Quantitative Economics

Trinity 2010

Worksheet 6: Applied Macro

Quantitative Economics
Macro-econometrics Tutorial Exercises

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1 General issues for tutorial discussion

1. Describe some of the characteristics of a stochastic trend process. What are the implications of differencing a time series, x_t, with a stochastic trend to create Δx_t?

2. Discuss the notions of cointegration and common trends. How can two time series, y_t, z_t with stochastic trends be systematically related?

3. Why should we expect temporal dependence in time-series data, so that $\mathsf{corr}(x_t, x_{t-s}) \neq 0$ for $s = 1, 2, ...$? What implications would your explanation have for $\mathsf{corr}(\Delta x_t, \Delta x_{t-s})$?

4. Do common trends also help explain the high correlations observed between many macroeconomic variables? Describe a simple way to check your explanation.

5. Why do wage and price inflation not trend over the past 150 years? Does that imply they will be easy to model? What implications does that finding have for real wages?

6. What are some of the characteristics of a process with a location shift? Describe how to 'model' a single location shift in y_t at a time $T_1 < T$ by an indicator variable $1_{\{t>T_1\}}$.

7. Discuss the notion of co-breaking. Describe a simple model with a single location shift in each of (y_t, z_t) where they co-break.

8. Carefully describe all the features of wages and prices that are eliminated simply by calculating real-wage growth.

9. Compare the taming of those problems by instead calculating the log of real wages relative to output per worker.

10. Discuss what, if anything, is learned from finding that a simple relation between nominal wage inflation and unemployment (Phillips curve) shifts.

2 Exercises for tutorial

1. Consider the simple autoregression:

$$y_t = \rho y_{t-1} + \epsilon_t \quad t = 1, 2, \ldots, T \tag{1}$$

where $y_0 = 0$, $\epsilon_t \sim \mathsf{IN}[0, \sigma_\epsilon^2]$ and $|\rho| < 1$. Derive the formula for the least squares estimator of ρ.

2. Consider the simple regression over $t = 1, 2, \ldots, T$:

$$y_t = \lambda 1_{\{t > T_1\}} + \epsilon_t \tag{2}$$

where $\epsilon_t \sim \mathsf{IN}[0, \sigma_\epsilon^2]$ and $T_1 < T$.
(a) Derive the formula for the least squares estimator $\widehat{\lambda}$ of λ.
(b) interpret its impact on the residuals $\widehat{\epsilon}_t = y_t - \widehat{\lambda} 1_{\{t > T_1\}}$ from (2).

3. Consider the following model over $t = 1, 2, \ldots, T$:

$$\begin{aligned} y_t &= \mu_0 + \lambda x_t + e_t \\ z_t &= \mu_1 + x_t + \nu_t \end{aligned} \tag{3}$$

where:

$$x_t = x_{t-1} + \beta + \epsilon_t \tag{4}$$

and x_0 is a fixed number.
(a) Express x_t in terms of current and past values of ϵs, x_0, and β. What kind of process is $\{x_t\}$?
(b) Express a relationship between y_t and z_t that does not depend on x_t or current and past values of ϵs.
(c) Is it possible to prove that such a relationship between y_t and z_t is unique?

3 Data graphs for tutorial

Using the data set QEHistTutData.xls, check you have the time series for wages=W, prices=P, unemployment rate=Ur, log of GDP=g, log of employment=l, log of real capital stock=k, 3-month Treasury bill rate=R_s, long-term interest rate (yield on 20 year bonds)=R_l.

1. Graph the time series for wages, W, and prices, P, and briefly discuss.
2. Graph the time series for the logarithms of wages, w, and prices, p, and briefly discuss.
3. Graph the time series for the logarithm of real wages, $w - p$, and briefly discuss.
4. Graph the time series for the changes in the logarithm of wages, Δw, prices Δp, and real wages, $\Delta(w - p)$, and briefly discuss.
5. Summarize what you have learned about wages, prices and real wages and their relationships from these four graphs.
6. Plot log real wages, $w - p$, against GDP per person employed, $y - l$, and briefly describe their relationship.
7. Show a regression line on that plot of log real wages against GDP per person employed and discuss how well it describes their relationship.
8. Graph the time series for the unemployment rate, U_r, and briefly discuss.
9. Plot wage inflation, Δw, against the unemployment rate, U_r, and briefly describe their relationship (Phillips curve).
   Is a linear regression a good description of the scatter?
10. Show a regression line on that plot of wage inflation, Δw, against the unemployment rate, U_r, and discuss how well it describes their relationship.

4 Empirical modelling for tutorial

Using the data set QEHistTutData.xls:

1. Estimate the regression of the log of wages, w, on that of prices, p, and briefly discuss the resulting coefficient.
   Plot some graphs that would help you interpret how well the regression describes w.

2. Estimate the regression of wage inflation, Δw, on price inflation, Δp, and briefly discuss the resulting coefficient.
   Plot some graphs that would help you interpret how well the regression describes Δw.

3. Estimate the regression of wage inflation, Δw, on the unemployment rate, U_r, and briefly discuss the resulting coefficient.
   Plot some graphs that would help you interpret how well the regression describes Δw.

5 General issues for vacation

1. Discuss the neo-classical model of wage determination equating marginal revenue per worker with marginal costs. What are the implications of a rise in productivity per worker? How well does the theory match the empirical evidence for the UK over the last 150 years?

2. Discuss the four key problems facing an economist wishing to build an econometric model of a macroeconomic time series. (dynamics, relevant variables, breaks, and trends) Is it sensible to proceed without tackling **all** of these? Is it possible to develop methods for handling them all jointly?

3. What are the respective roles of economic-theory models and statistical models in the explanation of macroeconomic data? Discuss the possible roles of theory and data in empirical macroeconometric modelling.

4. Discuss the difference between a data-generation process like $\mathsf{D}_{\mathsf{y}}(y_1 \dots y_T | z_1 \dots z_T, \boldsymbol{\theta})$ and an econometric model like $\mathsf{f}_{\mathsf{y}}(y_1 \dots y_T | z_1 \dots z_T, \boldsymbol{\beta})$. Are all models of $\mathsf{D}_{\mathsf{y}}(\cdot)$ equally useful? (Unknown parameters $\boldsymbol{\beta}$ must be estimated from data, and 'best' methods for doing so, and resulting estimator distributions, assume the statistical model is the DGP.)

5. Discuss some graphical statistics that help reveal how well an estimated model describes the time series evidence.

6. Discuss the five conditions of congruence.

7. Why are mis-specification tests important?

8. How many mis-specification tests should one conduct? Show why a smaller number than you propose would miss testing important assumptions of the statistical model, but why a larger number would add little extra insight.

9. How would you try to ascertain the causes underlying a theory model being rejected by data evidence? Can there ever be a unique answer?

10. Given the alternative of under-specified models, discuss the general-to-simple approach (Gets) of starting an empirical analysis from a general unrestricted model (GUM) that includes all the relevant variables, their dynamics, breaks and trends based on the available economic theory. What are its main advantages and drawbacks?

11. Discuss the advantages of transforming dynamic equations to equilibrium-correction models (EqCMs). (to interpret and eliminate stochastic trends)

12. Outline how to allow for multiple location shifts by impulse indicators.

13. How might economic agents form expectations when the data-generating process is subject to unanticipated location shifts?

6 Exercises for vacation

1. Consider the following model over $t = 1, 2, \ldots, T$:

$$y_t = \beta_1 z_t + \beta_2 y_{t-1} + \beta_3 z_{t-1} + \epsilon_{1,t} \tag{5}$$

where $|\beta_2| < 1$,and for $|\rho| < 1$, z_t is generated by:

$$z_t = \rho z_{t-1} + \epsilon_{2,t} \tag{6}$$

with $\epsilon_{1,t} \sim \mathsf{IN}[0, \sigma_{11}]$, $\epsilon_{2,t} \sim \mathsf{IN}[0, \sigma_{22}]$, when $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are independent, and all parameters are constant.
(a) Express (5) in equilibrium-correction form.
(b) What happens to the equilibrium relation when $\beta_1 + \beta_2 + \beta_3 = 1$?
(c) Propose a macroeconomic equation where such a parameter restriction might hold.

2. If $\rho = 1$ in (6), z_t becomes a random walk.
(a) Are y_t and z_t cointegrated?
(b) What is the effect of $\rho = 1$ on the equilibrium-correction form derived from (5)?
(c) What is the effect of $\rho = 1$ on the equilibrium-correction form when $\beta_1 + \beta_2 + \beta_3 = 1$ holds?

7 Data graphs for vacation

Using the data set QEHistTutData.xls:

1. Plot the wage share, $(w - p - g + l)$, against the unemployment rate, U_r, and briefly describe their relationship.

2. Show a regression line on that plot of the wage share, $(w - p - g + l)$, against the unemployment rate, U_r, and discuss how well it describes their relationship.
   Would it make less or more sense to regress in the other direction of U_r on $(w - p - g + l)$?

3. Plot the current unemployment rate, $U_{r,t}$, against its lagged value, $U_{r,t-1}$, and briefly describe their relationship.
   How well does $U_{r,t-1}$ describe $U_{r,t}$? How should such a connection be interpreted?

4. Plot the change in current unemployment rate, $\Delta U_{r,t}$, against $U_{r,t-1}$. Is the result what you expected to find?

8 Empirical modelling for vacation

Using the data set QEHistTutData.xls:

1. Estimate the regression of real-wage inflation, $\Delta(w - p)$, on the unemployment rate, U_r, and briefly discuss the resulting coefficient.
   Plot some graphs that would help you interpret how well the regression describes $\Delta(w - p)$. Compare the outcome with that from the previous regressions of wage inflation, Δw on Δp and on U_r.

2. Estimate the regression of the real wage share, $(w - p - g + l)_t$, on the unemployment rate, $U_{r,t}$ and $(w - p - g + l)_{t-1}$. briefly discuss the resulting coefficients.
   Plot some graphs that would help you interpret how well the regression describes $\Delta(w-p-g+l)$. How might you compare the outcome with that from the model of $\Delta(w - p)$?

3. Contrast the previous result with that of a regression of $\Delta(w - p - g + l)_t$ on $U_{r,t}$.
   How can you compare the outcome with that from the model of $(w - p - g + l)_t$?