Equilibrium in the Jungle*

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Abstract

In the jungle, power and coercion govern the exchange of resources. We study a simple, stylized model of the jungle that mirrors an exchange economy. We define the notion of jungle equilibrium and demonstrate that a number of standard results of competitive markets hold in the jungle.

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1 Introduction

In the typical analysis of an exchange economy, agents are involved in consumption and exchange goods voluntarily when mutually beneficial. The endowments and the preferences are the primitives of the model. The distribution of consumption in society is determined endogenously through trade.

This paper is motivated by a complementary perspective on human interaction. Throughout the history of mankind, it has been quite common (and we suspect that it will remain so in the future) that economic agents, individually or collectively, use power to seize control of assets held by others. Power relationships, either formal or informal, are pervasive in every society and take several forms. Often, power is purely physical. Sometimes, however, power is more gentle. In the male-female "market", for example, charm and attraction play a key role in obtaining a favorite mate. In an office parking lot, social conventions such as seniority allow control of the preferred parking places. The power of persuasion enables some to convince others to take actions against their own interest.

We introduce and analyze an elementary model of a society, called the *jungle,* in which economic transactions are governed by coercion. The jungle consists of a set of individuals, each having exogenous preferences over bundles of goods and exogenous "strength". The ranking of agents according to strength is unambiguous and known to all. Power, in our model, has a simple and strict meaning: a stronger agent is able to take resources from a weaker agent.

The framework mirrors a basic exchange economy. The total endowment of goods is exogenous. The distribution of power in the jungle is the counterpart of the distribution of endowments in the market and is also exogenous. As the incentives to produce or collect the goods are ignored in an exchange economy, so are the incentives to build strength in the jungle.

We define a *jungle equilibrium* as a feasible allocation of goods such that no agent would like and is able to take goods held by a weaker agent. We demonstrate several properties that the allocation rule of the jungle shares with the allocation rule of an exchange economy. In particular, we will show that under standard assumptions a *jungle equilibrium* exits, is Pareto efficient and there exist prices that "almost" support the jungle equilibrium as a competitive equilibrium.

The observation that mainstream economic models ignore a variety of human activities that involve power and that are relevant to the production and distribution of wealth is not new. Economists such as Sam Bowles, Herbert Gintis, Jack Hirshleifer, and Herschel Grossman have emphasized this point for long time. Here are some examples. Hirshleifer (1994), for example, says that "...the mainline Marshallian tradition has nevertheless almost entirely overlooked what I will call the dark side of the force - to wit, crime, war, and politics" and "Appropriating, grabbing, confiscating what you want-and, on the flip side, defending, protecting, sequestering what you already have-that's economic activity too". Grossman (1995) develops a model where agents decide how much effort to put in production and how much to embed in "extralegal appropriative activities" and studies its equilibrium. Bowles and Gintis (1992) emphasize that "power is based on the capacity of some agents to influence the behavior of others to their advantage through the threat of imposing sanction" and analyzed markets, especially labor markets, where the terms of transactions are determined through a non Walrasian process where an agent's wealth affects his power.

These authors have argued for the importance of involuntary exchange in the economy and its inclusion in standard economic theory. In contrast, our main goal is to construct a formal model of involuntary exchange that is similar to the Walrasian model and yields comparable properties.

2 The Jungle

In this section, we introduce a simple model in which resources become available for distribution among a group of agents. Power and coercion governs the allocation of these resources. Each of the agents has exogenous preferences over bundles of goods. Agents are ranked according to their strength. This ranking is unambiguous and known to all. A stronger agent is able to take resources from a weaker agent. We present and study an equilibrium concept that we call the jungle equilibrium and investigate its properties.

We consider a model with commodities labeled $1, ..., K$, and a set of agents, $I = \{1, ..., N\}$. An aggregate bundle $w = (w_1, ..., w_K) >> 0$ is available for distribution among the agents. Each agent i is characterized by a preference relation $\succsim^i$ on the set of bundles $\mathcal{R}_+^K$ and by a consumption set $X^i \subseteq \mathcal{R}_+^K$. The interpretation of X^i is that agent i has bounds on his ability to consume. We assume that X^i is compact and convex, and satisfies free disposal, that is, $x^i \in X^i$, $y \in \mathcal{R}_+^K$ and $y \leq x^i$ implies that $y \in X^i$.

The preferences of each agent satisfy the standard assumptions of strong monotonicity, continuity, and strict convexity. The distribution of resources in the jungle is determined by the relative power of the agents. We choose a particularly simple notion of power. The agents are ordered by a strength relation S. We assume that the binary relation S is a linear ordering (irreflexive, asymmetric, complete, and transitive), and without loss of generality, that $1S2$,$2S3$,...,$(N-1)SN$. The assumptions on S are not merely technical. One can easily envisage power relationships for which they are not satisfied. The statement iSj is interpreted as "i is stronger than j". If iSj, i can take from j anything that j has. Finally, a *jungle* is defined as a tuple $< N, K, \{\succsim^i\}_{i\in I}, \{X^i\}_{i\in I}, S >$.

A *feasible allocation* is a vector of non-negative bundles $z = (z^0, z^1, z^2, ..., z^N)$ such that $z^0 \in \mathcal{R}^K_+$, $z^i \in X^i$ for $i = 1, .., N$, and $\sum_{i=0}^N z^i = w$. The vector z^0 represents the goods that are not allocated to any agent. A feasible allocation is *efficient* if there is no other feasible allocation for which at least one agent is strictly better off and none of the other agents is worse off.

A *jungle equilibrium* is an allocation such that no agent can assemble a more preferred bundle from the bundles that are freely available or held by himself and by an agent who is weaker than him. Formally, a *jungle equilibrium* is a feasible allocation z such that there is no agents i and j, iSj, and a bundle $y^i \leq z^0 + z^i$ or $y^i \leq z^i + z^j$ such that $y^i \in X^i$ and $y^i \succ^i z^i$.

Proposition 1 *A jungle equilibrium exists.*

Proof: Construct the allocation $\hat{z} = (\hat{z}^0, \hat{z}^1, \hat{z}^2, ..., \hat{z}^N)$ as follows. Let $\hat{z}^1$ be agent 1's best bundle in the set $\{x^1 \in X^1 \mid x^1 \leq w\}$. Define inductively $\hat{z}^i$ to be agent i's best bundle in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} \hat{z}^j\}$ and $\hat{z}^0 = w - \sum_{j=1}^N \hat{z}^j$. The allocation $\hat{z}$ is obviously a jungle equilibrium. QED

Note that the procedure in the proof of Proposition 1 generates a unique allocation as preferences are strictly convex and X^i is convex. The definition of jungle equilibrium appears rather weak in so far as an agent can appropriate goods belonging to only one weaker agent despite other weaker agents possibly owning desirable commodities. However, as the next lemma shows, under smoothness assumptions on the preferences and consumption sets of the agents, a jungle equilibrium is unique and is such that no agent can assemble a more desirable bundle from goods that are freely available or belong to all weaker agents. We say that a jungle is *smooth* if, for each agent i, the preferences are represented by a strictly increasing, strictly quasiconcave, and differentiable utility function $u^i : \mathcal{R}_+^K \to \mathcal{R}$, $(\nabla u^i \gg 0)$ and there exists a convex, strictly increasing, and differentiable function g^i $(\nabla g^i \gg 0)$ such that $X^i = \{x^i \in \mathcal{R}_+^K \mid g^i(x^i) \leq 0\}$. First, we show a simple, technical result.

Lemma 2 *Let $a = (a_1, ..., a_n)$ and $b = (b_1, ..., b_n)$ be strictly positive vectors, and suppose that $ax > 0$ and $bx < 0$, for some vector $x = (x_1, ..., x_n)$ such that $x_i \neq 0$ for any i. Then, there exist numbers $c > 0$ and $d > 0$ such that $ca_i - da_j > 0$ and $cb_i - db_j < 0$ for some i for which $x_i > 0$ and some j.*

Proof: See Appendix

Proposition 3 *If a jungle is smooth, $\hat{z}$ is the unique jungle equilibrium.*

Proof: Let z be a jungle equilibrium. We will show that z^i is i's best bundle in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} z^j\}$. Since preferences are strictly convex, it will then follow that $z = \hat{z}$. If $z^i = w - \sum_{j=1}^{i-1} z^j$, the claim is obvious. If $z^i \neq w - \sum_{j=1}^{i-1} z^j$, note that, by strict monotonicity, $g^i(z^i) = 0$. Suppose there exists a bundle $\bar{x}^i$ such that $g^i(\bar{x}^i) \leq 0$ and $u^i(\bar{x}^i) > u^i(z^i)$. Note that, by convexity of g^i, $\nabla g^i(z^i)(\bar{x}^i - z^i) \leq 0$ and, by strict quasiconcavity and monotonicity $\nabla u^i(z^i)(\bar{x}^i - z^i) > 0$ (Mas Colel et al. p. 934). Clearly, we can choose $\bar{x}^i$ so that $\nabla g^i(z^i)(\bar{x}^i - z^i) < 0$. Now, applying the Lemma above, there exist numbers $c > 0$ and $d > 0$ such that $c \left.\frac{\partial u^i(x^i)}{\partial x_k^i}\right|_{x^i=z^i} - d \left.\frac{\partial u^i(x^i)}{\partial x_j^i}\right|_{x^i=z^i} > 0$ and $c \left.\frac{\partial g^i(x^i)}{\partial x_k^i}\right|_{x^i=z^i} - d \left.\frac{\partial g^i(x^i)}{\partial x_j^i}\right|_{x^i=z^i} < 0$ for some k for which $(\bar{x}_k^i - z_k^i) > 0$ and for some j. Hence, for small $\epsilon > 0$, adding ϵc units of commodity k and subtracting ϵd units of commodity j keeps agent i within his consumption set, strictly improves agent i's utility, and can be obtained if ϵc units of commodity k are freely available or are held by a weaker agent. QED

One important normative justification for the competitive equilibrium is provided by the "First Fundamental Welfare Theorem" which states that any competitive allocation is efficient. As the next result shows, efficiency also holds in the jungle.

Proposition 4 *The allocation $\hat{z}$ is efficient.*

Proof: Suppose not and let $(y^0, y^1, ..., y^N)$ be a feasible allocation such that $y^i \succsim^i \hat{z}^i$ for every agent i and $y^j \succ_j \hat{z}^j$ for some j. Since the number of agents is finite, we will obtain a contradiction by showing that for every j for whom $y^j \neq \hat{z}^j$ there must be an agent j' stronger than j for whom $y^{j'} \neq \hat{z}^{j'}$. To see this note that, if $y^j \neq \hat{z}^j$ and $y^j \succsim^j \hat{z}^j$, then by the uniqueness of the optimal bundle of j in the convex set $\{x^j \in X^j \mid x^j \leq w - \sum_{h=1}^{j-1} \hat{z}^h\}$ we have that $y^j \notin \{x^j \in X^j \mid x^j \leq w - \sum_{h=1}^{j-1} \hat{z}^h\}$, that is, there is at least one good k for which $y^j_k > w_k - \sum_{j<i} \hat{z}^j_k$. Hence, there must be an agent j' stronger than j for whom $y^{j'} \neq \hat{z}^{j'}$. QED

The main argument of this Proposition also appears in Ghosal and Polemarchakis (1999, Lemma 1).

Corollary 5 *If a jungle equilibrium is unique, it is efficient.*

Proof: It follows from Proposition 4 and the observation that allocation $\hat{z}$ is always a jungle equilibrium.

3 The Jungle and the Exchange Economy

In this section, we compare the jungle to the exchange economy. The standard model of an exchange economy is defined as a tuple $< N, K, \{\succsim^i\}_{i\in I}, \{X^i\}_{i\in I}, \{w^i\}_{i\in I} >$, where w^i is the initial endowment of agent i and $\sum_{i=1}^N w^i = w$. The jungle differs from the exchange economy in one component. Instead of initial endowments, the jungle includes a specification of the power relation. A *competitive equilibrium* for the exchange economy is a pair (z, p), where z is a feasible allocation and $p = (p_{1,...,}p_K)$ a non-negative price vector such that, for every agent i, z^i is i's most preferred bundle in the set $\{x^i \in X^i \mid px^i \leq pw^i\}$.

3.1 Examples

Example 1: Allocation of Houses

Consider a jungle with a set of agents $I = \{1, ...N\}$ and a set $\{1, ..., N\}$ of indivisible commodities, referred to as houses. Each agent i can consume only one house, that is, X^i contains the null and the unit vectors, has a strict ordering over X^i, and strictly prefers having any house to having no house.

For any initial endowment that assigns one house to one agent, a competitive equilibrium exists (Kaneko (1983)). Assume without loss of generality that agent i owns initially house i. First construct a partition $\{I_1, .., I_l, .., I_L\}$ of I as follows. Start with agent $i_0 = 1$ (this choice is arbitrary). Define i_{k+1} as i_k's most preferred house. Continue until $i_{k+1} = i_q$ for some $q \leq k$. Choose $I_1 = \{i_q, ...i_k\}$. Continue with all remaining agents and houses until a partition is completed. Choose a sequence of numbers $p_1 > p_2 > ... > p_L > 0$ and assign to the houses in I_l the price p_l. Clearly this is a competitive equilibrium price vector. An agent in I_l buys a house in I_l and, if he prefers an house not in I_l, it must be a house in I_t, $t < l$, which he cannot afford.

In this construction, some agents in each round exchange their houses and receive their best house from the set of houses not allocated in earlier rounds. The group of agents who obtain a house in each round has the property that each of its members can obtain his preferred house (among those that have not been allocated in earlier rounds) by reallocating the houses held within the group.

Since preferences are strict ordering, the allocation $\hat{z}$ is the unique and efficient jungle equilibrium. In the construction of the jungle equilibrium, houses are also allocated in rounds. Only one agent makes a choice in each round: he is the strongest agent among those agents who have not made their choices earlier.

Example 2: Allocation of Houses and Gold

This example is a modification of Example 1. Consider a world with a set H of N houses held by an agent (the chief) who is interested only in gold and N merchants who own gold but are interested only in houses. Denote the chief as agent $N+1$ and gold as good $N+1$. Gold is divisible but the houses are not. The initial bundle of the chief is $(1, 1, ..., 1, 0)$ containing 0 units of gold and 1 unit of each house. His consumption set contains all bundles of the form $(0, 0, ..., 0, m)$. Agent $i \neq N+1$ owns an initial amount $m^i > 0$ of gold, can consume only one house, and has a strict ordering over the houses.

In this example, gold is the mirror image of strength. Define house k_i inductively as i's most preferred house in the set $H - \{k_1, ..., k_{i-1}\}$. If $m^1 > m^2 > ... > m^N > 0$, then in any competitive equilibrium agent i obtains house k_i. To see this, suppose that in some competitive equilibrium agent i does not obtain k_i but any $j < i$ gets k_j. Then, some agent $t > i$ gets k_i. By definition $m_t < m_i$. The price of k_t cannot exceed m_t and thus k_i is in agent i's budget set, a contradiction. Analogously, consider a jungle in which $1S2....SNSN+1$. Suppose that in a jungle equilibrium agent i does not get k_i but any $j < i$ gets k_j. Then, some agent $t > i$ gets k_i. Agent t is weaker than i and thus agent i can seize k_i, a contradiction. Hence, in any jungle equilibrium agent i obtains house k_i.

3.2 The Second Welfare Theorem

The second fundamental welfare theorem asserts that, under suitable assumptions, any efficient allocation is a competitive equilibrium allocation for some initial endowment. Recall that the difference between the exchange economy and the jungle is that in the former we specify the distribution of initial endowments and in the latter the distribution of strength. Thus, an adaptation of the second welfare theorem to the jungle would state that any efficient allocation is a jungle equilibrium for some distribution of strength. Clearly, such an assertion cannot hold in general since the number of power relations is finite and the number of efficient allocations is infinite.

In Example 1, however, every efficient allocation is a jungle equilibrium for some strength relation S: if agent j prefers the house owned by i, select iSj; it can be easily shown that this binary relation does not have cycles and can be completed on the set of agents. An analogous result was obtained by Abdulkadiroglu and Sonmez (1998) who show that the set of allocations obtained by serial dictatorship of some order, the set of a competitive allocations, and the set the efficient allocations coincide.

In general, a weaker claim does hold. Following Varian (1974), given a feasible allocation z, we say that i *envies* j if exists $x^i \leq z^j$ such that $x^i \in X^i$ and $x^i \succ^i z^i$. In the next proposition we show that, for any efficient allocation, there exists a power relation such that no agent "envies" the bundle of a weaker agent. In other words, for any efficient allocation, there exists a power relation such that the efficient allocation is a jungle equilibrium if we reinterpret iSj to mean that i is able to force j to exchange bundles.

Proposition 6 *Given any efficient allocation z, there exists a strength relation S such that iSj implies i does not envy j.*

Proof: Let z be an efficient allocation. Define the incomplete relation T such that iTj if j envies i. Note that the relation T does not have a cycle. If $i_1 T i_2 T ... i_L T i_1$, then define $y^i = z^i$ for any $i \notin \{i_1, ..., i_L\}$ and $y^{i_l} = x^{i_l} \leq z^{i_{l-1}}$, $x^{i_l} \in X^{i_l}$ and $x^{i_l} \succ^{i_l} z^{i_l}$, for all $l = 1, ..., L$. The vector $(y^0, y^1, ..., y^N)$, where $y^0 = w - \sum_{i=1}^N y^i$, is a feasible allocation, contradicting the efficiency of z. To complete the proof take the binary relation S to be a completion of T on I.

The argument in the above proof is similar to the argument in Varian (1974) which shows that, for any efficient allocation, there exists an agent who does not envy any agents.

3.3 Jungle Prices

One interpretation of the second fundamental welfare theorem is that equilibrium prices provide a measure for assessing the distribution of wealth corresponding to an efficient allocation. This interpretation leads to the investigation of the distribution of wealth corresponding to a jungle equilibrium and, in particular, to the question as to whether a stronger agent is also a wealthier agent.

In Example 1, the jungle equilibrium allocation is a competitive equilibrium allocation for the exchange economy in which the jungle equilibrium is the initial endowment. Any price vector for which the price of the house assigned to agent i is greater than the price of the house assigned to agent j whenever iSj is an equilibrium price vector. At these prices, being stronger implies being wealthier. Of course, other equilibrium prices might exist. In particular, if the strongest agent ranks the highest a house that all other agent rank the lowest, there exists a competittive equilibrium price vector in which the strongest agent is the poorest agent.

When agents have the same preferences and consumption set and prices supporting a jungle equilibrium as a competitive equilibrium exist, the relationship between power and wealth is unambiguous: the value of an agent's jungle equilibrium bundle increases with his strength. To see this, consider a jungle equilibrium z and suppose that (z, p) is a competitive equilibrium.

If iSj then $z^i \succsim^i z^j$ and thus also $z^i \succsim^j z^j$. Since j chooses z^j it must that $pz^i \geq pz^j$ since, if $pz^i < pz^j$, then $(z^i + z^j)/2 \in X^j$, $p(z^i + z^j)/2 \leq pz^j$ and, by strict convexity, $(z^i + z^j)/2 \succ^j z^j$.

A major problem for the application of the second fundamantal welfare theorem to our setting is that the existence of competitive prices supporting the jungle equilibrium is not guaranteed when consumption sets are bounded and exhibit a satiation point. Under our assumptions about preferences and consumption sets, an exchange economy might not have a competitive equilibrium. Suppose that $N = 2$ and that the utility of agent i is $3x_1^i + 2\sqrt{3 + x_2^i} + 2\sqrt{3 + x_3^i}$ and his consumption set is defined by $x_1^i + x_2^i + x_3^i - 3 \leq 0$.

The aggregate initial endowment is $(1, \frac{5}{2}, 2)$. First note that $(1, 1, 1)$ is the (unique) optimal bundle of agent 1 among all feasible bundles. The Kuhn-Tucker condition are

$$3 - \mu^1 - \gamma^1 = 0$$
$$\frac{1}{\sqrt{x_2^1 + 3}} - \mu^1 = 0$$
$$\frac{1}{\sqrt{x_3^1 + 3}} - \mu^1 = 0$$

Hence, plugging $(x_1^1, x_2^1, x_3^1) = (1, 1, 1)$, we get $\mu^1 = \frac{1}{2}$ and $\gamma^1 = \frac{5}{2}$. Then, if we assign $(0, \frac{3}{2}, 1)$ to agent 2, we have the jungle equilibrium $\hat{z}$. Then, if a competitive equilibrium exists when $\hat{z}$ is the initial endowment, $\hat{z}$ must also be a competitive equilibrium allocation as it is efficient. Agent 1's first order conditions are

$$3 - \theta^1 - \lambda^1 p_1 = 0$$
$$\frac{1}{\sqrt{\hat{z}_2^1 + 3}} - \theta^1 - \lambda^1 p_2 = 0$$
$$\frac{1}{\sqrt{\hat{z}_3^1 + 3}} - \theta^1 - \lambda^1 p_3 = 0$$

Note that $\lambda^1 > 0$. Now, since $\hat{z}_2^1 = 1$, $\hat{z}_3^1 = 1$, we must have $p_2 = p_3$. However, agent 2's consumption set is not binding at $\hat{z}^2$. Hence, since $\hat{z}_2^2 = \frac{1}{2}$,

$\hat{z}_3^2 = 1$, $p_2 \neq p_3$.

Despite non existence of equilibrium prices, one can find a sequence of price vectors p^n such that the sequence of demands given p^n converges to $\hat{z}^i$ for $i = 1, 2$. This is easily seen taking p^n such that $p_1^n \equiv 1$, p_2^n and p_3^n converge to zero, and $p_2^n / p_3^n \equiv 2\sqrt{2}/3$, the marginal rate of substitution of agent 2.

More generally, we establish the following an approximation result:

Proposition 7 *Suppose that the jungle is smoooth. In the exchange economy in which $w^i = \hat{z}^i$, $i = 1, ...N$, there exists a sequence of price vectors p^n such that, for every agent i, the sequence of demands of agent i given p^n converges to $\hat{z}^i$.*

Proof: See Appendix.

In the proof of Proposition 7, we construct a sequence of price vectors which "almost support" the jungle equilibrium as a competitive equilibrium. Far enough in the sequence, the demand of each agent is arbitrarily close to his consumption in the jungle equilibrium. The price vector has the limit property that if iSj then the prices of the goods exhausted by agent i are in relative terms "infinitely larger" than the prices of the goods exhausted by agent j. The wealth distribution that supports jungle equilibria as competitive equilibria appears, in a sense, to be extreme: a more powerful agent in the jungle corresponds to infinitely wealthier agent in the exchange economy.

4 Concluding comments

We do not have a common view regarding the wider interpretation of the findings of this paper. Our conclusions are separate.

4.1 Concluding comments by MP

The aim of this paper is to investigate theoretically an environment in which transactions are governed by coercion. Its main goal is to demonstrate at an abstract level the richness of analysis when the allocation rule is driven by agents using power to appropriate resources.

We have emphasized the analogy between the initial endowments in the exchange economy and the initial distribution of power in the jungle for the determination of the final distribution of commodities among the agents. I wish to add a few simple remarks.

In an exchange economy, the interior efficient allocations can be supported as a competitive equilibrium allocation for some redistribution of the endowments. In so far as the jungle precludes "redistribution" of power, it also precludes redistribution of resources. One possible avenue for a more flexible definition of power is to interpret the consumption sets as restrictions not only on the ability to consume but also on the ability to seize resources held by others. Such an interpretation, however, will in general undermine the efficiency of the jungle equilibrium.

One notable omission is production. The inclusion of production processes in this model can also invalidate the efficiency of equilibria. Suppose, for example, that $N = 2$ and that there are several ways of producing the satiation bundle of agent 1. The inputs mix commandeered by agent 1 for the production of his satiation bundle does not necessarily maximize agent 2's utility subject to agent 1 consuming his satiation bundle. An efficient jungle equilibrium exists but it may not be achieved if agent 1 does not include in his considerations the utility level of agent 2. A similar problem arises in the case of no production when the utility of agent 1 is linear. Production, however, seems to make inefficiency more generic.

4.2 Concluding comments by AR

My work in economic theory is seldom motivated by real life issues. However, this time, I am motivated by current economic controversy. As in many other countries, people in Israel are questioning the basic economic apparatuses and the prevailing myths about the function of the free market. he rhetoric which is widely heard has its roots in economic textbooks which explicitly or implicitly hail the market as the ultimate economic mechanism. I have therefore come to realize that as a teacher of Microeconomics I am participating in a political and ideological debate whether I wish to or not.

Economic theory is the study of mechanisms by which society organizes its economic activities and, in particular, distributes its resources. In some cases, we allocate goods through a lottery although a lottery outcome is likely to be inefficient and in any case we aren't too enthusiastic about luck determining our fate. In other cases, we use central committees to determine the allocation of goods. These committees are supposed to take into account the needs and preferences of individuals though they are often criticized as being corrupt. Most of economic theory deals with variants of the market system in which goods enter the world with attached ownership rights and are allocated by means of prices. One way in which to view our paper is as a study of an additional mechanism which is not uncommon in real life, even in the jungle.

However, in my view, this paper is not purely a study of the jungle economy. I view it as a rhetorical exercise which sharpens the issues in standard microeconomics. Being faithful to the classical economic tradition, we constructed a model which is close to the standard exchange economy. We used terminology that is familiar to any economics student. After having defined the notion of jungle equilibrium, we conducted the same type of analysis which can be found in any microeconomic textbook on competitive equilibrium. We showed existence and then discussed the first and second fundamental welfare theorems. We emphasized the analogy between the initial endowments of an exchange economy and the initial distribution of power in the jungle as determining the distribution of commodities among the agents. Were I teaching this model, I would also add the standard comments regarding externalities and the place for government intervention.

There are arguments which attempt to dismiss the comparison between markets and jungles. One can argue that the market has the virtue of providing incentives to "produce" and to enlarge the size of the "pie" to be distributed among the agents. One can also argue, however, that the jungle provides incentives to develop power (physical, intellectual or mental) which is an important social asset. Agents make efforts to produce more goods. Agents who wish to be stronger are an asset for a society which can then defend itself against invaders or evade others in order to accumulate resources.

One might argue that market mechanisms save the resources that would have been wasted in conflicts. Note, however, that under complete information a stronger agent can persuade a weaker agent to part with his goods with no resistance. Societies often create rituals which aid people in recognizing relative power and thereby avoid the costs of conflict. Under incomplete information, the market also wastes resources. And finally, I have not mentioned the obvious trade costs which are also associated with market institutions.

One might argue that labour is a good which should be treated differently.

The long history of slavery shows this to be inaccurate.

One also might argue that in the market system no individual wishes to leave the market while in the jungle a weak individual can simply leave the society. Note, however, that the market model specifies only what the individual does within the market itself. The existence of the option of immigration is not included in the standard market model either.

Alternatively, one might argue that the virtue of the market system is that it utilizes people's natural desire to acquire wealth. On the other hand, the jungle uses the people's natural willingness to exercise power and to dominate without employing central government.

Obviously, I am not arguing in favor of adopting the jungle system. Overall, the relative comparison of the jungle and market mechanisms depends on our assessment of the characteristics with which agents enter the model. If the distribution of the initial holdings in the market reflects social values which we wish to promote, we might regard the market outcome as acceptable. However, if the initial wealth is allocated unfairly, dishonestly or arbitrarily, then we may not favor the market system. Similarly, if power is desirable we might accept the jungle system but if the distribution of power reflects brutal force which threatens our lives we would clearly not be in favor. Thus, although the paper is a rhetorical exercise, it is one which sheds some light on the implicit message students are receiving from us.

5 Appendix

Proof of Lemma 2: First note that $x_j < 0$ for some j and $x_i > 0$ for some i. Second note that, if $\frac{a_j}{b_j} < \frac{a_i}{b_i}$ for some i for which $x_i > 0$ and some j, the claim follows by choosing $\frac{b_i}{b_j} < \frac{d}{c} < \frac{a_i}{a_j}$. Finally, we show that it is impossible that $\frac{a_j}{b_j} \geq \frac{a_i}{b_i}$ whenever $x_j < 0$ and $x_i > 0$. If so, let $\frac{a_k}{b_k}$ be the lowest $\frac{a_j}{b_j}$ associated with $x_j < 0$. Then $ax = \sum_{i=1}^{n} \left(\frac{a_i}{b_i}\right) x_i b_i \leq \sum_{i=1}^{n} \left(\frac{a_k}{b_k}\right) x_i b_i = \frac{a_k}{b_k} bx$, a contradiction as $ax > 0$ and $bx < 0$. QED

Proof of Proposition 7: Without loss of generality, we label the commodities so that all commodities except for $1, ..., t^i$ are exhausted by agents stronger than i, that is, $w_j - \sum_{h=1}^{i-1} \hat{z}_j^h = 0$ if and only if $j > t^i$. The Kuhn-Tucker conditions for agent i are

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} = 0 \text{ for } j = 1, ..., t^{i+1}$$
$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \gamma_j^i = 0 \text{ for } j = t^{i+1} + 1, ..., t^i$$

with non negative multipliers λ^i for the consumption set and γ_j^i for the binding resource constraints.

The first set of equations refers to the commodities that are not exhausted by agent i. The second set of equations refers to the commodities that are exhausted by agent i.

The constraints are necessary for agent i's maximization. If agent i exhausts all the goods, one can set $\lambda^i = 0$ and $\gamma_j^i = \frac{\partial u^i(\hat{z}^i)}{\partial x_j}$. If not, $g^i(\hat{z}^i) = 0$ because of strict monotonicity of preferences and the "constraint qualification" condition is satisfied: the gradients (the derivatives with respect to the commodities $1,..,t^i$) of the constraints ($\hat{z}_j^i - w + \sum_{h=1}^{i-1} \hat{z}_j^h \leq 0$, $j = 1, ..., t^i$ and $g^i(\hat{z}^i) \leq 0$) that are satisfied with equality are linearly independent. In fact, the gradient of g^i (recall that $g^i(\hat{z}^i) = 0$) is a vector of t^i positive numbers and the gradients of the remaining constraint holding with equality are at most $t^i - 1$ unit vectors.

For later use, we add a set of residual conditions which is trivially satisfied

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \gamma^i_j \leq 0 \text{ for } j = t^i + 1, ..., K$$

Consider now an exchange economy with $w^i = \hat{z}^i$, $i = 1, ..., N$. For $\hat{z}^i$ to be i' optimal choice given a price vector $(p_1, ..., p_K)$, the Kuhn-Tucker sufficient conditions are

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j = 0 \text{ for } j = 1, ..., t^{i+1}$$
$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j = 0 \text{ for } j = t^{i+1} + 1, ..., t^i$$
$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j \leq 0 \text{ for } j = t^i + 1, ..., K$$

Hence, $\hat{z}^i$ is an optimal choice given any price vector p^i such that $p^i_j = 0$, $j = 1, ..., t^{i+1}$, and $p^i_j = a\gamma^i_j$, $j = t^{i+1} + 1, ..., t^i$, $p^i_j \geq a\gamma^i_j$, $j = t^i + 1, ..., K$, where $a > 0$.

In general, one cannot construct a price vector for which the demands of every agent i are equal to $\hat{z}^i$. However, we construct a sequence (p^ε) of price vectors such that the demands of every agent i converge to $\hat{z}^i$ as $\varepsilon \to 0$.

Let $d(j)$ be the first player i for whom $\sum_{h=1}^{i} \hat{z}^h_j = w_j$ (agent $d(j)$ exhausts commodity j) and define $p^\varepsilon_j = \varepsilon^{d(j)} \gamma^{d(j)}_j$ if $\gamma^{d(j)}_j > 0$ and $p^\varepsilon_j = \varepsilon^{d(j)+\frac{1}{2}}$ if $\gamma^{d(j)}_j = 0$. If commodity j is never exhausted then set $p^\varepsilon_j = \varepsilon^{N+1}$. This price vector is such that the price of a good which is exhausted by player i is by an order of magnitude larger than the price of a good which is exhausted by agent $i+1$, and the price ratio of any two goods j and k which are exhausted by the same agent i is γ^i_j/γ^i_k if both $\gamma^i_j > 0$ and $\gamma^i_k > 0$.

We will show that, as $\varepsilon \to 0$, the demand of agent i given the price vector p^ε converges to $\hat{z}^i$ for each i. Note that a direct application of the continuity of i's demand is not sufficient as p^ε converges to the null vector. Consider the price vector $\hat{p}^{\varepsilon i}$ obtained from p^ε by replacing p^ε_j, $j = t^i + 1, ..., K$, with $\varepsilon^i \gamma^i_j$. Clearly i's demands given $\frac{1}{\varepsilon^i}\hat{p}^{\varepsilon i}$ converges to $\hat{z}^i$ by continuity. Suppose that the vector of demands given the price vector $\frac{1}{\varepsilon^i}p^\varepsilon$ converges along some subsequence to $\tilde{z}^i \neq \hat{z}^i$. Note that $\frac{1}{\varepsilon^i}\hat{p}^{\varepsilon i}\hat{z}^i = \frac{1}{\varepsilon^i}p^\varepsilon \hat{z}^i$ (since $\hat{p}^{\varepsilon i}$ differ from p^ε only in goods which were exhausted by stronger players) and $\frac{1}{\varepsilon^i}\hat{p}^{\varepsilon i} \leq \frac{1}{\varepsilon^i}p^\varepsilon$ for small ε. Since $\tilde{z}^i$ is in the budget constraint of agent i when the price vector is equal to $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon^i}\hat{p}^{\varepsilon i}$, strict convexity implies that $u^i(\tilde{z}^i) < u^i(\hat{z}^i)$.

A contradiction is thus obtained noting that $\hat{z}^i$ is in the budget constraint given $\frac{1}{\varepsilon^i}p^\varepsilon$ for any ε. QED

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