picks up both auction-specific seller behavior u_t, and the error introduced by replacing the expectation of S_t by its actual value. If the variation of the price shock between items within an auction is relatively small compared with the variation in the average price shock between auctions, the loss of efficiency from estimating (14) rather than the Probit model will also be small. Moreover, this model can be used for the Impressionist dataset, for which we have prices only for sold items, if we replace ps_t by ps_t^{sold}, the average price shock for sold items only. However, since ps_t^{sold} is an upwardly-biased measure of ps_t, we would expect this to introduce an upward bias into the estimate of $\overline{\theta}$.

4.3. Estimation

Using the two approaches to estimation outlined above, we can derive estimates for σ and $\overline{\theta}$. Estimating $\overline{\theta}$ is of special interest, since it tells us how far below the estimate, on average, the ”secret” reserve is set. Knowing $\overline{\theta}$, or even a range for $\overline{\theta}$, could potentially be of use for bidders.

We first estimate the probit model, which, as explained above, can be done only for contemporary art. In column 1 and 3 of Table 3, the price shock $ps_{it} \equiv \ln p_{it} - \ln \hat{p}_{it}$ is calculated using the midpoint of the low and the high estimate for $\hat{p}_{it}$ (as in the graphs above). In columns 2 and 4, $\hat{p}_{it}$ is the low estimate. The probit models in columns 1 and 2 are special cases of the random-effects probit model, with the auction-specific contribution to the variance in the reserve factor set to zero, for which the contribution to the likelihood for item *it* is:

$$\Pr[y_{it} = 1] = \Phi\left(\frac{ps_{it} - \ln\overline{\theta}}{\sigma_\omega}\right)$$

A random-effects probit model is then estimated in columns 3 and 4. The coefficients are highly significant in both models, and the results for both the standard probit and the random-effects probit indicate that the reserve price is on average 71% of the expected price. The estimates of the standard deviation between sellers, σ_ω, are also very similar in the two models, and equal to .35 in the models using the low-estimates. In the random effects probit model, the variance of the auction specific shocks, σ_u, is estimated to be about .08. The estimate of ρ, the correlation between reserve prices of sellers in a particular auction, is .042. While statistically significant, this indicates that the auction-specific variance in seller reserve factors actually contributes very little to the total variance in reserve factors in the model.

Table 3
Sale Rates and Unexpected Price Changes

	Probit Contemporary Art (All Items)		RE Probit Contemporary Art (All Items)		OLS (Dependent Variable: Inverse norm of the sale rate) Contemporary Art (All Items)		Contemporary Art (Sold Items)		Impressionist and Modern Art (Sold Items)	
	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate
	1	2	3	4	5	6	7	8	9	10
$\ln P_t/E(P_t)$	2.64	2.85	2.67	2.89	1.68	1.68	1.75	1.74	1.57	1.51
	(0.08)	(0.09)	(0.09)	(0.09)	(0.16)	(0.16)	(0.23)	(0.24)	(0.22)	(0.22)
					[.15]	[.15]	[.15]	[.15]	[.37]	[.37]
constant	1.36	0.97	1.39	0.99	0.88	0.60	0.67	0.38	0.51	0.29
	(0.04)	(0.03)	(0.05)	(0.05)	(0.03)	(0.03)	(0.04)	(0.06)	(0.03)	(0.05)
					[.05]	[.06]	[.05]	[.06]	[.05]	[.10]
σ_ω and $\sigma_{\omega+v}$	0.38	0.35	0.37	0.35	0.59	0.60	0.57	0.57	0.64	0.66
θ	0.60	0.71	0.59	0.71	0.59	0.70	0.68	0.80	0.72	0.83
Log-Likelihood	-1535	-1485	-1518	-1470						
R^2					0.78	0.77	0.63	0.62	0.25	0.25
Obs.	4299	4299	4299	4299	35	35	35	35	151	151

Note: Standard erors in parentheses. Huber/White standard errors are in brackets. The estimated standard deviation of the auction effects in the random-effects probit model of columns 3 and 4 is .08. The estimate of rho, the proportion of the toal variance contributed by the auction-level variances, is .042. A likelihood ratio test of the significant of rho indicates that rho is highly significant.

We then estimate the ordinary least squares model

$$\Phi^{-1}(S_t) = \tfrac{1}{\sigma_{\omega+v}}\left(ps_t - \ln\overline{\theta}\right) + \pi_t$$

both for contemporary art and for impressionist art. For contemporary art, we can estimate the equation both using the entire sample, and using the sample of sold items. For impressionist and modern art, we can use only the sample of sold items, and thus our results will be biased upwards. Our estimation consists of an ordinary least squares regression of the inverse normal of the sale rate on the price shocks. In columns 5 and 6, ps_t is the average (for auction t) of the individual price shocks for all paintings (again, for items that were not sold, we use the high bid). In columns 7-10, ps_t is the average of the price shocks for sold paintings.[6]

From the results of all regressions, one can see that the coefficient on the unexpected change in price and the constant is highly significant in both the impressionist and modern art and the contemporary art datasets. Furthermore, the model appears to explain variations in the sales rates quite well. Using only sold items, for impressionist art the R-squared is around .25. For contemporary art, the R-squareds from the other regressions range from .62 to .78. When all items are used in the contemporary art dataset, the R-squared increases from .62 to around .77. Clearly, there is information contained in the prices of items that were not sold.

As would be expected, the OLS estimate of $\overline{\theta}$ when all items are used in the contemporary art data set is similar to the probit regressions (.60 vs. .59 for the average estimate, and .71 vs. .70 for the low estimates). Also as would be expected, the estimated standard deviation is much lower in the Probit estimates than in the OLS estimates. The standard deviation in the Probit estimates is σ_ω, representing the variation in reserve factors between sellers. The standard deviation given by the OLS

[6] For contemporary art, auction sessions (i.e. morning and afternoon sessions) that were held on the same day were grouped as one auction. Generally, auctions for contemporary art would be held three to six months apart. For impressionist and modern art, there are often auction sessions are consecutive days. We grouped sessions that were held on consecutive days into one auction. We felt this was reasonable as buyers often go long distances to attend an auction, and as pre-sale catalogues for all

estimates is $\sigma_{\omega+\upsilon}$, representing the combined variation of sellers' reserve factors and item-specific price shocks as described in section 4.2.

For impressionist art, we have estimated that on average the secret reserve price to be set at 83% of the low estimate. For contemporary art, using only sold items, we estimate the reserve to be 80% of the low estimate. As noted above, the estimates using sold items only are likely to be biased upward, and using all items for contemporary art we estimate the discount to be 70% of the low estimate. The relative estimates of $\bar{\theta}$ for impressionist art and contemporary art are consistent with the higher sale rate of contemporary y art, than impressionist art. The variation in prices appears to be lower for contemporary art. Our estimates indicate that the higher sale rate for contemporary art is driven both by a lower reserve price for contemporary art and a lower variance in the prices and reserves.

How reasonable are our estimates of $\bar{\theta}$? Reserve prices for art are clearly set below the low estimate. In contemporary art, out of a sample 3295 sold items, 1263 items sold at or below the low estimate. Of the items that sold at or below the low estimate, these items sold an average of 87% below the low estimate. In impressionist and modern art, out of 11544 sold items, 1898 items sold at or below the low estimate. In this sample the mean percentage below the low estimate was 88%. The only evidence we could find on any actual reserve prices is contained in a book by Peter Watson that documents the selling of *Portrait of Dr. Gatchet.* For this picture, the secret reserve was $35,000,000, 87.5% below the low estimate of $40,000,000.[7]

4.4 Implications for the Discount Factor

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auctions on consecutive days would be available before the start of the first auction day.

[7] In another context, McAfee, Quan, and Vincent (2000) construct a theoretical model and find that for real estate, the optimal reserve for buildings should be at least 75% of the appraised value, despite the Resolution Trust Corporation (RTC) and The Federal Deposit Insurance Corporation (FDIC) using

Using the estimated $\overline{\theta}$, we can use our theoretical model to recover $\delta\gamma$, which is the effective discount factor between auctions after allowing for expected price growth. In order to do this we assume that the price shocks for all items are identically normally distributed, and estimate ψ directly as the sample standard deviation of the price shocks ps_{it} for contemporary art. The estimate is $\hat{\psi} = 0.505$ for both measures of $\hat{p}_{it}$.

Then for $\overline{\theta} = 0.7$ the theoretical model (equations (A2) and (A3)) implies $\delta\gamma$= 0.66 (this is not at all sensitive to $\hat{\psi}$); the highest and lowest estimates of $\overline{\theta}$ in Table 5 imply $\delta\gamma$= 0.75 and $\delta\gamma$= 0.58, respectively. These estimates imply either that the seller greatly discounts the future, or that the seller expects a much lower price for his painting, if the painting does not sell in the current auction, i.e. he believes his painting is "burned." For example, if a seller's discount factor between auctions is .95, then a reserve factor ($\overline{\theta}$) of .7 implies that he expects the future value of the painting to only be about 80% of its current value if it does not sell in the upcoming auction. Ashenfelter and Graddy (2003) find evidence that paintings are "burned" and this is widely believed in the art trade.

5. Testing the Model

Our first test of the model concerns whether the probability of sale is actually lower with a higher price variance, and unrelated to the level of prices, as predicted by the theoretical model. We test this proposition in two ways: firstly, by looking at the spread between the high and the low price estimates for paintings and secondly, by looking at the way in which the percentage sold by value varies with the sale rate across items, using the data in Table 2.

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reserve prices of between 50-70%.

If the auctioneer's estimates were purely exogenous, one might suppose that the spread between the high estimate and the low estimate is related to the auctioneer's estimate of the variance for each individual painting. One would suspect that a dealer would provide a wider range of high-low estimates when he is uncertain about what price he may fetch at auction. Suppose that the high estimate is interpreted as the mean plus a multiple of the standard deviation ($H = \mu + r\sigma$), and the low estimate as the mean minus a multiple of the standard deviation ($L = \mu - r\sigma$). Then, the high estimate minus the low estimate divided by 2 is proportional to the standard deviation ($\frac{H-L}{2} = r\sigma$) and the average of the high estimate and the low estimate would just be the mean ($\frac{H+L}{2} = \mu$). A large difference between the high estimate and the low estimate would therefore signal a high price variance and a low probability of sale, and the mean should not effect the probability of sale. This reasoning, however, relies on the spread being a good estimate of the auctioneer's opinion of variance in price. In actuality, the spread is set in conjunction with the seller's idea of his reserve price (which in theory should depend on his estimation of the variance along with his effective discount rate). Because of the convention that the reserve price is always set at or below the low estimate, if the seller has a low discount rate and therefore a high reserve price, one of two things could happen. Either the auctioneer convinces the seller to lower his reserve price, or the auctioneer increases the lower bound on his estimates. If the latter were to happen frequently enough, then a small difference in the high estimate and the low estimate would translate into a low probability of sale and a higher mean would translate into a lower probability of sale. Therefore correlating the difference between the high estimate and the low estimate does not present a clear test of our theory. Nonetheless, we regress the sale rates on the mean of the high and low estimates and the difference between the high and the low estimates (divided by 2), where the high and the low estimates are expressed in logs. The results are presented in Table 4 below, and strongly support the interpretation of the low estimate being a reflection of the seller's reserve price, rather than an exogenous reflection of the auctioneer's estimate of the variance. Hence, our model fails this first test, but we believe there is a reasonable explanation for this failure.

Table 4
Sale Rates and Estimate Spreads
Sold= 1 (Probit)

	Contemporary Art	Impressionist and Modern Art
(ln(high est)-ln(low est))/2	2.070 (0.485)	0.643 (0.192)
ln(average estimate)	-0.009 (0.013)	-0.017 (0.006)
constant	0.502 (0.151)	0.658 (0.077)
Psuedo R-squared	0.0045	0.0011
no. of observations	4299	16226

Our second measure of variance is inferred by comparing the percentage sold by value, *V*, reported in Table 2, with the sale rate *S*. Intuitively, when variability is high, the average price of sold items will be high compared with the average price (i.e. the highest bids) for all items in the auction. This means that the ratio *V*/*S* for a particular type of item depends positively on the price variance for that item (we show this formally in the appendix). Since, in addition, the sale rate depends negatively on the price variance, we should find that the elasticity of the percentage sold by value with respect to the sale rate is strictly less than 1. We test this prediction by regressing ln *V* on ln *S* using the data on different departments as presented in Table

2. We find a coefficient on ln *S* of .625 with a standard error of .083. These estimates confirm that the elasticity of the percentage sold by value with respect to the sale rate is strictly less than 1, and hence that the variation in sale rates across departments is consistent with the predictions of our model, with lower sale rates for some types of items explained by high price variability.

Our second test relates to the specification of the models in Table 3, which restricts the coefficient on the price and expected price variables to be identical. If the coefficients were not identical, that could indicate, for example, that certain types of art (grouped by value) had different sale rates. In the context of our model, it could also indicate that the reserve price is not proportional to the expected price. Table 5 presents regressions that test this restriction.

In all cases other than in columns 1 and 3, we cannot reject this restriction. While the probit estimates that use the average of the high and low estimates as the expected price do reject this restriction, it is not rejected when the low estimate is used as the expected price. It is interesting that the reserve price is often talked about as being proportional to the low estimate.

Table 5
Sale Rates and Unexpected Price Changes (Restriction Testing))

	Probit		RE Probit		OLS (Dependent Variable: Inverse norm of the sale rate)					
	Contemporary Art		Contemporary Art		Contemporary Art		Contemporary Art		Impressionist and Modern Art	
	(All Items)		(All Items)		(All Items)		(Sold Items)		(Sold Items)	
	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate
	1	2	3	4	5	6	7	8	9	10
ln P_t	2.65	2.85	2.69	2.90	1.71	1.72	1.81	1.81	1.58	1.53
	(0.08)	(0.09)	(0.09)	(0.09)	(0.16)	(0.16)	(0.24)	(0.25)	(0.22)	(0.22)
					{0.15}	{0.15}	{0.20}	{0.20}	{0.36}	{0.36}
ln $E(P_t)$	-2.68	-2.87	-2.74	-2.93	-1.69	-1.69	-1.78	-1.78	-1.55	-1.49
	(0.09)	(0.09)	(0.09)	(0.10)	(0.16)	(0.16)	(0.24)	(0.24)	(0.22)	(0.22)
					{0.15}	{0.15}	{0.19}	{0.19}	{0.37}	{0.25}
constant	1.66	1.14	1.82	1.27	0.69	0.28	0.38	0.06	0.21	-0.07
	(0.15)	(0.14)	(0.19)	(0.18)	(0.24)	(0.24)	(0.33)	(0.34)	(0.26)	(0.26)
					{0.20}	{0.21}	{0.31}	{0.33}	{0.25}	{0.10}
ln Pt=E(Pt)										
Chi2(1)	4.35	1.47	5.54	2.42						
F(1,32) or F(1,148)					0.61	0.88	0.78	0.92	1.37	1.99
					{0.84}	{1.18}	{0.85}	{0.96}	{1.47}	{2.14}
Log-Likelihood	-1533	-1484	-1516	-1468						
R^2					0.78	0.78	0.64	0.63	0.26	0.26
Obs.	4299	4299	4299	4299	35	35	35	35	151	151

Note: Standard erors in parentheses. Huber/White standard errors are in brackets. The estimated standard deviation of the auction effects in the random-effects probit model of columns 3 and 4 is .08. The estimate of rho, the proportion of the toal variance contributed by the auction-level variances, is .046. A likelihood ratio test of the significant of rho indicates that rho is highly significant.

Finally, we test for persistence in the buy-in rate that cannot be explained by price shocks. In order to test whether there is persistence in the buy-in rate that cannot be explained by price shocks (or the price level), we use the residuals from the regressions in columns 5-10 of Table 2 and regress these residuals on lagged residuals from these regressions. These results are presented in Table 6. For impressionist and modern art, we do find unexplained persistence. Although the coefficients on the lagged residuals are also positive in the contemporary art regressions, the coefficients are not significant. These results do suggest that there may be other variables influencing the sale rate that are not captured by the model.

Table 6
Unexplained Persistence in the Sale Rate
Dependent Variable: Residuals from Regressions in Table 1

	Impressionist and Modern Art (Sold Items)		Contemporary Art (Sold Items)		Contemporary Art (All Items)	
	Average Estimate	Low Estimate	Average Estimate	Low Estimate	Average Estimate	Low Estimate
	1	2	3	4	5	6
Lagged Residual	0.53	0.53	0.22	0.23	0.29	0.31
	(0.06)	(0.06)	(0.17)	(0.17)	(0.17)	(0.17)
constant	-0.01	-0.01	0.00	0.00	0.00	0.00
	(0.02)	(0.02)	(0.04)	(0.04)	(0.03)	(0.03)
R^2	0.32	0.32	0.05	0.05	0.09	0.09
Obs.	151	151	35	35	35	35

Note: Standard errors in parentheses.

One might also be concerned about correlation between the error terms and the price shocks. However, there is little reason to believe that the error terms in the above regression are correlated with the price shock. In almost all auctions, the reserve prices and the estimates are set at the same time. During extraordinary downturns, the auction houses may telephone the sellers after the pre-sale catalogues have been published but prior to the sale, in order to convince the sellers to revise their secret reserve prices. This is a rare event but may have occurred during the sharp downturn in 1990. This may induce slight correlation that could bias downward the estimate of the coefficient on the price shock. However, as this is likely to have occurred only rarely, it is unlikely to have had much effect on our estimates.

6. Conclusion

In this paper we have developed an empirical model of auction sale rates by using an approach similar to that in the labor economics literature. We find that auction sale-rates can be explained by price shocks and a constant, or a ”natural sale rate.” This natural sale rate depends upon discount rates of individuals and the variance of log prices. Using detailed data to estimate the model, we can recover the average amount below the low estimate that the secret reserve price is likely to be set in different types of auctions. We have estimated that the average amount below the low estimate that the secret reserve price is set is between 70% and 80% below the low estimate in auctions of Contemporary and Impressionist art.

While we have identified that sale rates depend upon the discount rates of individual sellers and the variance in log prices for the individual items, this is only the first step to an answer of the question why sale rates differ so dramatically across items. The next step in this research should be to ask what underlying characteristics of items sold in different departments lead them to have different variances or different discount rates which result in the widely different “natural sale rates” across departments.

Appendix

Notation: ε is a random variable with mean 0 and variance ψ^2.

$X \equiv \varepsilon / \psi$ has mean 0, variance 1, distribution function $G(x)$ and density $g(x)$.

$Z \equiv e^{\varepsilon}$ has distribution function $F(z)$ and density $f(z)$.

PROOF OF PROPOSITION 1: Rewriting (4): $r_t = \delta E_t\left[r_{t+1} + \int_{r_{t+1}} (1 - F_{t+1}(p))dp\right]$.

From (1), the price distribution $F_t(p) = F(z)$ where $z = e^{-\mu_t} p$.

Defining $q_t \equiv e^{-\mu_t} r_t$, and using (2), this equation can be written:

$$q_t = \delta E_t\left[e^{\eta_t}\left(q_{t+1} + \int_{q_{t+1}} (1 - F(z))dz\right)\right] \tag{A1}$$

Let $\underline{z}$ satisfy

$$\underline{z} = \delta\gamma\left(\underline{z} + \int_{\underline{z}} (1 - F(z))dz\right) \tag{A2}$$

(recall that $\gamma = E_t[e^{\eta_t}]$). Provided that $\delta\gamma$<1, this equation has a solution $\underline{z}$ >0. Then $q_t = \underline{z}$ is a solution of (A1) and it can be verified that it is the only non-explosive solution. Hence $r_t = e^{\mu_t}\underline{z}$ is the solution of (4). But $E_t[p_t] = e^{\mu_t} E[Z]$, so:

$$r_t = \theta E_t[p_t] \quad \text{where} \quad \theta = \underline{z} / E[Z] \tag{A3}$$

For the comparative statics results, first differentiate (A2) with respect to δ:

$$\frac{\partial \underline{z}}{\partial \delta}\left(1 - \gamma\delta F(\underline{z})\right) = \frac{\underline{z}}{\delta} > 0 \ . \ \text{Hence} \frac{\partial \theta}{\partial \delta} > 0 \text{ and similarly} \frac{\partial \theta}{\partial \gamma} > 0 .$$

To see how $\underline{z}$ depends on ψ, rewrite (A2) in terms of the distribution G :

$$\underline{z}\left(1 - \delta\gamma G(\underline{x})\right) = \delta\gamma \int_{\underline{x}} e^{\psi x} g(x)dx \qquad \text{where } \underline{x} = \tfrac{\ln \underline{z}}{\psi} \tag{A4}$$

Differentiating, the terms involving $\partial \underline{x} / \partial \psi$ cancel out, so we have:

$$\frac{\partial \underline{z}}{\partial \psi}\left(1 - \delta\gamma G(\underline{x})\right) = \delta\gamma \int_{\underline{x}} x e^{\psi x} g(x)dx \tag{A5}$$

Now $E[Z] = \int e^{\psi x} g(x)dx$, so $\dfrac{\partial \ln \theta}{\partial \psi} = \dfrac{1}{\underline{z}}\dfrac{\partial \underline{z}}{\partial \psi} - \dfrac{\int x e^{\psi x} g(x)dx}{\int e^{\psi x} g(x)dx}$ and using (A4) and (A5):

$$\frac{\partial \ln \theta}{\partial \psi} = \frac{\int_{\underline{x}} x e^{\psi x} g(x)dx}{\int_{\underline{x}} e^{\psi x} g(x)dx} - \frac{\int x e^{\psi x} g(x)dx}{\int e^{\psi x} g(x)dx} . \qquad \text{Define } h(x) \equiv \frac{e^{\psi x} g(x)}{\int e^{\psi x} g(x)dx} .$$

h has the required properties for a density function. So we can write the previous equation as: $\partial \ln\theta / \partial\psi = E[Y \mid Y \geq \underline{x}] - E[Y]$ for a random variable Y with density $h(.)$. This is positive, and the proof is complete.

PROOF OF PROPOSITION 2: From the proof of Proposition 1, the reserve price is $r_t = e^{\mu_t}\underline{z}$, where $\underline{z}$ is the solution of (A2). The painting is sold if and only if $\ln p_t > \ln r_t$, which (using (1)) is equivalent to $\varepsilon_t > \ln \underline{z}$.

Thus the threshold value of ε_t is $\underline{\varepsilon} \equiv \ln \underline{z}(\delta,\gamma,\psi^2)$.

The probability of sale is $P = 1\text{-}G(\ln \underline{z} / \psi)$, so $\partial P / \partial\delta < 0$ and $\partial P / \partial\gamma < 0$ follow immediately from the properties of $\underline{z}$ established previously.

Finally, $\frac{\partial P}{\partial\psi} \overset{\text{sgn}}{=} -\frac{\partial \underline{x}}{\partial\psi}$ where $\underline{x} = \frac{\ln \underline{z}}{\psi}$ as before. Again using the results from the previous proof, $\frac{\partial \underline{x}}{\partial\psi} = \frac{1}{\psi}\left(\frac{1}{\underline{z}}\frac{\partial \underline{z}}{\partial\psi} - \underline{x}\right) = \frac{1}{\psi}\left(E[Y \mid Y \geq \underline{x}] - \underline{x}\right) > 0$.

PROPOSITION 3: *If the distribution of log-prices for the n items for sale at an auction have different means μ_i, but the same variance ψ, and the sellers at the auction use the same reserve factor θ, then conditional on the sale rate S, the percentage sold by value V converges in probability (as n increases) to $v(\psi)S$, where v is an increasing function.*

PROOF: $$S = \tfrac{1}{n}\sum_{i=1}^{n} y_i \quad \text{and } V = \frac{\tfrac{1}{n}\sum y_i p_i}{\tfrac{1}{n}\sum p_i}$$

where $\ln p_i = \mu_i + \varepsilon_i$ and $\varepsilon_i \sim D(0,\psi^2)$, and $y_i = 1$ iff $\varepsilon_i \geq \underline{\varepsilon}$ (Proposition 2). All items are equally likely to be sold, so conditional on the sale rate S, $\Pr(y_i = 1) = S$.

Then, conditional on S:

$$\text{plim}\,\tfrac{1}{n}\sum y_i p_i = \tfrac{1}{n}\sum \exp(\mu_i) E(e^{\varepsilon_i} \mid y_i = 1) S$$

$$\text{and plim}\,\tfrac{1}{n}\sum p_i = \tfrac{1}{n}\sum \exp(\mu_i) E(e^{\varepsilon_i})$$

Hence $\text{plim}\, V = v(\psi) S$ where: $$v(\psi) \equiv \frac{E(e^{\varepsilon} \mid \varepsilon > \underline{\varepsilon})}{E(e^{\varepsilon})} = \frac{\int_{\underline{x}} e^{\psi x} g(x)\,dx}{\int_{\underline{x}} g(x)\,dx \int e^{\psi x} g(x)\,dx}$$

(where g is the density function of ε/ψ as before).

Differentiating with respect to ψ and using the result from the proof of Proposition 2 that $\underline{x}$ is increasing in ψ gives the required result.

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