



INTRODUCTORY ECONOMICS
Michaelmas/Hilary Terms 09/10
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WEEK 5.

Reading. This week the main topic is differentiation and optimisation with applications to consumer theory. This is partly designed to consolidate the lecture material on calculus and optimisation and partly to consolidate/extend the material on consumer theory. The relevant chapters in the course textbooks remain Varian Chapters 6, 7, 8 and 9; Perloff Chapter 4.1-4.3 and 5.5 but you now need to pay close attention to the Appendices in Varian and the algebra in Perloff (no skipping). The relevant Chapters of the Workbook are 5 to 8.

Problem:

Consider the utility function

$$u = x_1^\beta x_2^{1-\beta}$$

where $\beta \in [0, 1]$. Suppose that prices are $\{p_1, p_2\}$ and the consumer's budget is m .

PART A

1. Derive an expression for the consumer's marginal rate of substitution between good 2 and good 1 ($MRS_{2,1}$).
2. Solve the consumer's optimisation problem

$$\max_{x_1, x_2} u = x_1^\beta x_2^{1-\beta} \text{ such that } p_1 x_1 + p_2 x_2 = m$$

and derive the consumer's Marshallian demands for good 1 and good 2.

3. For both goods derive expressions for: the own price elasticity of demand, the cross-price elasticity of demand, the budget/income elasticity of demand
4. For both goods find all of the terms of the own-price Slutsky equations, identify them and show that the substitution effect is negative.
5. Derive an expression for the maximum utility available to the consumer given the prices they face and their income [Hint: plug the Marshallian demands back into the utility function. This expression is called the indirect utility function.]

PART B

1. Solve the problem

$$\min_{x_1, x_2} m = p_1 x_1 + p_2 x_2 \text{ such that } x_1^\beta x_2^{1-\beta} = u$$

and derive the consumer's Hicksian demands for good 1 and good 2. [Hint: a "Hicksian demand" relates the demand for a good to the prices and the utility level. Proceed analogously to the way you would tackle the problem of finding a Marshallian demand function]

2. Partially differentiate the Hicksian demand for good 2 with respect to its own price and show that the result is the same as the substitution effect for good 2 which you calculated in Question A(4). Explain why this is the case.
3. Derive an expression for the minimum expenditure needed to reach any given utility given the prices the consumer faces [Hint: plug the Hicksian demands back into the budget constraint. This expression is called the expenditure function.]
4. Show that the indirect utility function and the expenditure function are each the respective inverses of the other.

....and relax.