INTRODUCTORY MICROECONOMICS - Solutions
Collection Paper for
Philosophy, Politics and Economics, Economics and Management, Engineering, Economics and Management
Hilary Term 2010

1. (i) This can be solved by the method of Lagrange Multipliers, by substitution or by using the MRS condition. What follows is the substitution method.
The maximisation problem is

$$\max_{x_1,x_2} u(x_1,x_2) = \ln(x_1) + x_2 \text{ subject to } p_1x_1 + p_2x_2 = m$$

Solve the budget constraint for x_2 in terms of (p_1, p_2, m) and substitute into the utility function

$$u = \ln(x_1) + \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

Now the problem an unconstrained one in one variable (x_1):

$$\max_{x_1} u(x_1) = \ln(x_1) + \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

First order condition:

$$\frac{du}{dx_1} = \frac{1}{x_1} - \frac{p_1}{p_2} = 0$$

Which implies

$$x_1 = \frac{p_2}{p_1}$$

Substituting into the budget constraint gives

$$p_1\left[\frac{p_2}{p_1}\right] + p_2x_2 = m$$

which implies

$$x_2 = \frac{m}{p_2} - 1$$

So the answers are

$$x_1 = \frac{p_2}{p_1} \quad \text{and} \quad x_2 = \frac{m}{p_2} - 1$$

NB if $m/p_2 < 1$ then $x_2 = 0$ and $x_1 = m/p_1$.

1 (ii) This requires the Slustsky equation

$$\frac{\partial x_1(p_1,p_2,m)}{\partial p_1} = \frac{\partial x_1(p_1,p_2,u)}{\partial p_1} - \frac{\partial x_1(p_1,p_2,m)}{\partial m}x_1$$

You are asked to show that for these preferences/demands

$$\begin{aligned} \frac{\partial x_1(p_1,p_2,m)}{\partial p_1} &= \frac{\partial x_1(p_1,p_2,u)}{\partial p_1} \\ Uncomp.Effect &= Comp(Subs)Effect \end{aligned}$$

The easiest way to show this is to show that since

$$x_1 = \frac{p_2}{p_1}$$

then

$$\frac{\partial x_1(p_1,p_2,m)}{\partial m} = 0$$

so the last term in the Slutsky equation (the income effect) is zero. The reason why the total (uncompensated effect) is the same as the substitution effect is therefore that, for these (quasi-linear) preferences, there are no income effects for x_1 – the indifference curves in (x_2, x_1)-space with x_1 on the horizontal axis are vertical translations of each other.

1(iii) You're going to need the expenditure function. Substitute the Marshallian demands into the utility function to get the indirect utility function:

$$v(p_1,p_2,m) = \ln\left(\frac{p_2}{p_1}\right) + \frac{m}{p_2} - 1$$

and invert for the expenditure function

$$m(p_1,p_2,u) = \left[u - \ln\left(\frac{p_2}{p_1}\right) + 1\right] p_2$$

Then

$$\begin{aligned} CV &= m(2p_1,p_2,v(2p_1,p_2,m)) - m(2p_1,p_2,v(p_1,p_2,m)) \\ &= \left[\left[\ln\left(\frac{p_2}{2p_1}\right) + \frac{m}{p_2} - 1\right] - \ln\left(\frac{p_2}{2p_1}\right) + 1\right] p_2 - \left[\left[\ln\left(\frac{p_2}{p_1}\right) + \frac{m}{p_2} - 1\right] - \ln\left(\frac{p_2}{2p_1}\right) + 1\right] p_2 \\ &= -p_2 \ln 2 \end{aligned}$$

$$\begin{aligned} EV &= m(p_1,p_2,v(2p_1,p_2,m)) - m(p_1,p_2,v(p_1,p_2,m)) \\ &= \left[\left[\ln\left(\frac{p_2}{2p_1}\right) + \frac{m}{p_2} - 1\right] - \ln\left(\frac{p_2}{p_1}\right) + 1\right] p_2 - \left[\left[\ln\left(\frac{p_2}{p_1}\right) + \frac{m}{p_2} - 1\right] - \ln\left(\frac{p_2}{p_1}\right) + 1\right] p_2 \\ &= -p_2 \ln 2 \end{aligned}$$

So EV and CV are both the same (in *this* case so if you just point that out that's fine. In any case since the question doesn't specify it's not necessary to do both). This is a consequence of quasi-linear preferences and a price change relating to the good which has no income effects.

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2(i). The production function is

$$y = 1.52L^{0.6}K^{0.4}$$

so scale the inputs by λ

$$1.52(\lambda L)^{0.6}(\lambda K)^{0.4} = 1.52L^{0.6}K^{0.4}\left(\lambda^{0.6}\lambda^{0.4}\right) = y\left(\lambda^{0.6+0.4}\right) = y\lambda$$

2(ii).
(a) Plug $\overline{K} = 100$ into the production function $y = 1.52L^{0.6}100^{0.4} \approx 9.59L^{0.6}$ which implies that

$$L(y) = \left(\frac{y}{9.59}\right)^{1.67}$$

(b) Only labour is variable so we just need to compute the cost of the labour input:

$$\begin{aligned} v(c) &= wL(y) = 24\left(\frac{y}{9.59}\right)^{1.67} = 0.55y^{1.67} \\ AVC(y) &= \frac{wL(y)}{y} = \frac{0.55y^{1.67}}{y} = 0.55y^{0.67} \end{aligned}$$

(c) The fixed cost is the cost of $\overline{K} = 100$ which is

$$F = 800 \qquad \Rightarrow \qquad AFC(y) = \frac{800}{y}$$

(d) Use

$$c(y) = F + v(c) \qquad \Rightarrow \qquad \frac{c(v)}{y} = \frac{F}{y} + \frac{v(c)}{y}$$

so

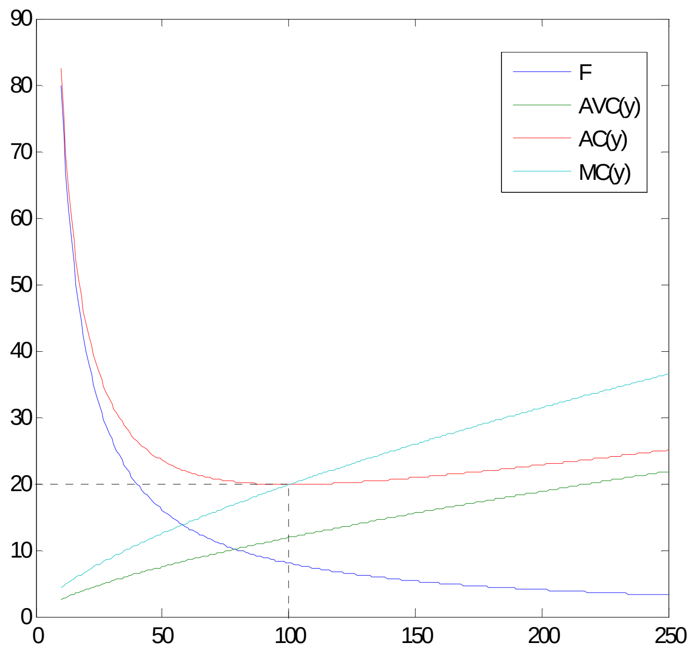
$$AC(y) = \frac{800}{y} + 0.55y^{0.67}$$

(e) Since

$$c(y) = F + v(c)$$
$$c(y) = 800 + 0.55y^{1.67}$$

so

$$MC(y) = c'(y) = 0.92y^{0.67}$$



(f) The rule is "supply y such that $p = MC(y)$ and $dMC(y)/dy > 0$ if $p > AVC(y)$. Otherwise supply $y = 0$". The price = marginal cost condition is $p = 0.92y^{0.67}$ which we invert for the supply function

$$y(p) = \left(\frac{p}{0.92}\right)^{1.5}$$

We need to check the other conditions. You can do this algebraically or with reference to the figure (assuming you've drawn it right) where you can see that.the MC curve is upwards sloping everywhere and that $MC(y) > AVC(y)$.

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3 (i) This is the Stackleberg game which we solve by backwards induction.
The Follower's problem is

$$\begin{aligned}
\max_{y_B} \pi_B &= [a - by_A - by_B]\, y_B - c_B y_B \\
\frac{\partial \pi_B}{\partial y_B} &= a - by_A - 2by_B - c_B = 0 \\
&\Rightarrow \; y_B = \frac{a}{2b} - \frac{c_B}{2b} - \frac{y_A}{2}
\end{aligned}$$

This is B's reaction function.
The Leader's problem is (NB we already know the Follower's reaction function so this has been plugged in for y_B)

$$\begin{aligned}
\max_{y_A} \pi_A &= \left[a - by_A - b\left(\frac{a}{2b} - \frac{c_B}{2b} - \frac{y_A}{2}\right)\right] y_A - c_A y_A \\
\frac{\partial \pi_A}{\partial y_A} &= -by_A + \frac{a}{2} + \frac{c_B}{2} - c_A = 0 \\
\Rightarrow y_A &= \frac{a}{2b} + \frac{c_B}{2b} - \frac{c_A}{b}
\end{aligned}$$

This gives us the Leader's equilibrium output (NB it does not depend on y_B, it is *not* a reaction function). Plug the Leader's output in to the Follower's reaction function to solve for the Follower's equilibrium output.

$$\begin{aligned}
y_B &= \frac{a}{2b} - \frac{c_B}{2b} - \frac{1}{2}\left(\frac{a}{2b} + \frac{c_B}{2b} - \frac{c_A}{b}\right) \\
y_B &= \frac{a}{4b} - \frac{3c_B}{4b} + \frac{c_A}{2b}
\end{aligned}$$

3(ii) Now think about the **Cournot (simultaneous) game**. We already have B's reaction function

$$y_B = \frac{a}{2b} - \frac{c_B}{2b} - \frac{y_A}{2}$$

Because the firms are symmetric in terms of functional forms we know

$$y_A = \frac{a}{2b} - \frac{c_A}{2b} - \frac{y_B}{2}$$

So solve for the Nash equilibirum by sub'ing y_B into the reaction function for A

$$\begin{aligned}
y_A &= \frac{a}{2b} - \frac{c_A}{2b} - \frac{1}{2}\left(\frac{a}{2b} - \frac{c_B}{2b} - \frac{y_A}{2}\right) \\
y_A &= \frac{1}{3b}\left(a + c_B - 2c_A\right)
\end{aligned}$$

Again since the firms are symmetric

$$y_B = \frac{1}{3b}\left(a + c_A - 2c_B\right)$$

So comparing the Stackelberg and Cournot equilibria shows the first-mover advantage.

$$\begin{aligned}
y_A^S &= \frac{1}{2b}\left(a + c_B - 2c_A\right) > \frac{1}{3b}\left(a + c_B - 2c_A\right) = y_A^C \\
y_B^S &= \frac{1}{4b}\left(a - 3c_B + 2c_A\right) < \frac{1}{3b}\left(a + c_A - 2c_B\right) = y_B^C
\end{aligned}$$

3 (iii) Let $c_B = 2c$ then $c_A = c$

$$\begin{aligned}
y_A^C &= \frac{1}{3b}\left(a + c - 2c\right) \\
y_B^C &= \frac{1}{3b}\left(a + c - 4c\right)
\end{aligned}$$

So

$$y_A^C > y_B^C$$

Firm A will grab more of the market and hence make greater profits than Firm B.