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Output and Welfare Effects in the Classic Monopoly Price Discrimination Problem

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Abstract

This paper uses convexity arguments to determine the effects of monopolistic third-degree price discrimination on total output and welfare We focus on benchmark cases, including constant demand elasticities, with constant curvature of inverse demand σ. We show how the effects of price discrimination depend on (a) the degree of curvature σ relative to zero (for output) and one (for welfare), and (b) whether low-price markets have greater curvature than high-price markets.

JEL classification: D42, L12, L13

Keywords: price discrimination, monopoly.

1 Introduction

When a profit-maximizing monopolist can charge different prices in separate markets for the same product, are total output and welfare higher or lower than under uniform pricing? This question has a long pedigree going back to Pigou (1920) and especially Robinson (1933). It is well-known that its answer is ambiguous. In the benchmark case of linear demands and all markets served, the effect on total output is zero. Rather broadly, an increase in

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output is a necessary (but not sufficient) condition for discrimination to increase welfare because with discrimination output is inefficiently distributed across markets. In the benchmark linear case, therefore, discrimination reduces welfare. Even with linear demands, however, this conclusion can be reversed if discrimination causes some market(s) to be served that would not be with uniform pricing; or if, in a dynamic setting, freedom to discriminate weakens the monopolist's ability to commit to high prices.

There is now an extensive literature on third-degree price discrimination – i.e. different (linear) prices in different markets – by a profit-maximizing monopolist. Schmalensee (1981) appraised and developed formally the analysis of Joan Robinson for the classic case of distinct demands and constant marginal cost. Varian (1985), using neat convexity arguments, extended the analysis to interdependent demands and also more general cost conditions – a point taken further by Schwartz (1990).[1] Other directions of generalization have included the opening of new markets; dynamics and commitment power; regulation; imperfect competition; and nonlinear ('second-degree') price discrimination.

In this paper we return to the classic static setting of independent demands, all markets served, and constant marginal cost. We derive output and welfare results for some non-linear demand benchmarks using convexity arguments. Though some of our methods yield results more generally, we focus mainly on non-linear demand functions characterized by constant curvature of inverse demand, where by 'curvature' we mean the elasticity of the slope of a function. Thus the curvature of the inverse demand function $P(x)$, which gives price P as a function of output x, is defined as $\sigma(x) \equiv -xP''/P'$.[2] In our benchmark cases, σ is constant in each market.

[1] Schwartz shows under rather general cost conditions that output increase is necessary for discrimination to increase welfare. But that need not be so if there are demand externalities.

[2] The σ term plays an important role in the comparative statics of imperfect competition.

We begin, in the next section, with the well-known benchmark case of linear demands (which corresponds to $\sigma = 0$). A simple convexity argument shows the result due to Joan Robinson that (i) discrimination increases total output if demand is convex in low-price (known as 'weak') markets and concave in high-price ('strong') markets, (ii) decreases total output in the opposite case, and (iii) has zero net effect with all demands linear.

In section 3 we examine the benchmark of a constant elasticity of demand η in each market (which implies that $\sigma = 1 + 1/\eta$). Aguirre (2006) has recently shown that the output effect of discrimination is always positive in this case. We obtain that output result using a simpler (but more general) proof based on a convexity argument based on the *reciprocal* of price. We then extend our method to find a welfare result: when all demands have constant elasticities discrimination reduces welfare if the difference between the highest and lowest price elasticity is no greater than one.

A quite different convexity argument is used in section 4 to address the more general case of constant curvature of inverse demand – i.e. constant σ – in each market. We show in a series of results how the effects of discrimination on output and welfare depend on the combined effect of two influences: (a) the absolute magnitude of σ, and (b) the relative magnitudes of σ across markets. In the special case where σ is *common* across markets[3], (b) is absent so (a) is what matters. Then the output effect of discrimination has the sign of σ, and the welfare effect has the sign of $(\sigma - 1)$ if discriminatory prices are not far apart. As to (b), variation of curvature across markets is a positive influence of discrimination – for output and welfare – if σ is greater in low-price markets than in high-price markets, with the reverse being true in the opposite case. This echoes Joan Robinson's finding, but in

[3] In this special case (of which linear demands are one instance) where σ is common across markets, inverse demand in one market is an affine transformation of inverse demand in another. In a recent paper Cowan (2007) considers the welfare effects of price discrimination when (direct) demand in one market is an affine transformation of demand in another market. The present paper complements that analysis.

a considerably more general setting, and with results for welfare as well as output. Concluding remarks and a schematic summary of our results are in section 5.

2 The Model and Linear Demand Benchmark

A pure monopolist with constant unit cost $c > 0$ supplies $n \geq 2$ markets. Demand in a representative market is denoted $x(p)$, where p is price in that market and $x'(p) < 0$. Consumer surplus is $v(p)$, which satisfies $v'(p) = -x(p)$. Welfare in a market, $w(p)$, is the sum of consumer surplus and profit so $w(p) = v(p) + (p - c)x(p)$. The price elasticity of demand is $\eta(p) \equiv -\frac{px'}{x}$ and the curvature of demand with respect to price is denoted $\zeta(p) \equiv -\frac{px''}{x'}$. The Lerner index is $\mu(p) \equiv \frac{p-c}{p}$. Define $\sigma(p) \equiv \frac{\zeta(p)}{\eta(p)} = \frac{xx''}{x'x'}$, which is the curvature – that is to say elasticity of slope – of inverse demand $P(x)$ with respect to output, as can be seen by differentiating twice the identity $P(x(p)) \equiv p$ to obtain

$$\sigma = \frac{xx''}{x'x'} = -\frac{xP''}{P'}.$$

Total output and welfare are $X \equiv \sum x$ and $W \equiv \sum w$, where summation is across all markets. (Market subscripts are omitted to avoid clutter.)

With price discrimination allowed (regime D), the firm maximizes profit $\pi(p) \equiv (p - c)x(p)$ in each market. We assume that $\sigma < 2$ everywhere so $\pi(p)$ is quasi-concave (and profit as a function of output is concave). The profit-maximizing price p_D in a market is given by the condition

$$0 = \pi'(p_D) = (p_D - c)x'(p_D) + x(p_D) \tag{1}$$

or

$$\mu(p_D)\eta(p_D) = 1. \tag{2}$$

We assume that the prices in regime D are not all equal, in other words that the firm does want to discriminate. With price discrimination not allowed (regime N), the firm chooses p to maximize $\sum \pi(p) = (p-c)\sum x(p)$. Thus the non-discriminatory or uniform price is given by

$$0 = \sum \pi'(p_N) = (p_N - c)\sum x'(p_N) + X(p_N). \qquad (3)$$

It is assumed that p_N is such that $x(p_N) > 0$ in every market, so all markets are served in both regimes. Markets in which $p_D > p_N$ are said to be 'strong', and those in which $p_D < p_N$ are 'weak'. Rearranging (3) gives

$$\mu(p_N) = \frac{\sum x(p_N)}{\sum \eta(p_N)x(p_N)} ,$$

so the Lerner Index is the inverse of the weighted average price elasticity, with the weights being shares of the aggregate demand at the non-discriminatory price. The profit function for regime N need not be quasi-concave, though it is if the profit functions are concave since concavity is preserved by addition. Theorem 1 of Nahata et al (1990), however, shows that quasi-concavity of each profit function implies that p_N is bounded above by the highest p_D and is bounded below by the lowest p_D and this is all that is needed for the output and welfare results that follow.

We first state a well-known result due to Robinson (1933) and developed by Schmalensee (1981).

Proposition 1 *Suppose that demand over the relevant range is convex ($x'' \geq 0$) in weak markets and concave ($x'' \leq 0$) in strong markets. Then total output is no less with discrimination than without. The reverse is true if demand is convex in strong markets and concave in weak markets. So with linear demands, total output is the same with and without discrimination.*

Proof. If $x'' \geq 0$ in weak markets and $x'' \leq 0$ in strong markets, then $x'(p_N) \geq x'(p_D)$ in all markets. In weak markets convexity implies that

$$\begin{aligned} x(p_D) - x(p_N) &\geq (p_D - p_N)x'(p_N) \\ &= (p_D - c)x'(p_N) - (p_N - c)x'(p_N) \\ &\geq (p_D - c)x'(p_D) - (p_N - c)x'(p_N), \end{aligned}$$

whereas in strong markets concavity implies that

$$\begin{aligned} x(p_D) - x(p_N) &\geq (p_D - p_N)x'(p_D) \\ &= (p_D - c)x'(p_D) - (p_N - c)x'(p_D) \\ &\geq (p_D - c)x'(p_D) - (p_N - c)x'(p_N). \end{aligned}$$

Adding across all markets yields that

$$X_D - X_N \geq \sum (p_D - c)x'(p_D) - \sum (p_N - c)x'(p_N) = X_N - X_D.$$

by (1) and (3). Therefore $X_D \geq X_N$ under the stated conditions, with the inequality strict if convexity/concavity is strict in any market. The reverse inequality holds by the same reasoning if weak markets are concave and strong ones are convex. It follows that with linear demands, which are both (weakly) convex and concave, $X_D = X_N$. ■

Schmalensee (1981) obtained Proposition 1 by Lagrangian methods, and Shih et al (1988) by the mean-value theorem[4]. The proof here is a little simpler, and relies only on the properties of convex and concave functions and the first-order conditions under D and N. A straightforward intuition for it can be derived by considering the shape of the demand functions. Start at the non-discriminatory price. When discrimination is allowed the price cuts in the weak markets cause demand to increase considerably because the demand functions are convex, while in the strong markets demand falls only moderately due to concavity. Proposition 1 does not directly yield a welfare comparison, but it implies that discrimination lowers welfare if demand concave in strong markets and concave in weak markets, because in that case demand is both (weakly) lower with discrimination and inefficiently distributed across markets.

[4]Cheung and Wang (1994) extend the 'adjusted concavity' results of Shih et al.

3 Constant Demand Elasticity Benchmark

Beyond the linear case, Proposition 1 does not apply if demands are either all convex or all concave. A standard case of convex demand is the constant elasticity function $x(p) = Ap^{-\eta}$ for $A > 0$. An interior solution to the profit maximization problem requires that demand is elastic ($\eta > 1$). Since $\sigma = 1 + 1/\eta$ it follows that $\sigma < 2$ and thus that the profit function for regime D is quasi-concave. The demand function never touches either axis so all markets are automatically served when the non-discriminatory price is set. Denote the highest elasticity by η_H and the lowest by η_L. To explore the output effect, let $r \equiv 1/p$ be the reciprocal of price – i.e. the relative price of the outside good – and define the function $y(r) \equiv x(1/r)$. Note that

$$y'(r) = -(1/r)^2 x'(1/r)$$

and

$$y''(r) = (1/r)^3(2x' + (1/r)x'') = -(1/r)^3 x'(\zeta - 2),$$

which has the sign of $(\zeta - 2)$. With constant elasticity $\zeta = 1 + \eta > 2$ so $y(r)$ is strictly convex. We can now state

Proposition 2 *When demand elasticities are constant total output is greater with discrimination than without.*

Proof. With constant elasticities $y(r)$ is strictly convex so

$$\begin{aligned} x(p_D) - x(p_N) &= y(\frac{1}{p_D}) - y(\frac{1}{p_N}) > [\frac{1}{p_D} - \frac{1}{p_N}]y'(\frac{1}{p_N}) \\ &= -[\frac{1}{p_D} - \frac{1}{p_N}]p_N^2 x'(p_N) \\ &= [\frac{p_D - c}{p_D} - \frac{p_N - c}{p_N}]\frac{p_N^2 x'(p_N)}{c} \\ &= [\frac{p_N x'(p_N)}{\eta(p_D)} - (p_N - c)x'(p_N)]\frac{p_N}{c} \\ &= [-x(p_N) - (p_N - c)x'(p_N)]\frac{p_N}{c}. \end{aligned}$$

Adding across markets, and using (3), yields $X_D > X_N$. ■

The result that discrimination increases output under constant elasticity conditions was shown for small differences in the elasticities by Formby et al (1983) using Lagrangian methods, and Aguirre (2006) showed this more generally using an inequality due to Bernouilli. Proposition 2 is, perhaps, simpler to prove. Moreover the same method of proof can be used to generalize the proposition to the case where (i) $\zeta > 2$ in each market, so $y(r)$ is still convex, and (ii) each elasticity at the uniform price is no greater than that at the discriminatory price.

More importantly, a similar method can be used to derive a welfare result. Define $z(r) \equiv w(1/r)$. Note that

$$z'(r) = -(1/r)^2 w'(1/r) = -(1/r)^2(1/r - c)x'(1/r)$$

and

$$z''(r) = (2/r^3)w' + (1/r^4)w'' = -(1/r^4)x'[(\zeta - 2)\mu - 1].$$

We look for conditions which imply that $(\zeta - 2)\mu - 1 < 0$ in all markets for prices between p_N and p_D so that $z(r)$ is concave. With constant elasticity $\zeta = 1 + \eta$ so $(\zeta - 2)\mu - 1 = (\eta - 1)\mu - 1$, so if $\eta \leq 2$ then $(\eta - 1)\mu - 1 < 0$ everywhere and $z(r)$ is concave. More generally with constant elasticities we have the following lemma.

Lemma 3 *If the difference between the highest and the lowest elasticity is no more than 1, so* $\eta_H - \eta_L \leq 1$, *then* $z(r)$ *is concave in every market in the relevant region.*

Proof. At the discriminatory optimum $\mu\eta = 1$ so $(\eta - 1)\mu - 1 = -1/\eta < 0$ and locally z is concave. In strong markets it is always true that z is concave in the region $[p_N, p_D]$ because concavity holds at p_D and as the price falls towards p_N the term $(\eta - 1)\mu$ decreases. The expression $(\eta - 1)\mu - 1$ could only be positive if the market is weak and if the non-discriminatory price is much higher than the discriminatory one. This possibility is ruled out if $(\eta_H - 1)\mu(p_N) < 1$. We have $\eta_H - \sum \eta x(p_N)/\sum x(p_N) < \eta_H - \eta_L \leq 1$ because the weighted average elasticity exceeds the lowest elasticity. By the first-order condition for regime N we have $\sum \eta x(p_N)/\sum x(p_N) = 1/\mu(p_N)$ so $\eta_H - 1/\mu(p_N) < 1$, which implies that $(\eta_H - 1)\mu(p_N) < 1$. For all other weak markets we have $(\eta - 1)\mu(p_N) - 1 < (\eta_H - 1)\mu(p_N) - 1 < 0$. Thus $z'' < 0$ for prices in the relevant range. ■

Proposition 4 *If demand elasticities are constant, and the highest and lowest elasticity differ by no more than one, total welfare is lower with discrimination than without.*

Proof. With the condition on the elasticities $z'' < 0$ so

$$\begin{aligned} w(p_D) - w(p_N) &= z(\frac{1}{p_D}) - z(\frac{1}{p_N}) < [\frac{1}{p_D} - \frac{1}{p_N}]z'(\frac{1}{p_N}) \\ &= [-x(p_N) - (p_N - c)x'(p_N)]\frac{(p_N - c)p_N}{c} \end{aligned}$$

as in the proof of Proposition 2. Adding across markets yields $W_D < W_N$.

■

Propositions 2 and 4 say that discrimination increases output when elasticities are constant, but reduces welfare if $w(1/r)$ is concave in r, as is quite widely the case. However, when $w(1/r)$ is not concave over the relevant range the welfare effect of discrimination can be positive. Market size then becomes important. Suppose that with one strong market and one weak one the elasticities differ by more than one and that welfare is lower with discrimination. A multiplicative shift of direct demand in the base case in the strong market will not change the discriminatory price, but it will increase the non-discriminatory price. As the strong market expands in scale p_N tends to the discriminatory price in the strong market and eventually the welfare level with discrimination goes above that without discrimination. Ippolito (1980) presents an early numerical analysis and shows the importance of the relative sizes of the market for signs of the welfare effects. What Proposition 4 gives is a sufficient condition, of wide but not universal applicability, that gives a determinate welfare effect for the case of constant demand elasticities whatever the relative market sizes.

4 Constant Inverse Demand Curvature Benchmark

The results so far may cause one to wonder whether any general conditions for welfare-increasing price discrimination can be found in the classic model with all markets served. The benchmark case of a *common* constant curvature of inverse demand σ shows, among other things, that this is indeed possible. More generally, the aim of this section is to use another kind of convexity argument – this time based around the discriminatory prices p_D – to compare total output and welfare when inverse demand in each market has constant curvature.

The strategy is to find suitable functions $f(x)$ and $g(w)$ of output and welfare such that $\sum f = 0$ and $\sum g = 0$ at both N and D. So, for output, define

$$f(x) \equiv \pi'(P(x)) = x + [P(x) - c]x'(P(x))$$

and observe that $f(x_D) = 0$ in every market,

$$\sum f(x_N) = 0,$$

$$f' = 2 + (p - c)\frac{x''}{x'} = 2 - \mu\zeta = 2 - \mu\eta\sigma,$$

so $f'(x_D) = 2 - \sigma$, and that

$$f'' = -\sigma\frac{d}{dx}(\mu\eta) - \mu\eta\frac{d\sigma}{dx}. \tag{4}$$

The final term is zero if σ is constant. To pursue this case, let inverse demand have the constant curvature form $P(x) = a + bx^{1-\sigma}/(\sigma - 1)$ for $\sigma \neq$

1, with $P(x) = a - b\log(x)$ when $\sigma = 1$, where $b > 0$ and $(a-c)(1-\sigma) > 0$ to ensure finite demand when $p = c$. We allow a, b and σ to vary across markets. It is straightforward to show that $\frac{d}{dx}(\mu\eta)$ has the sign of $-[1 + \mu\eta(1-\sigma)]$ generally and that this is negative when σ is constant. Therefore f'' has the same sign as σ.

Proposition 5 *If* σ *is constant and positive (negative) in each market, then*

$$\sum(2-\sigma)(x_D - x_N) > (<)0. \tag{5}$$

Proof. Assume $\sigma > 0$ in all markets. Then $f'' > 0$, so

$$f(x_N) - f(x_D) > (x_N - x_D)f'(x_D) = (2-\sigma)(x_N - x_D).$$

Summing across markets yields (5). Likewise with inequalities reversed if $\sigma < 0$ in all markets. ■

Corollary 6 *Assume that* σ *is constant and positive in each market, and that there exists* $\bar{\sigma}$ *such that* $\sigma \leq \bar{\sigma}$ *in all strong markets and* $\sigma \geq \bar{\sigma}$ *in all weak markets. Then* $X_D > X_N$. *If, on the other hand,* σ *is constant and negative in each market, and there exists* $\bar{\sigma}$ *such that* $\sigma \geq \bar{\sigma}$ *in all strong markets and* $\sigma \leq \bar{\sigma}$ *in all weak markets, then* $X_N > X_D$.

Proof. Given the assumption in the first part of the Corollary, it is true in both strong and weak markets that

$$(2-\sigma)(x_N - x_D) \geq (2-\bar{\sigma})(x_N - x_D)$$

because $x_N - x_D > (<)0$ in strong (weak) markets. With (5) this implies

$$0 > \sum(2-\sigma)(x_N - x_D) \geq (2-\bar{\sigma})\sum(x_N - x_D) = (2-\bar{\sigma})(X_N - X_D)$$

and so $X_D > X_N$. The second part of the Corollary follows similarly. ■

A special case of Corollary 6 is that of σ common across markets, in which case discrimination raises output if demand is strictly convex and lowers it when demand is strictly concave. Shih et al (1988) also prove this particular output result. More generally, the Corollary says that a sufficient condition for total output to increase is that convexity, as measured by σ, is positive and constant in each market, and more so in weak markets than strong ones. But total output decreases if σ is negative and constant in each market, and more so in weak markets than strong ones.

A similar argument works for the welfare comparison. Let $\rho(w)$ be the price associated with welfare level w in a representative market, and define

$$g(w) \equiv (\rho(w) - c)\pi'(\rho(w)) = (\rho(w) - c)[x + (\rho(w) - c)x'(\rho(w))]$$

In every market $g(w_D) = 0$ and

$$\sum g(w_N) = 0.$$

Using the fact that $1/\rho' = w' = (p - c)x'$ we obtain

$$g' = 3 + \frac{x}{(p-c)x'} + (p-c)\frac{x''}{x'} = 3 - \frac{1}{\mu\eta} - \mu\zeta = 3 - \frac{1}{\mu\eta} - \mu\eta\sigma,$$

so $g'(w_D) = 2 - \sigma$, and

$$g'' = \left[\left(\frac{1}{\mu\eta}\right)^2 - \sigma\right] \frac{d}{dw}(\mu\eta) - \mu\eta\frac{d\sigma}{dw}. \tag{6}$$

The final term is zero if σ is constant. Then g'' has the sign of $[\sigma(\mu\eta)^2 - 1]$, so $g''(w_D)$ simply has the sign of $(\sigma - 1)$. Thus we have

Proposition 7 *If* σ *is constant and greater than* $\max\{1, [\mu_N\eta(p_N)]^{-2}\}$ *in each market, then*

$$\sum (2 - \sigma)(w_D - w_N) > 0. \tag{7}$$

These inequalities are reversed if σ *is constant and less than* $\min\{1, [\mu_N\eta(p_N)]^{-2}\}$ *in each market.*

Proof. If $\sigma \geq \max\{1, [\mu_N \eta(p_N)]^{-2}\}$ and is constant, then $g'' > 0$, so

$$g(w_N) - g(w_D) > (w_N - w_D)g'(w_D) = (2 - \sigma)(w_N - w_D).$$

Summing across markets yields (7). Likewise with inequalities reversed if $\sigma \leq \min\{1, [\mu_N \eta(p_N)]^{-2}\}$ and is constant. ■

Corollary 8 *Assume that* $\sigma \geq \max\{1, [\mu_N \eta(p_N)]^{-2}\}$ *is constant in each market and that there exists* $\bar{\sigma}$ *such that* $\sigma \leq \bar{\sigma}$ *in all strong markets and* $\sigma \geq \bar{\sigma}$ *in all weak markets. Then* $W_D > W_N$. *If, on the other hand,* σ *is constant and less than* $\min\{1, [\mu_N \eta(p_N)]^{-2}\}$ *in each market, and there exists* $\bar{\sigma}$ *such that* $\sigma \geq \bar{\sigma}$ *in all strong markets and* $\sigma \leq \bar{\sigma}$ *in all weak markets, then* $W_N > W_D$.

Proof. Given the assumption in the first part of the Corollary, it is true in both strong and weak markets that

$$(2 - \sigma)(w_N - w_D) \geq (2 - \bar{\sigma})(w_N - w_D).$$

With (7) this implies

$$0 > \sum (2 - \sigma)(w_N - w_D) \geq (2 - \bar{\sigma}) \sum (w_N - w_D) = (2 - \bar{\sigma})(W_N - W_D)$$

as required. The second part of the Corollary follows similarly. ■

If $\sigma > 1$, the condition that $\sigma \geq \max\{1, [\mu_N \eta(p_N)]^{-2}\}$ holds if p_N exceeds or is not much less than p_D. And if $\sigma < 1$, the condition that $\sigma \leq \min\{1, [\mu_N \eta(p_N)]^{-2}\}$ holds if p_N is less than or is not much above p_D. So if discriminatory prices are sufficiently close to the uniform price, and if $\sigma \neq 1$ is constant and common across markets, then $W_D - W_N$ simply has the sign of $(\sigma - 1)$. More generally, Corollary 8 says that total welfare increases if convexity as measured by σ exceeds one and is constant in each market, is greater in weak markets than strong ones, and discriminatory prices are sufficiently close to the uniform price. The Corollary also yields a corresponding sufficient condition for total welfare to decrease with discrimination, a special case of which is when all demand functions are linear. Recalling Proposition 4, note that Corollary 8 does not apply when all demands have constant elasticities because from $\sigma = 1 + 1/\eta$, we then have that $\sigma > 1$ but that weak markets have lower values of σ than strong ones.

Corollary 8 does not cover the case where some $\sigma > 1$ and some $\sigma < 1$. However, for that case we can state

Proposition 9 *If σ is constant and greater than one in each weak market, and σ is constant and less than one in each strong market, then*

$$\sum \left[3 - \frac{1}{\mu_N \eta_N} - \mu_N \eta_N \sigma \right] (w_D - w_N) > 0. \tag{8}$$

(The inequality is reversed if 'weak' and 'strong' are swapped in the condition above.)

Proof. Under the stated conditions $g'' > 0$ over the relevant range of prices in weak markets and $g'' < 0$ in strong markets. It follows that in all markets

$$g(w_D) - g(w_N) < (w_D - w_N) g'(w_N) = (w_D - w_N) \left[3 - \frac{1}{\mu_N \eta_N} - \mu_N \eta_N \sigma \right].$$

Adding across markets yields (8). Likewise for the reverse inequality. ■

Corollary 10 *If, given the condition at the start of Proposition 9, discriminatory prices are close enough to the uniform price that there exists $\gamma > 0$ such that $g'(w_N) \leq \gamma$ in weak markets and $g'(w_N) \geq \gamma$ in strong markets, then $W_D > W_N$. (The opposite holds under the conditions at the end of the Proposition if discriminatory prices are likewise close enough to the uniform price.)*

Proof. Under the stated conditions $0 < \sum (w_D - w_N) g'(w_N) \leq (W_D - W_N)\gamma$, and so $W_D > W_N$. ■

As to the closeness condition in Corollary 10, note that if $p_D \approx p_N$, then $g'(w_N) \approx g'(w_D) = 2 - \sigma$, which is by the assumption in Proposition 9 lower in weak markets than in strong markets.

Propositions 7 and 9 and their Corollaries together give conditions that are sufficient to sign welfare changes when curvature is constant. The sufficient conditions for welfare to rise with discrimination are fairly stringent: curvature must be sufficiently great in absolute size, at least as high in weak markets as in strong markets, and the prices must not be too far apart.

5 Conclusion

In this paper we have derived some new results on the output and welfare effects of third-degree price discrimination by a monopolist with constant unit costs. We have focussed mainly on the class of demand conditions characterized by constant curvature of inverse demand, which includes, among much else, the sub-case of constant demand elasticities as well as the familiar linear demands case. Using convexity arguments – which may be of some independent interest – we have given sufficient conditions to sign both output and welfare effects of price discrimination. As well as some new findings, our methods also provided a degree of synthesis of known results.

Our results can be summarised in terms of two influences: (a) the absolute level of curvature of inverse demand as measured by σ, and (b) relative curvature as between markets. Let us say that relative curvature is 'positive' if curvature is greater in weak markets than strong markets, and 'negative' in the opposite case. Then the table below gives a schematic summary of our results.

	Neg curvature	Low pos curvature	High curvature
Neg rel curvature	$X \downarrow$ and $W \downarrow$	$W \downarrow$	
Pos rel curvature		$X \uparrow$	$X \uparrow$ and $W \uparrow$

Discrimination is negative (as indicated by the down arrows) for both output and welfare if absolute and relative curvature are both negative. It is positive for both if relative curvature is positive and absolute curvature is not only positive but high in the sense of curvature exceeding unity (sufficiently). When curvature is positive but not high, discrimination increases output if relative curvature is positive but decreases welfare if it is negative. The linear demands case has zero absolute and relative curvature, so in accordance with the schema, the output effect of discrimination is zero and the welfare effect is negative. The case of constant elasticities of demand has high curvature but negative relative curvature – so it is an instance of the (empty) top-right box, for which we do not have general results. Nevertheless the output effect is positive, and we showed the welfare effect often to be negative, for that class of demand functions.

Overall our analysis is consistent with the view that monopolistic third-degree price discrimination, the effects of which are well known to be ambiguous, has a *tendency* to be detrimental to welfare *if* all markets are served at the non-discriminatory optimum. We do not suggest, however, that public policy should therefore be hostile to monopolistic third-degree price discrimination.

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