Microeconomic Theory
Lecture 2: Consumer Theory

Marcel Fafchamps
Oxford University

1. Consumer Theory

Readings: Varian, Microeconomic Analysis, Chapters 7-9

1.1. Axioms

Consumers are assumed to have preferences over bundles of goods, and to be rational about their preferences. This rationality is summarized in a few axioms:

1. Preferences are complete: consumers can rank all bundles.
2. Preferences are reflexive: a bundle A is equivalent to the same bundle A.
3. Preferences are transitive: if a consumer prefers A to B and B to C, then he/she prefers A to C.
4. Preferences are continuous: no big jumps in preference orderings.

If preferences satisfy axioms 1 to 4, then they can be represented by a utility function $u(x)$ such that, if a bundle x_1 is preferred to a bundle x_2, then $u(x_1) > u(x_2)$.

The utility function is unusual in the sense that the value it takes is unimportant; all that matters is the utility value of a particular bundle *relative* to the utility value of another bundle. The difference in utility between the two is *not* a measure of the degree of unhappiness associated with less or more goods. The fact that the utility difference between bundles 1 and 2 is 23 and the utility difference between bundles 3 and 4 is 56 does NOT mean that the difference in happiness between the first two bundles is half the difference in happiness between the last two. This idea is sometimes expressed by saying that the utility function is an ordinal function, i.e., all that matters is the order of the bundles, nothing else.[1] The utility function is nothing but a ranking of bundles by order of preference. The utility is the rank number of the bundle.

An immediate corollary is that any monotonic transformation of the utility function does not affect the underlying preferences since the ranking of bundles is unaffected.

Other assumptions regarding the properties of the utility function or underlying preferences may be required to obtain nicely behaved consumption choices (see infra).

[1] This property – ordinality – is no longer sufficient when consumers are subject to uncertainty. In this case, additional assumptions must be made regarding the utility function.

1.2. Utility maximization

Consumers are assumed to choose the bundle they like most among the consumption bundles they can afford. This can be represented in mathematical form as:

$$\max u(x) \text{ subject to } px = m \tag{1.1}$$

where m represents the consumer's budget or *cash-in-hand.* The requirement that the bundles be affordable, i.e., $px = m$, is called the budget constraint. Maximization problem (1.1) is a constrained maximization problem.

To mathematically solve the consumer's optimization problem, form the Lagrangian $u(x) - \lambda(px - m)$ where λ is the Lagrange multiplier. Assuming function $u(x)$ is continuous, the optimum consumption bundle is found by solving the system of first order conditions:

$$\frac{\partial u(x)}{\partial x_i} - \lambda p_i = 0 \; \forall i \tag{1.2}$$

$$px - m = 0 \tag{1.3}$$

where the reader recognized the last first order condition as the budget constraint itself. A second order condition must also be satisfied. Given that utility maximization is a constrained maximization problem, the second order condition is slightly different. See Varian for details.

1.3. The indirect utility function

The solution to the utility optimization problem varies with p and m. All possible solutions can be written in the form of an *indirect utility function* $v(p, m)$. As with the profit function, the indirect utility function is a maximized function; it represents all the solutions to the utility maximization problem.

`Remarks:`

1. The indirect utility function is not function of consumption quantities. It is only function of consumption prices and cash-in-hand.
2. The indirect utility function should never be maximized since it is already the solution to a maximization problem.

`Properties of the indirect utility function:`

1. Nonincreasing in p (higher prices are bad); nondecreasing in m (higher cash-in-hand is good). Reason: obvious.
2. Homogeneous of degree 0 in prices and cash-in-hand. Reason: budget constraint is linear in prices and cash-in-hand.
3. Quasiconvex in p. Reason: result of optimization.
4. Continuous in p. Reason: utility is continuous, and budget constraint is linear.

Let $x_i(p, m)$ be the quantity of good i consumer if the price vector is p and the cash-in-hand is m. Function $x_i(p, m)$ is called the *Marshalian demand function.* By the so-called Roy's Identity, we have:

$$x_i(p, m) = -\frac{\frac{\partial v(p,m)}{\partial p_i}}{\frac{\partial v(p,m)}{\partial m}} \tag{1.4}$$

1.4. Expenditure function and compensated demand

Consider the following optimization problem:

$$\min px \text{ subject to } u(x) = u \tag{1.5}$$

The solutions to all the minimization problems of this type for all p and u can be summarized as a function $e(p, u)$. This function, called the expenditure function, gives the minimum cash-in-hand required for a consumer facing prices p to reach utility u. The consumption choices corresponding to the minimization of problem (1.5) are called Hicksian or *compensated demand functions* and are written $h(p, u)$. They are notional constructs, unlikely to be observed in practice. But, as we will see, they are useful theoretical tools. Hicksian demand functions are related to the expenditure function via a Roy-identity or Hotelling-lemma-type relationship:

$$h_i(p, u) = \frac{\partial e(p, u)}{\partial p_i} \tag{1.6}$$

Marshallian and Hicksian demand functions are related by the Slutsky equation:

$$\frac{\partial x_j(p, m)}{\partial p_i} = \frac{\partial h_j(p, u)}{\partial p_i} - \frac{\partial x_j(p, m)}{\partial m} x_i \tag{1.7}$$

1.5. Properties of demand functions

`Hicksian demand functions:`

Define the substitution matrix as constituted of elements $\frac{\partial h_j(p,u)}{\partial p_i}$.

1. The substitution matrix is symmetric. Reason:

$$\frac{\partial h_j(p, u)}{\partial p_i} = \frac{\partial^2 e(p, u)}{\partial p_i \partial p_j} = \frac{\partial h_i(p, u)}{\partial p_j} \tag{1.8}$$

   since the order of differentiation in $\frac{\partial^2 e(p,u)}{\partial p_i \partial p_j}$ does not matter and can be reversed.

2. The substitution matrix is negative semidefinite. Reason: from (1.8) and the fact that the expenditure function, being the result of optimization, has a negative semidefinite Hessian (the second order condition).

`Marshallian demand functions:`

1. Homogeneity of degree 0 in prices and cash-in-hand: if all prices and cash-in-hand are multiplied by a constant k, quantities consumed do not change. Reason: budget constraint is linear in prices and cash-in-hand. Example: measure prices in pounds or pennies.

2. From the Slutsky equation, the substitution matrix can be rewritten as the matrix with elements:

$$\frac{\partial x_j(p, m)}{\partial p_i} + \frac{\partial x_j(p, m)}{\partial m} x_i \tag{1.9}$$

   The substitution matrix (1.9) is symmetric and negative semidefinite. Reason: see Hicksian demand above.

2. Preferences

- Where do preferences come from? what is systematic, what is idiosyncratic, what is manipulable (propaganda, advertizing, raising awareness, education campaign).
- What if preferences change?
  - anticipated changes: e.g., old age
  - unanticipated changes:
    - (1) someone tries something new and is surprised to discover he likes it, or had become addicted to it;
    - (2) young do not understand that their risk preferences will change, e.g., start smoking when not yet risk averse, or engage in dangerous sports that leave permanent injuries
  - switch back and forth between different set of preferences:
    - (1) impulses: buys something on the spur of the moment;
    - (2) self-commitment problem, e.g., Dr Jekyll and Mr Hyde: during the day, want to save and behave, but at night want to spend it all on 'booze and cheap women'.

3. Applications

Here are some examples of areas in which producer and consumer theory have been used in development economics.

3.1. Consumer theory

1. Estimation of consumer demand systems
2. Estimation of demand for particular goods, including education, health, sanitation, environment goods (if traded)
3. Estimation of labor supply, including earnings functions
4. Estimation of preference for nutrition, e.g., hedonic model of demand for micro-nutrients.