Microeconomic Theory
Lecture 1: Producer Theory

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Warning: These notes are intended solely as an informal supplement to the readings. I cannot guarantee that they are free of mistakes. Use at your own risk and when in doubt, consult the readings. Examples of applications to development are given during the classes and lectures.

1. Fundamental Axioms

Economics as a science is based on two fundamental axioms:

1. People pursue their self-interest.
2. People act rationally.

These axioms obviously cannot be a complete description of human behavior. There are many instances where people selflessly seek to help others, their children for instance. There are also many instances where people are not rational, get angry, make mistakes, and the like. But they are certainly a better starting point for examining human interaction than assuming that people do NOT pursue their self-interest and do NOT think about the consequences of their actions.

The main reason why economists continue to base most of their thinking on these two axioms is probably due to Adam Smith. He remarked that, through market interaction, people's pursuing their self-interest results in the good of all. The cobbler makes shoes to earn money for food, and the farmer produces food to earn money for shoes, and in the end the work for each other without knowing it. The market keeps track of how much each has done for society by allocating money to each individual according to the level of their contribution to other people's welfare. To this day, this fundamental insight remains the foundation of economics as a science.

Another reason why economists maintain a fascination for models of the world that assume people to be selfish and rational is that policies based on such models are likely to be robust. We know, for instance, what happens to communists revolutions when they call upon individuals' revolutionary spirits to work hard for the good of society: it may work for a while, but the quality of the work suffers and a repressive apparatus is required to keep the system going. In contrast, the market works without coercion since everyone feels they are working in their own self-interest. We also know what happens to policies that rely on the irrational side of our nature: fear, envy, anger, lust. They often lead to wars and conflict – and a lot of destruction in property. Eighteenth century philosophers who invented economics as a science were looking for a better way of organizing society that would rely on logic and science, not lies and superstition. Hence their desire to organize economics around human rationality.

Finally, economists often feel that what distinguishes human beings from, say, animals is their capacity for reasoning. You can fool some people some of the time but not all people all of the time. We may not always reason right, but given time, example for others, and sufficiently high stakes, we are all capable of figuring out how best achieve a particular outcome. Thus, although rationality may be violated in small ways everyday, it should hold in the large, that is, for large enough stakes and a long enough time frame. Rational people cannot be fooled and policies and mechanisms that assume people to be rational are both more robust (they will not collapse when people realize they are being fooled) and more intellectually and ethically satisfying. Again, this is very much an eighteenth century 'enlightened' view of the world. It lives on in economic science.

This is not to say that economists never explore violations of these axioms. There is excellent work on altruism by economists, e.g., Becker. There is also excellent work on bounded rationality and various deviations from full rationality, such as time inconsistency, loss aversion, trembling hand equilibrium, etc. Economists have also managed to include practices such as addiction, crime, and mistakes in 'rational' models by adding high impatience or high costs of computing rational outcomes. Though these efforts may not always be fully convincing, they indicate that the economic paradigm is alive and well. Explanations of human behavior that simply rely on self-interest and rationality are inherently more intellectually satisfying than explanations that call for people to either *systematically* act against their own welfare or be *perpetually* fooled.

2. Producer theory

Readings: Varian, Microeconomic Analysis, Chapters 1-6

Most of economics is built upon three pillars: a simplified model of firms, a simplified model of individual workers/consumers, and a simplified model of how the two interact. These three pillars are the workhorse of economics. One cannot claim to be an economist without knowing them. We begin with producer theory. We continue with consumer theory.

2.1. The technology:

We begin with a simplified description of the production technology available to producers. The idea is that producers turn inputs x into outputs y. They have at their disposal different ways of turning inputs into outputs, i.e., different combinations of machines and workers of different types and skills. These different ways are sometimes called techniques of production. To the extent that these different techniques of production are divisible, producers can achieve a wide range of input and output mixes by combining multiple production techniques, e.g., do some by hand and some with a machine.

Economists are not interested in the details of the production process itself. They are only interested in the mapping from inputs to outputs. To describe the production possibilities open to producers, they often postulate the existence of a *production function* detailing how many units of output can be obtained with a certain combination of units of inputs. The production function is written as a mathematical relationship. In case of a single output, it is written $y = f(x)$ where x is typically a vector of inputs. Common functional forms chosen for the production function include:

1. The Cobb Douglas production function:

$$y = cx_1^a x_2^b \tag{2.1}$$

2. The Constant Elasticity of Substitution or CES production function:

$$y = c(a_1 x_1^{\rho} + a_2 x_2^{\rho})^{\frac{1}{\rho}} \tag{2.2}$$

3. The Leontieff production function:

$$y = min(ax_1, bx_2) \tag{2.3}$$

In case of multiple outputs, the production technology is summarized in implicit form as $G(y, x) = 0$. For instance, if a firm can produce white cloth or red cloth using the same amounts of x_1 and x_2, an example of joint production function may be:

$$y_1 + y_2 - x_1^a x_2^b = 0 \tag{2.4}$$

`Remarks:`

1. The production function is a physical relationship. It includes no assumption regarding the behavior of producers – except that of technical efficiency (i.e., that engineers know what they are doing and are not wasteful).

2. The production function is an approximation. The quality of the approximation depends on the divisibility of the production techniques.

3. The production function is specific to a particular firm or industry (if firms have access to the same types of technology).

2.2. Properties of the production function:

2.2.1. Returns to scale:

1. A production function $f(x)$ is said to exhibit *constant returns to scale globally* if $f(tx) = tf(x)$ for all x and t.

2. A production function $f(x)$ is said to exhibit *constant returns to scale locally* if $f(tx) = tf(x)$ for certain values of x and t.

3. A production function $f(x)$ is said to exhibit *decreasing returns to scale* if $f(tx) < tf(x)$ for all x and t.

4. A production function $f(x)$ is said to exhibit *increasing returns to scale* if $f(tx) > tf(x)$ for all x and t.

2.2.2. Homogeneity:

A function $f(x)$ is said to be homogeneous of degree k if and only if $f(tx) = t^k f(x)$. Consequently, a production function that exhibits constant returns to scale is homogeneous of degree one.

2.2.3. Discussion:

Constant returns to scale is a reasonable assumption if techniques of production can be duplicated. For example, if 100 meters of cloth can be produced with two workers and a machine, 200 meters of cloth can be produced with four workers and two machines.

Decreasing returns to scale is a reasonable assumptions if certain inputs/factors of production cannot be duplicated. Examples include: management skills of the entrepreneur, arable land, clear air.

Increasing returns to scale is a reasonable assumption if small scale production requires proportionally more inputs per unit of output. For instance, the amount of calories required to produce a kilowatt of electricity is higher with a hand held generator than with a diesel electric power plant. Increasing returns to scale also arise when a minimum threshold of inputs is required before production can even start, e.g., a building to put the machines in, a telephone line to take orders, etc.

2.3. Profit maximization

Producers pursue their self-interest and their self-interest is assumed to depend only on the level of their profits. In the standard model, on-the-job satisfaction and greed for power are ignored as possible determinants of producers' behavior. (They have, however, been included in extensions to the standard model.) The objective function of the producer may also differ from profit maximization in the case of a small farmer facing imperfect markets for food or labor. We will discuss this case in the lecture on household models.

Profit is equal to total revenues py minus total costs wx, where p stands for a vector of output prices, y for a vector of outputs, w for a vector of input prices, and x for inputs. Profit maximization is written:

$$\max \pi \equiv py - wx \text{ subject to } G(y, x) = 0 \tag{2.5}$$

In case of a single output, the above simplifies to:

$$\max \pi \equiv pf(x) - wx \tag{2.6}$$

The solution to the above optimization problem is a value of profits π. This value varies with output prices p and input costs w. If we were to solve the profit maximization problem for all p and w, we would get a maximized value of profits π for each p and w. Let $\pi(p, w)$ describe this relationship. We call function $\pi(p, w)$ the *profit function*. Those familiar with dynamic programming will notice the similarity with the value function. It should not be confused with the profit definition $\pi = py - wx$.

It might also be interesting to consider the cost minimization problem:

$$\min wx \text{ subject to } f(x) = y \tag{2.7}$$

where y is exogenously given. The solution to the above minimization problem is the minimum input cost required to produce quantity y. The solution to this optimization problem for any arbitrary values of w and y is called the cost function $c(w, y)$.

`Remarks:`

1. The profit function is not function of output and input quantities. It is only function of output and input prices.

2. The profit function should never be maximized since it is already the solution to a maximization problem.

3. There exist restricted versions of the profit and cost functions where certain factors, e.g., capital are held constant. In this case, the profit function will be function of (1) output prices (2) price of variable inputs (3) quantities of inputs that are held constant.

From the (mathematical) theory of optimization, we know that the solution to the profit maximization problem can be found by solving a system of first order conditions of the form:

$$p\frac{\partial f(x_i)}{\partial x_i} = w_i \tag{2.8}$$

Equation (2.8) must be satisfied in case of an interior solution, that is, if input x is used for production. A more general form exists that allows for corner solutions (that is, the possibility that certain inputs are not used at all). These more general conditions, know as Kuhn-Tucker conditions are:

$$p\frac{\partial f(x_i)}{\partial x_i} \leq w_i \text{ if } x_i = 0 \tag{2.9}$$

$$p\frac{\partial f(x_i)}{\partial x_i} = w_i \text{ if } x_i > 0 \tag{2.10}$$

To be an optimum, the solution to the above Kuhn-Tucker conditions must also satisfy the so-called second order condition. This is because the first order condition can identify either a minimum or a maximum of the profit maximization problem. The second-order condition checks that the solution is indeed a maximum. To this effect, the second order condition essentially examines the curvature of the optimized function around the optimum and checks that it bends in the right direction. For a single input, single output production function, the second order condition boils down to:

$$\frac{\partial^2 f(x)}{\partial x^2} \leq 0 \tag{2.11}$$

In the general case, it requires that the Hessian matrix (matrix of second order derivatives of $f(x)$) be negative semidefinite.

2.4. Input demand and output supply

Profit maximization also yields optimum output and input quantities that correspond to a particular set of output and input prices. The relationship between optimum output and prices is called the *output supply function* and is written $y(p, w)$. Similarly, the relationship between optimum input levels and prices is called the *input demand function* and is written $x(p, w)$.

Input demand and output supply functions have some properties that follow from the fact that they are solutions to a particular type of optimization problem. Input demand and output supply are homogeneous of degree zero in all output and input prices. This means that if all prices are multiplied by the same constant, the quantities of output and input do not change. This property follows automatically from the definition of profits as $py - wx$: clearly, maximizing $py - wx$ or maximizing $100py - 100pw$ yields the same choice of y and x.[1] (Think of expressing prices in pounds or in pennies: this should not matter for which level of output is chosen.)

[1] This idea is indeed formalized by a theorem in optimization theory that says that multiplying the objective function by a scalar does not change the solution to the optimization.

Define the *substitution matrix* as:

$$\begin{bmatrix} \frac{\partial x_1}{\partial w_1} & \frac{\partial x_1}{\partial w_2} \\ \frac{\partial x_2}{\partial w_1} & \frac{\partial x_2}{\partial w_2} \end{bmatrix} \tag{2.12}$$

The substitution matrix is symmetric and negative definite. This follows from the fact that the substitution matrix is the inverse of the Hessian matrix and that the Hessian matrix must be symmetric and negative definite for the second order condition to be satisfied. From this we get two important properties:

1. Input demand decreases when input price goes up, i.e., $\frac{\partial x_i(p,w)}{\partial w_i} \leq 0$.

2. Symmetry: $\frac{\partial x_i}{\partial w_j} = \frac{\partial x_j}{\partial w_i}$.

2.5. Profit function

Properties of the profit function $\pi(p, w)$:

1. Nondecreasing in output prices p_i (high output price is good), nonincreasing in input prices w_i (high input costs is bad). Reason: see definition of profits.

2. Homogeneous of degree 1 in all prices p and w. Reason: the definition of profits is linear in prices.

3. Convex in p and w. Reason: the profit function is the result of optimization.

4. Continuous in p and w. Reason: profits are linear in prices.

Hotelling's lemma (derivative property):

$$y_i(p, w) = \frac{\partial \pi(p, w)}{\partial p_i} \tag{2.13}$$

$$x_j(p, w) = -\frac{\partial \pi(p, w)}{\partial w_j} \tag{2.14}$$

3. Applications

Here are some examples of areas in which producer and consumer theory have been used in development economics.

1. Estimation of individual production functions, e.g., returns to fertilizer
2. Estimation of individual efficiency, e.g., are small farmers efficient? are exporters more efficient?
3. Estimation of aggregate production functions, e.g., growth regressions
4. Estimation of demand for new technology, e.g., seeds or fertilizer.
5. Estimation of output supply/input demand systems, e.g., to predict how will agriculture respond to a devaluation/changes in relative prices.

4. Exercises

1. Define the technical rate of substitution.
2. Define the elasticity of substitution.
3. Consider a Cobb-Douglass production function. Under what conditions is it constant returns to scale? Decreasing returns to scale? Increasing returns to scale?