# Gravity *Redux*: Structural Estimation of Gravity Equations with Asymmetric Bilateral Trade Costs*

Jeffrey H. Bergstrand†, Peter Egger‡, and Mario Larch§

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**Abstract**

Theoretical foundations for estimating gravity equations were enhanced recently in Anderson and van Wincoop (2003). Though elegant, the model assumes *symmetric* bilateral trade costs to generate an estimable set of structural equations. In reality, however, trade costs (and trade flows) are not bilaterally symmetric. We use the simple workhorse Krugman-type monopolistic-competition/increasing-returns-to-scale model of trade assuming only multilateral trade balance to allow for *asymmetric* bilateral trade costs. A Monte Carlo analysis of our general equilibrium model demonstrates – in the presence of asymmetric bilateral trade costs – that the bias of the Anderson-van Wincoop approach is at least an order-of-magnitude larger than that using our approach for computing general equilibrium comparative statics. We then confirm empirically the difference of our approach and that of Anderson and van Wincoop in the Canadian-U.S. case allowing asymmetric effects of national borders. Furthermore, we apply our approach empirically to the more general case of trade flows among 67 countries in the presence of asymmetric bilateral tariff rates.

**Key words:** Gravity model; Trade costs; Structural estimation

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† *Affiliation:* Department of Finance, Mendoza College of Business, University of Notre Dame, Kellogg Institute for International Studies, and CESifo Munich. *Address:* Department of Finance, Mendoza College of Business, University of Notre Dame, Notre Dame, IN 46556 USA. *E-mail:* bergstrand.1@nd.edu.

‡ *Affiliation:* Ifo Institute for Economic Research, Ludwig-Maximilian University of Munich, CESifo, and Centre for Globalization and Economic Policy, University of Nottingham. *Address:* Ifo Institute for Economic Research, Poschingerstr. 5, 81679 Munich, Germany. *E-mail:* egger@ifo.de.

§ *Affiliation:* Ifo Institute for Economic Research and CESifo. *Address:* Ifo Institute for Economic Research, Poschingerstr. 5, 81679 Munich, Germany. *E-mail:* larch@ifo.de.

# 1 Introduction

*"Our analysis suggests that inferential identification of the asymmetry [in bilateral trade costs] is problematic."*
(Anderson and van Wincoop, 2003, p. 175)

For nearly a half century, the "gravity equation" has been used to explain econometrically the *ex post* effects of economic integration agreements, national borders, currency unions, language, and other measures of trade costs on bilateral trade flows. While two early formal theoretical foundations for the gravity equation with trade costs – first Anderson (1979) and later Bergstrand (1985) – addressed the role of "multilateral prices", Anderson and van Wincoop (2003) refined the theoretical foundations for the gravity equation to emphasize the importance of accounting properly for the *endogeneity* of prices. Two major conclusions surfaced from the now seminal Anderson and van Wincoop (henceforth, A-vW) study, "Gravity with *Gravitas*". First, a complete derivation of a standard Armington (conditional) general equilibrium model of bilateral trade in a multi-region ($N > 2$) setting with iceberg trade costs suggests that traditional cross-section empirical gravity equations have been misspecified owing to the omission of theoretically-motivated *multilateral* (price) resistance terms for exporting and importing regions. Second, to estimate properly the full general equilibrium comparative-static effects of a national border or an economic integration agreement, one needs to estimate these multilateral resistance (MR) terms for any two regions with and without a border or agreement, respectively, in a manner consistent with theory. Due to the underlying nonlinearity of the structural relationships, A-vW suggest that estimation requires a custom nonlinear least squares (NLS) program to account properly for the endogeneity of prices and/or estimate the comparative static effects of a trade cost.

However, though A-vW (2003) is elegant and motivated by only four assumptions, one necessary assumption is that every pair of regions has *perfectly symmetric* bilateral trade costs.[1] Hence, in a world trade setting with $N$ countries, the tariff rate (and *ad valorem*-

[1] The other three assumptions are that all goods are produced in an endowment economy and are

equivalent non-tariff rate) on products from Japan to the United States equals exactly that on products from the United States to Japan, and so forth. Clearly, this assumption is grossly at odds with reality; data supporting this is provided in Figure 1, using bilateral tariff data from the Global Trade Analysis Project (GTAP) on 67 economies in 2001. There is large heterogeneity bilaterally in tariff rates. In Figure 1, only 42 percent of the bilateral tariff rates are symmetric; 58 percent are not. Also, the figure illustrates that the asymmetry can be as large as 150 percent. Moreover, an important implication of this assumption is that every pair of countries' bilateral trade will be balanced. This is also grossly at odds with reality. However, the symmetric bilateral trade costs (SBTC) assumption was useful to derive an elegant system of structural equations that provided a logically-consistent formal theoretical foundation for proper estimation of a gravity model.

Since the SBTC assumption is grossly violated in the real world, we address three questions in this paper. First, is there a set of plausible *alternative* assumptions that can generate a theoretical foundation for the gravity equation without SBTC? Second, in a world where we know the true data-generating process, can this alternative theoretical foundation provide unbiased coefficient estimates and *precisely-estimated* general equilibrium comparative statics? Third, in a world with asymmetric bilateral trade costs, does the A-vW approach which assumes SBTC yield biased coefficient estimates and comparative statics, and are such biases avoided under the alternative approach?

In this paper, we suggest two fairly standard assumptions as alternatives to SBTC to motivate a theoretical foundation for the gravity equation. First, we assume the simple Krugman (1980) model of increasing returns to scale with monopolistic competition (IR-MC), as summarized in Baier and Bergstrand (2001) and Feenstra (2004), that has become the workhorse for studying bilateral intra-industry trade. This workhorse IR-MC model pins down the relationship between the exporting country's economic size and the number of varieties consumed by the representative consumer in the importing country. The second assumption is multilateral trade balance. Of course, this assumption has a long history in the pure theory of international trade, unlike the assumption of bilateral trade

differentiated by origin, preferences are CES, and market clearance holds.

balance implied by symmetric bilateral trade costs in the A-vW model. Even open-economy macroeconomics models assume *multilateral* trade balance in the long run. By assuming multilateral trade balance, we can address the endogeneity of prices raised by A-vW *without* assuming symmetric bilateral trade costs.

We show in this paper that replacing A-vW's endowment economy under symmetric bilateral trade costs with a Krugman IR-MC economy assuming only multilateral trade balance generates a theoretical foundation for the gravity equation where structural estimation of the model yields both unbiased coefficient estimates and *even more precisely* estimated general equilibrium comparative statics than A-vW's model – when bilateral trade costs are asymmetric. We demonstrate this in the context of a Monte Carlo analysis allowing either symmetric or asymmetric bilateral trade costs. Finally, we apply the approach in the context of two widely-recognized empirical examples with symmetric and asymmetric trade costs.

The remainder of this paper is as follows. Section 2 establishes the theoretical framework. Section 3 presents the Monte Carlo analysis. Section 4 provides empirical analyses. Section 5 concludes.

# 2 Gravity *Redux*

The purpose of this section is to show that the theoretical model of Krugman (1980), summarized in Baier and Bergstrand (2001) and Feenstra (2004, Ch. 5), generates a straightforward gravity equation for bilateral trade flows allowing for endogeneity of prices and GDPs without assuming symmetric bilateral trade costs.

## 2.1 Utility

Following Krugman (1980), Baier and Bergstrand (2001), and Feenstra (2004), there exists a single industry where preferences are constant-elasticity-of-substitution (CES).

We assume that utility of consumers in country $j$ is given by:

$$U_j = \left[ \sum_{i=1}^{N} \sum_{k=1}^{n_i} c_{ijk}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $c_{ijk}$ is the consumption of consumers in country $j$ of variety $k$ from country $i$, $n_i$ is the number of varieties of the single good produced in country $i$, which is endogenous in the model, and $N$ is the number of countries (or regions). Whereas A-vW assumed an exogenous (arbitrary) distribution parameter in their utility function, we assume (as typical in this Dixit-Stiglitz, 1977, class of models) that preferences are determined by the "love of variety".

As typical, we assume iceberg transport costs and symmetric firms within each country, and hence all products in country $i$ sell at the same price, $p_i$. Consequently, the utility function simplifies to:

$$U_j = \left[ \sum_{i=1}^{N} n_i c_{ij}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

Maximizing equation (2) subject to the budget constraint:

$$Y_j = \sum_{i=1}^{N} n_i p_i t_{ij} c_{ij}, \tag{3}$$

where $t_{ij}$ is one plus the iceberg trade costs (the latter a fraction) and $Y_i$ is national income, yields the demand functions:

$$c_{ij} = \left( \frac{p_i t_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j}, \tag{4}$$

where $P_j$ is the CES price index:

$$P_j = \left[ \sum_{i=1}^{N} n_i (p_i t_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{5}$$

As in Krugman (1980), Baier and Bergstrand (2001), and Feenstra (2004), the value of aggregate exports from country $i$ to country $j$, $X_{ij}$, equals $n_i p_i t_{ij} c_{ij}$. Substituting equation (4) into this expression for $X_{ij}$ yields:

$$X_{ij} = n_i Y_j \left( \frac{p_i t_{ij}}{P_j} \right)^{1-\sigma}, \tag{6}$$

which is identical to equation (5.26) in Feenstra (2004, p. 153).

## 2.2 Production: Alternative Assumption 1

The assumption of a monopolistically competitive market with increasing returns to scale in production (internal to the firm) and a single factor (labor) is sufficient to identify the exporting countries' number of varieties, cf., Krugman (1980), Baier and Bergstrand (2001), and Feenstra (2004). The representative firm in country $i$ is assumed to maximize profits subject to the workhorse linear cost function:

$$l_i = \alpha + \phi y_i, \tag{7}$$

where $l_i$ denotes labor used by the representative firm in country $i$ and $y_i$ denotes the output of the firm.

Two conditions characterize equilibrium in this class of models. First, profit maximization ensures that prices are a markup over marginal costs:

$$p_i = \frac{\sigma}{\sigma - 1} \phi w_i, \tag{8}$$

where $w_i$ is the wage rate in country $i$, determining the marginal cost of production.[2] Second, under monopolistic competition, zero economic profits in equilibrium ensures:

$$y_i = \frac{\alpha}{\phi}(\sigma - 1) \equiv \bar{y}, \tag{9}$$

so that the output of each firm is a constant, $\bar{y}$.

[2]The wage rate in country 1 serves as the numeraire.

An assumption of full employment of labor in each country ensures that the size of the exogenous factor endowment, $L_i$, determines the number of varieties:

$$n_i = \frac{L_i}{\alpha + \phi \bar{y}}. \tag{10}$$

We can now derive a gravity equation. First, we can show that the trade flow from $i$ to $j$ is a function of GDPs, labor endowments, and trade costs. With labor the only factor of production, $Y_i = w_i L_i$ or $w_i = Y_i/L_i$. Using equations (8) and (10), we can substitute $\sigma\phi w_i/(\sigma - 1)$ for $p_i$ in equation (6) and substitute $Y_i/L_i$ for $w_i$ in the resulting equation to yield:

$$X_{ij} = Y_i Y_j \frac{(Y_i/L_i)^{-\sigma} t_{ij}^{1-\sigma}}{\left[\sum_{k=1}^{N} Y_k (Y_k/L_k)^{-\sigma} t_{kj}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}. \tag{11}$$

However, we can easily show that equation (11) is identical to the gravity equation in Feenstra (2004) with GDPs and prices. Using equation (8), we can substitute $p_i/[(\sigma\phi)/(\sigma-1)]$ for $w_i$ in $L_i = Y_i/w_i$ and then substitute the resulting equation, $Y_i/[(\sigma - 1)p_i/(\sigma\phi)]$ , for $L_i$ in equation (10) to yield:

$$n_i = \gamma \frac{Y_i}{p_i}, \tag{12}$$

where $\gamma = \phi\sigma/[(\sigma - 1)(\alpha + \phi\bar{y})]$. Substituting equation (12) into equation (6) yields:

$$X_{ij} = \frac{Y_i Y_j p_i^{-\sigma} t_{ij}^{1-\sigma}}{\sum_{k=1}^{N} Y_k p_k^{-\sigma} t_{kj}^{1-\sigma}}. \tag{13}$$

which is identical to equation (5.26') in Feenstra (2004, p. 154).[3]

## 2.3 Multilateral Trade Balance: Alternative Assumption 2

Equation (13) is a standard representation of the gravity equation. Feenstra (2004) summarized the three methods that have been used up to this point in the literature to address the role of prices. The first approach, used in Bergstrand (1985, 1989) and Baier

[3] To see this, note that – using our notation – the denominator of (13) is identical to $\bar{y}\sum_{k=1}^{N} n_k (p_k t_{kj})^{1-\sigma}$.

and Bergstrand (2001), was to assume that prices are exogenous and use available price index data to account for the role of prices. This method is now acknowledged to work poorly for two reasons, the first is that conceptually such prices are endogenous and the second is that available price indexes are crude approximations. The second approach has been to account for the price terms using region-specific fixed effects. While such fixed effects can account for the influence of the price terms in estimation, the shortcoming of this method is that – without estimates of the prices before and after the counterfactual experiment – one cannot calculate the appropriate general equilibrium comparative statics using fixed effects (or method 1 above). The third method is to estimate a structural set of nonlinear price equations – under the assumption of symmetric bilateral trade costs (SBTC) – which then generate multilateral price terms before and after the counterfactual experiment to conduct finally the general equilibrium comparative statics, cf., A-vW (2003). While this approach provides unbiased estimates and general equilibrium comparative statics, it does so under the SBTC assumption, which also implies bilateral trade balance, cf., A-vW (2003, eq. 13) for $x_{ij}$ and $x_{ji}$. Both considerations are typically violated in the real world.

An alternative assumption, which has a long history in the pure theory of international trade, is to assume *multilateral* trade balance. While also violated in the real world, it is less restrictive than *bilateral* trade balance. Multilateral trade balance is ensured by assuming $N$ equations:

$$\sum_{j=1}^{N} X_{ij} = \sum_{j=1}^{N} X_{ji} \qquad i = 1, ..., N. \tag{14}$$

Hence, our gravity model is equations (11) subject to (14), analogous to A-vW's equations (12) and (13). Our $N(N-1)$ equations (11) along with $N$ equations (14) comprise a system of $N^2$ equations in $N(N-1)$ endogenous bilateral trade flows, $X_{ij}$ (excluding as in A-vW a country's internal trade), and $N$ GDPs, $Y_i$. However, unlike A-vW, we do not assume symmetric bilateral trade costs. Rather, we arrive at our system of equations using the Krugman IR-MC market structure to identify the preference parameter combined with the (less restrictive) multilateral trade balance assumption.[4]

## 2.4 Estimating Elasticities of Substitution and Comparative Statics

An important aspect of the gravity equation literature is going beyond just estimation of unbiased coefficient estimates (or the partial effects of trade costs); country fixed effects can be used to obtain unbiased bilateral trade cost parameter estimates. Rather, the unique feature of this literature is calculating general equilibrium comparative statics – and potentially welfare effects. A-vW (2003) went beyond estimation to compute comparative statics using actual and counterfactual MR terms. However, estimates of comparative static effects require an assumption regarding elasticities of substitution, because the elasticities could not be estimated, cf., A-vW (2001).

In our approach, the elasticities of substitution can be estimated. Given data on GDPs, populations and cif-fob factors and given estimates of trade cost parameters, then in our model minimizing the absolute values of the differences of exports and imports for all $N$ countries yields an estimate of the elasticity of substitution. These will be provided.[5] Consequently, the comparative static effects of trade-cost changes can be estimated using the estimated elasticities that surface from our approach. Using the estimated elasticities of substitution, we provide estimates of two comparative statics. One is the change in trade relative to the products of GDPs, $X_{ij}/(Y_i Y_j / Y_W)$. The other is the welfare effect due to a change in trade costs, based on the equivalent variation for country $i$ ($EV_i$), defined as:

$$EV_i = 100 \cdot \left[ \frac{Y_i^c}{Y_i} \left( \frac{\sum_{k=1}^{N} Y_k (Y_k/L_k)^{-\sigma} (t_{ki})^{1-\sigma}}{\sum_{k=1}^{N} Y_k^c (Y_k^c/L_k)^{-\sigma} (t_{ki}^c)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} - 1 \right], \tag{15}$$

[4] While the assumption of bilateral trade balance is very restrictive, some recent evidence that the assumption of multilateral trade balance is not very restrictive is found in Dekle, Eaton and Kortum (2007). In that paper, the authors use a calibrated general equilibrium model of world trade to consider how much would wage rates and prices have to change from current levels if all multilateral trade balances were eliminated (the counterfactual). The authors find that wage rates and prices do not change very much. For instance, elimination of China's and the United States' multilateral trade imbalances requires wage rate adjustments of less than 10 percent.

[5] Appendix A describes an alternative method; both approaches yield consistent estimates.

where superscript $c$ indicates counterfactual values of trade costs and GDP.

The remainder of our paper demonstrates our approach under both symmetric and asymmetric bilateral trade costs. In the following section, we provide a Monte Carlo analysis to demonstrate our approach relative to A-vW's (to avoid data measurement issues). Section 4 applies our approach to two widely-recognized empirical contexts.

# 3 Monte Carlo Analysis

To avoid data measurement issues, we conduct a large-scale Monte Carlo study to evaluate our approach relative to several alternatives: A-vW, a traditional OLS gravity specification without multilateral resistance terms (labeled, for brevity, OLS), and a recent linear-approximation approach suggested by Baier and Bergstrand (2006)(described below and referred to henceforth, for brevity, as BV-OLS).

The Monte Carlo analysis proceeds in two steps. First, we use alternative sets of parameter values described in detail below to generate sets of all endogenous variables of the theoretical model ($Y_i$, $p_i$, $w_i$, $n_i$, $X_{ij}$) as functions of the model's exogenous variables, $L_i$, and $t_{ij}$, in a baseline general equilibrium. Then, we change exogenous bilateral trade costs $t_{ij}$, holding the model parameters and all $L_i$ constant to obtain counterfactual values for all the endogenous variables.

In a second step, we use these generated general equilibrium data and add a stochastic error term as in traditional Monte Carlo studies.[6] The major advantage of this procedure is that the true parameters *and* the comparative static effects are known so that one can infer the biases of alternative estimation strategies and the consequent comparative statics in a "laboratory" setting.

For robustness, we consider three alternative configurations of the world to capture the typical contexts for gravity equations – analyzing world trade flows. We consider three world sizes of $N$ equal to 10, 20 and 40; this allows us to study the performance

[6] An additive log-linear error term is conventional to the general-equilibrium-based literature on gravity-model estimation, cf., Anderson and van Wincoop (2003). In particular, it seems to be a suitable assumption in the absence of zero trade flows, as in our application.

of alternative techniques for estimation and comparative statics as sample size increases. There are only three parameters in the theoretical model ($\sigma$, $\alpha$, and $\phi$); without loss of generality, we set the fixed cost ($\alpha$) and marginal cost ($\phi$) parameters equal to unity ($\alpha = \phi = 1$). However, we will consider three alternative values for the elasticity of substitution ($\sigma$) – 3, 5, and 10 – to allow us to study the role of "curvature" for estimation and comparative statics. Hence, with three alternative elasticity values and three alternative numbers of countries, we have nine alternative combinations of $N$ and $\sigma$. For each of these nine, we use 10 different draws from the set of empirical values for populations, $L_i$, and (observable) bilateral trade costs – bilateral cif-fob factors, $cf_{ij}$ – where we assume $t_{ij} = cf_{ij}^{\rho}$ , with $\rho$ denoting the "tariff-equivalent" parameter for $cf_{ij}$ which is assumed to be $\rho = 2$. Population endowments ($L_i$) are drawn from the empirical realizations of population data for the year 2003 across 207 economies covered by the World Bank's *World Development Indicators* (2005).[7] Bilateral cif-fob factors are drawn from the empirical realizations of the cif-fob factors in the 25th-75th percentiles of the distribution using the cif and fob bilateral trade flows from the International Monetary Fund's *Direction of Trade Statistics* (2003).[8] These data generate 90 (9 scenarios $\times$ 10 draws) alternative *baseline* equilibria of bilateral trade flows, GDPs, prices, wage rates and numbers of varieties consistent with general equilibrium (before any counterfactuals are introduced).

## 3.1 Symmetric Bilateral Trade Costs (SBTC)

Initially, we evaluate our approach ("Suggested model") relative to the approaches of A-vW, BV-OLS and traditional OLS under the case of symmetric bilateral trade costs. Hence, for each of the 90 alternative baseline equilibria, we ensure that the restriction $cf_{ij} = cf_{ji}$ (and, hence, $t_{ij} = t_{ji}$) holds in the draws from the empirical distribution. Also, we ensure the same restriction holds when altering the trade cost for the counterfactual exercise.

[7] Average population size across the 207 economies is $30,042,094$, the standard deviation is $119,909,488$, and the minimum and maximum are $20,000$ and $1,290,000,000$, respectively.

[8] The average cif-fob ratio is $\frac{1}{N(N-1)} \sum_i^N \sum_{j \neq i} cf_{ij} = 1.196$, the standard deviation of that measure is 0.067, and the corresponding minimum and maximum are 1.010 and 1.455, respectively.

We consider two alternative error structures in the Monte Carlo simulations. We assume that the error terms, $u_{ij}$, are given by $u_{ij} = \mu_i + \nu_j + \xi_{ij}$. We assume in all cases that $\xi_{ij}$ is normally distributed $\mathcal{N}(0, 0.35 s_\xi^2)$, where $s_\xi^2$ denotes the variance of $\xi_{ij}$. First, we assume that the error terms ($u_{ijt}$) are uncorrelated with the right-hand-side variables. In the tables, this error structure is labeled "uncorrelated". In this case, $\mu_i$ and $\nu_j$ are each distributed as $\mathcal{N}(0, 0.15 s_\xi^2)$. We made 2000 draws for the error terms under this structure. Second, we also consider an error structure where we know the $u_{ij}$ are correlated with the bilateral trade cost variable, $t_{ij}$. To do this, we define two terms, $\ln cf_i$ and $\ln cf_j$, where $\ln cf_i = (1/N) \sum_{j=1}^{N} \ln cf_{ij}$ and $\ln cf_j = (1/N) \sum_{i=1}^{N} \ln cf_{ij}$. To generate correlated error terms, in the second case we assume $\mu_i$ is distributed $\mathcal{N}(\ln cf_i, 0.15 s_\xi^2)$ and $\nu_j$ is distributed $\mathcal{N}(\ln cf_j, 0.15 s_\xi^2)$. In the tables, this error structure is labeled "correlated". We made $2,000$ draws for the error terms under this structure also.

Table 1 (assuming $\sigma = 5$) provides the Monte Carlo results for the alternative world configurations of 10, 20, and 40 countries. The table has three panels top to bottom corresponding to alternative configurations of 10, 20, and 40 countries, respectively. Each panel has four rows. The first row is the coefficient estimate of $\ln cf_{ij}$, $(1-\sigma)\rho$. The second row is the estimate of $\sigma$. The third row is the estimate of the effects on bilateral trade flows relative to GDP products, or "scaled trade flows," of changing trade costs exogenously. The fourth row is the change in welfare (measured by equivalent variation) of the same change in trade costs. Values in the third and fourth rows in each panel of these tables are the results of a change in trade costs represented by two random draws from the world distribution of cif-fob factors described earlier.[9] Since we do $20,000$ draws (10 draws from the empirical distribution of $L_i$ and $cf_{ij}$ times $2,000$ error-structure draws), we report only mean effects, their standard deviations, and their average absolute bias.

Table 1 has 12 columns. The first column provides the names of the four estimates of interest for each panel (corresponding to the four rows). The second column indi-

[9]Note that this implies that some country-pairs will have larger and others smaller trade barriers in the counterfactual situation than in the benchmark equilibrium. Moreover, the associated changes in trade costs are eventually quite large for some of the dyads.

cates the "true" values. For $(1-\sigma)\rho$ and $\sigma$, these are the true values specified a priori (hence, no standard deviation or bias is relevant). For the bilateral trade-flow-effect and equivalent-variation estimates, the "true" values are the means and standard deviations of the comparative statics in response to the change in trade costs based upon the calibrated general equilibrium model. The remaining columns $3-12$ present the estimates of the two parameters $(1-\sigma)\rho$ and, if retrievable, $\sigma$, and the two comparative-static effects (trade, welfare) using each of five alternative techniques, with each technique applied twice: first with our uncorrelated error structure (odd-numbered columns) and second with correlated errors (even-numbered columns). Columns (3) and (4) present the estimates using our "suggested" model. For consistent parameter estimates of $(1-\sigma)\rho$ in the first stage, we use fixed effects, as has become the standard in the literature. Given a consistent estimate of $(1-\sigma)\rho$, we then use this parameter estimate with the $N$ (nonlinear) multilateral trade balance equations to obtain estimates of $N$ GDPs, and then obtain estimates of $\sigma$ (see Appendix A). Using exogenous changes in $cf_{ij} = cf_{ji}$, we can then generate the counterfactual GDPs and trade flows to estimate the two comparative statics.

Columns (5) and (6) present the estimates using the A-vW technique.[10] In this case, we use the same "structural" (iterative) estimation technique as in A-vW, under both error structures, from which $N$ multilateral resistance terms are estimated. Then using exogenous changes in $cf_{ij} = cf_{ji}$, we can generate the counterfactual multilateral resistance terms and trade flows to estimate the scaled trade-flow comparative statics, given an assumed value of $\sigma$ (say, 5). Finally, one can estimate the equivalent variation based on the same set of assumptions. In the case of uncorrelated errors, coefficient estimates using fixed effects in the first stage will generate asymptotically identical parameter estimates of $(1-\sigma)\rho$ to A-vW; this is not the case for correlated errors.

Columns (7) and (8) present the estimates using one of two techniques described in Baier and Bergstrand (2006), referred to here as BV-OLS-1. Columns (9) and (10) present the estimates using the other of the two techniques described in Baier and Bergstrand

[10] Since $\sigma$ is unknown under A-vW, we assume (as in A-vW) a value for $\sigma$ of 5, irrespective of the true value of $\sigma$. So in this case, we expect A-vW to perform well.

(2006), referred to here as BV-OLS-2. Columns (11) and (12) present the estimates using the traditional OLS gravity equation ignoring the role of endogenous prices. Baier and Bergstrand (2006) present two techniques for estimating gravity equation parameters and comparative statics accounting for the endogenous price terms by using a first-order log-linear Taylor-series expansion of the nonlinear price equations. The method results in estimating the coefficients using a (reduced-form) gravity equation and calculating the MR terms without having to solve a structural system of nonlinear equations.

Table 1 provides the results for an elasticity of substitution of 5 (which is chosen *specifically* to correspond to the *assumed* $\sigma$). Several points are worth noting. First, when the true value of $(1-\sigma)\rho = -4$, our suggested approach (both error structures), A-vW (with uncorrelated errors), and BV-OLS-1 (both error structures) provide coefficient estimates for $cf_{ij}$ that are virtually identical to the true value (see all panels). Moreover, both our approach and BV-OLS-1 share the minimum average absolute bias. Both BV-OLS-2 and traditional OLS gravity equations have notably larger biases.[11] We note that these same *relative* results hold as sample size grows from 10 to 20 to 40 countries, although as expected absolute biases decline with $N$.

The second row of each panel provides estimates of the elasticity of substitution, but only for our approach. Across sample sizes of $N$ countries, our approach provides very accurate estimates of $\sigma$. Moreover, $\sigma$ cannot be estimated using the other approaches.

The third row provides estimates of the comparative-static effect on scaled trade flows of a common trade-cost change. The most notable result is that in all three panels our suggested approach provides the *lowest biases* for the general equilibrium trade-effect estimates. We note three further results. First, the trade-effect comparative-static biases for our suggested approach and for A-vW are not notably different; this is to be expected since – under assumed symmetric bilateral trade costs – the two approaches should yield similar results. Second, BV-OLS-2 biases are much smaller than BV-OLS-1 biases (or OLS biases), since the former uses a GDP-weighted approach whereas the latter does

[11] BV-OLS-1 tends to have less bias because it uses approximations around the "mean", consistent with least squares estimation, cf., Baier and Bergstrand (2006).

not. Third, while the comparative-static estimates using BV-OLS-2 are considerably higher than using either our suggested approach or A-vW, they are also considerably less than those from *ignoring* multilateral resistance terms – as is typically done by empirical researchers.

In the fourth row of each panel, we provide two pieces of information. For our approach, we use the *estimated* elasticities of substitution to generates *estimates* of welfare-effect comparative statics. These are very close to the true values, not surprisingly, since the elasticity estimates using our approach are precise. The second piece of information is that – assuming $\sigma = 5$ (as in A-vW) – A-vW estimates of the welfare effects are also accurate. Again, this is not surprising because these estimates are based upon *assuming* the true value of $\sigma$, 5.

For robustness, we also ran the same Monte Carlo analysis for true values of $\sigma$ of 3 and 10. These estimates are provided in Appendix Tables A1 and A2. For brevity, we note three key findings. Most importantly, the overall findings summarized above hold also for the cases of $\sigma = 3$ and $\sigma = 10$; the results are robust. However, two issues are worth noting. First, the estimated welfare effects using our approach are now considerably less biased than those using A-vW. There is a simple explanation. Our approach uses *estimated* values of $\sigma$, and our method generates $\sigma$ estimates very close to the true values. By contrast, A-vW welfare estimates use an *assumed* value of $\sigma$. If the assumption for $\sigma$ is incorrect – in both tables for A-vW, $\sigma$ is assumed to equal 5, as in A-vW – the estimated welfare effects are very biased. This is another advantage of our approach. Second, when $\sigma = 3$ or $\sigma = 10$, the trade-effect comparative-static estimates are slightly smaller using A-vW's approach than ours.

In summary, we note two important conclusions regarding the comparative-static estimates from this Monte Carlo analysis. First, under the *assumption* of symmetric bilateral trade costs, neither our approach nor A-vW provides trade-effect estimates that are economically different from the true values. But this is not surprising: under SBTC, A-vW should be efficient. However, under *asymmetric* bilateral trade costs, we will see that things change. Second, even under SBTC, our approach provides precise *estimates* of the true elasticity of substitution, so that our welfare-effect estimates are also very precise. By contrast, A-vW assume a value of $\sigma$, so that if the $\sigma$ assumption is considerably different from the true value, A-vW welfare-effect estimates will be considerably biased.

## 3.2 Asymmetric Bilateral Trade Costs (ABTC)

We performed the same set of Monte Carlo simulations as before except now we admit *asymmetric* bilateral trade costs in the draws from the empirical distributions for cif-fob factors. Every other aspect was identical in these simulations as before, including the alternative error structures, configurations of countries, and parameter values.

We summarize the results in Table 2 for the case of $\sigma = 5$ (where A-vW assume the correct value); similar results hold for the two other elasticities (not reported, for brevity). Moreover, for brevity we focus on the results for our approach versus A-vW, ignoring the results for the two BV-OLS techniques and traditional OLS. Also for brevity, we report the results only for uncorrelated errors; hence, columns (3) and (4) in Table 2 both provide estimates from our approach and A-vW's, respectively.

Several points are worth noting. First, our method provides unbiased estimates of the $(1-\sigma)\rho$ and $\sigma$ – even in the presence of ABTC. Second, the trade-effect comparative statics and EV estimates using our approach *always* have a lower bias than A-vW (even though A-vW assumes the true $\sigma$). Moreover, the biases using A-vW in the presence of asymmetric bilateral trade costs are always at least one order-of-magnitude greater than those using our approach. Third, for trade-effect estimates, the bias tends to increase (decrease) as the number of countries in the world increase for the A-vW (our) approach. Thus, our approach – with a Krugman market structure and multilateral trade balance condition – performs better overall in the presence of ABTC relative to the A-vW technique assuming SBTC.

The fifth column in Table 2 provides comparable estimates using A-vW's approach, but allowing *asymmetric* multilateral resistance (MR) terms.[12] Footnote 11 in A-vW

[12] Instead of solving a set of $N$ nonlinear price equations for $P_i$, we solve a set of $2N$ nonlinear price equations for $P_i$ and $\Pi_i$ according to equations (10) and (11) in A-vW.

cautions us about the potential pitfalls of using A-vW if indeed the MR terms ($P_i$ and $\Pi_i$ in A-vW) are asymmetric. They note:

> *There are many equilibria with asymmetric barriers that lead to the same equilibrium trade flows as with symmetric barriers, so that empirically they are impossible to distinguish. . . . Our analysis suggests that inferential identification of the asymmetry is problematic* (A-vW, 2003, p. 175).

The fifth column of Table 2 confirms this issue. When we run A-vW allowing for $2N$ price equations, the third row reveals that the estimated trade-effect estimates are grossly biased, even more biased than *assuming* symmetric bilateral trade costs as in A-vW.[13]

# 4 Empirical Evidence

We now apply our technique and A-vW's technique to actual trade flow data. We consider two popular contexts: the U.S.-Canadian "border puzzle" case and a traditional gravity-equation case of world trade flows in the presence of asymmetric trade costs (in particular, asymmetric bilateral tariff rates).

## 4.1 The U.S.-Canadian Border Puzzle

McCallum (1995) inspired a cottage industry of gravity-equation analysis of the effects of a national border on the trade of Canadian provinces and U.S. states, including the seminal A-vW (2003). This section has two parts. We re-estimate the same specifications addressed in that literature, initially assuming SBTC (as assumed there). In the second part, we assume *asymmetric* national border barriers for Canada and the United States.

[13] For completeness, we note that, in the case of ABTC, both BV-OLS techniques and OLS yielded biases more than the suggested approach, but less than A-vW's technique.

#### 4.1.1 Symmetric Canadian-U.S. National Border Barriers

In this section, we present the results of re-estimating the analysis of A-vW using their nonlinear estimation technique, fixed effects, and our approach. The results are presented in Table 3.

The first panel of Table 3 presents the coefficient estimates under the three alternative estimation procedures. First, we confirm in the second column of the first panel of Table 3 the A-vW (2003) structural estimates of $-0.79$ and $-1.65$. Second, we confirm the fixed effects estimates of $-1.25$ and $-1.55$ found in that study, which we know to be unbiased. Third, we show that our approach yields *identical* coefficient estimates to those using fixed effects. Our method avoids coefficient estimate bias introduced using the SBTC-based A-vW method on actual trade data (which likely suffers from "correlated" errors). Fourth, the table reminds one that our method also generates an estimate of $\sigma$ from the data. Our method implies an estimate of $\sigma$ equal to approximately 12. While such an estimate is at the higher end of the range of recent estimates of $\sigma$, we will show shortly that – by allowing for *asymmetric* Canadian and U.S. national border coefficient estimates – the estimate of $\sigma$ falls right in the middle of the range of recent cross-country estimates of $\sigma$.

The second panel of Table 3 summarizes the trade-effect comparative-static estimates for pairings of provinces-provinces, states-states, and provinces-states. The important conclusion to draw from this panel is that — under the restriction that the border effect is symmetric — the standard deviation of the trade effects of border barriers is high using all estimation procedures.

The third panel of Table 3 presents the welfare effects of symmetric border barriers. Note that our method generates much smaller welfare effects of national barriers than A-vW or fixed effects. The reason is that our approach estimated an elasticity of substitution of nearly 12, while the estimates using A-vW or fixed effects *assume* a much lower elasticity, 5.

The fourth, fifth and sixth panels provide further results regarding the estimated average MR terms and impacts of border barriers on trade. The key aspect to note is that our method provides virtually identical results to those implied using coefficient estimates based upon fixed effects (which are unbiased) and then calculating the "border effects" using the A-vW system of equations. Again, under SBTC, we would expect A-vW to work as well as our approach (using parameter estimates derived from fixed effects).

### 4.1.2 Asymmetric Canadian-U.S. National Border Barriers

Table 4 reports the empirical results under the more plausible assumption that U.S. and Canadian national borders have *asymmetric* effects on trade. We introduce separate dummy variables for a Canadian national border and a U.S. national border. The first panel indicates that, as in the previous case, our method provides identical parameter estimates for the trade-cost variable's coefficient to fixed effects. However, note that with ABTC, the estimate of the elasticity of substitution from our model is equal to 6.4. This value is well within the range of recent estimates of this elasticity using cross-section trade data, cf., Baier and Bergstrand (2001), Head and Ries (2001), and Anderson and van Wincoop (2004).

The second panel of Table 4 confirms that – under an assumption of ABTC – our method yields border barrier effects that are lower and have considerably lower standard deviations than using the A-vW approach which assumes SBTC.

The third panel of Table 4 provides welfare-effect estimates of border barriers. In the second panel, recall that A-vW and fixed effects assume an elasticity of 5, whereas our approach estimates the elasticity at 6.4. The lower welfare effects using our approach are partly explained by the higher value of $\sigma$. However, the other relevant factor is that our coefficient estimates are unbiased, like those using fixed effects.

## 4.2 World Trade Flows and Asymmetric Bilateral Tariffs

In this final substantive section, we apply our estimation procedure on the case of world trade flows, tariffs, and dummy variables from the GTAP data set for the year 2001. Thus, we apply our approach to the most common context for the gravity equation, world trade flows. GTAP provides a data base of world trade flows among 67 countries, asymmetric bilateral tariff rates, populations, and numerous dummy variables to conduct general equilibrium comparative statics. This data set provides an excellent context in order to examine the usefulness of our procedure.

We run a country-fixed-effects gravity equation on "scaled" bilateral trade flows including, on the right-hand-side, the log of the gross bilateral tariff rate (*Tar*), the log of bilateral distance (*Dist*), and dummy variables for common language (*Comlang*), contiguity (*Contig*), former colony (*Colony*), and common colonial heritage (*Comcol*), often included in gravity specifications. Then, we employ the $N$ multilateral trade balance conditions to conduct the trade-effect and welfare-effect comparative statics.

As conventional to the gravity equation literature, we assume that the log of the gross trade-cost variable, $(t)$, is a linear function of the log of the gross bilateral tariff rate, log bilateral distance, and various dummies:

$$\begin{aligned}(1-\sigma)\ln t_{ij} &= -\sigma\kappa \ln(Tar_{ij}) + (1-\sigma)\rho \ln Dist_{ij} + (1-\sigma)\ln \beta_1 Comlang_{ij} \\ &+ (1-\sigma)\ln \beta_2 Contig_{ij} + (1-\sigma)\ln \beta_3 Colony_{ij} \\ &+ (1-\sigma)\ln \beta_4 Comcol_{ij}.\end{aligned}$$

The parameter of log gross import tariffs of importer $j$ against exporter $i$, $-\sigma\kappa$, has two components. The use of $-\sigma$, rather than $1-\sigma$, reflects an assumption of tariff-revenue redistribution to consumers. The term $\kappa$ reflects the influence of measurement error of *de jure* tariff rates.[14]

Table 5 presents the results. The first panel in Table 5 provides the coefficient estimates from the first stage fixed-effects regression. We obtain plausible parameter estimates and statistically significant effects, as is typical in such specifications. The top panel reports an estimate of $\sigma$ of 7.38, which is plausible in a data-set based on aggregate data.

The second panel reports the "trade-effect" estimates from a complete elimination of

[14] The impact of high *de jure* tariffs tends to be dampened by the misclassification of goods. Hence, we would expect that $\kappa < 1$. Evidence on this issue has been provided by Fisman and Wei (2004) and, more recently, by Javorcik and Narciso (2007).

bilateral tariff rates. Not surprisingly, given the sizable negative shock on tariffs, the increase in trade on average is fairly large. Also the standard deviation of the effects is quite large relative to the mean effect, indicating that the variation in tariffs among the developed and the developing economies is quite big.

The third panel reports the welfare-effect estimate of a complete elimination of bilateral tariff rates. This elimination raises welfare by about 6.6 percent. Such an estimate – *based upon empirical evidence* – is not out-of-line with estimates generally provided by CGE computations (see Francois and Martin, 2007, for a recent survey). Thus, our empirical model provides a welfare effect quite in line with existing estimates based upon "theory with numbers".

# 5 Conclusions

Theoretical foundations for estimating gravity equations were enhanced recently in Anderson and van Wincoop (2003). Though elegant, the model assumes *symmetric* bilateral trade costs to generate an estimable set of structural equations. In reality, however, trade costs (and trade flows) are not bilaterally symmetric. We use the simple workhorse Krugman-type monopolistic-competition/increasing-returns-to-scale model of trade assuming only multilateral trade balance to allow for *asymmetric* bilateral trade costs. A Monte Carlo analysis of our general equilibrium model demonstrates – in the presence of asymmetric bilateral trade costs – that the bias of the Anderson-van Wincoop approach is at least an order-of-magnitude larger than that using our approach for computing general equilibrium comparative statics. We then confirm empirically the difference of our approach from the one of Anderson and van Wincoop in the Canadian-U.S. case allowing asymmetric effects of national borders. Finally, we demonstrate that our approach works in the more general case of world trade flows in the presence of asymmetric bilateral tariff rates.

# Appendix A

Using $t_{ij} = cf_{ij}^{\rho}$ in (11) results in:

$$X_{ij} = Y_i Y_j \frac{(Y_i/L_i)^{-\sigma} cf_{ij}^{(1-\sigma)\rho}}{\left[\sum_{k=1}^{N} Y_k (Y_k/L_k)^{-\sigma} cf_{kj}^{(1-\sigma)\rho}\right]}. \tag{16}$$

Estimation of equation (16) subject to the $N$ multilateral trade balance constraints (14), after substituting in (16) for $X_{ij}$ and its analogue for $X_{ji}$ into (14), yields parameter estimates for $\sigma$ and $(1-\sigma)\rho$ using a nonlinear estimation technique. However, in empirical applications it will only rarely be the case that this iterative approach will not be rejected against a fixed country effects model, because the trade cost variables are typically correlated with the country-specific error terms.

Alternatively, one may employ fixed country effects in the estimation of (16); this is what we employ in the paper. This approach will obtain a consistent estimate of $(1-\sigma)\rho$, irrespective of whether $cf_{ij}$ is correlated with the country-specific error terms or not. However, with fixed effects $\sigma$ can not be directly estimated but can be retrieved in the following way. Use equation (16) to determine relative aggregate bilateral demand of consumers in market $j$:[15]

$$\frac{X_{ij}}{X_{kj}} = \frac{Y_i}{Y_k} \left(\frac{Y_i/L_i}{Y_k/L_k}\right)^{-\sigma} \left(\frac{cf_{ij}^{\rho}}{cf_{kj}^{\rho}}\right)^{1-\sigma}. \tag{17}$$

Following Eaton and Kortum (2002), the latter obtains an alternative way of estimating the elasticity of substitution among varieties by using the expression:

$$\widehat{\sigma} = -\frac{1}{N^2(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k \neq j} \left[ \left( \ln \frac{X_{ij}}{X_{kj}} - \ln \frac{Y_i}{Y_k} - \ln \frac{\widehat{cf_{ij}^{(1-\sigma)\rho}}}{\widehat{cf_{kj}^{(1-\sigma)\rho}}} \right) / \ln \left( \frac{Y_i/L_i}{Y_k/L_k} \right) \right]. \tag{18}$$

[15] Of course, the approach is also applicable with more than a single trade cost variable. Then, $cf_{ij}^{\rho}$ is a single element in a product which is represented by $t_{ij}$ in the main text.

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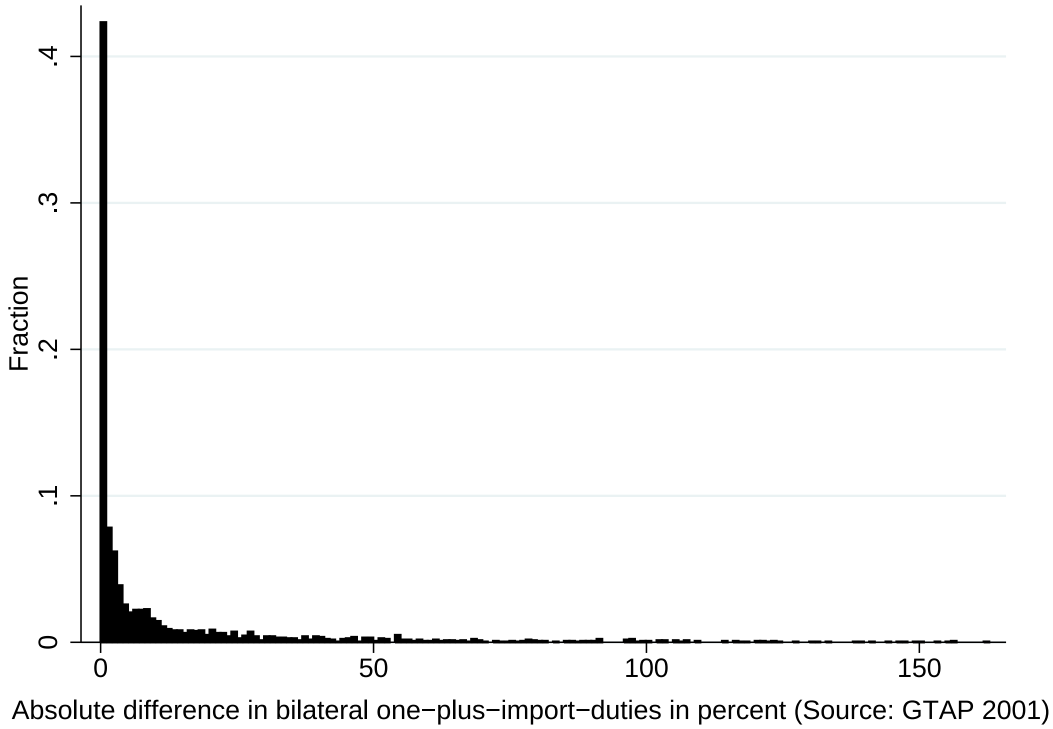


Figure 1: Asymmetry of bilateral import duties

Table 1: Monte Carlo results for model parameters, predicted trade flow changes and predicted welfare changes due to changing trade frictions in the case of a $\sigma = 5$

| Estimates | True | Suggested model | | A-vW | | BV-OLS 1 | | BV-OLS 2 | | OLS | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | | | | | 10-country-world, $\sigma = 5$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -8 | -8.0214 | -8.0354 | -8.0149 | -7.5514 | -8.0214 | -8.0354 | -7.7444 | -7.8193 | -7.6241 | -6.8319 |
| std. | - | 0.3232 | 0.3205 | 0.4738 | 0.5197 | 0.3232 | 0.3205 | 0.6343 | 0.6395 | 0.5502 | 0.6785 |
| bias | - | 3.2207 | 3.2069 | 4.6685 | 6.7944 | 3.2207 | 3.2069 | 6.6943 | 6.5789 | 6.7748 | 14.7083 |
| $\sigma$ | | | | | | | | | | | |
| mean | 5 | 5.0177 | 5.0264 | - | - | - | - | - | - | - | - |
| std. | - | 0.2487 | 0.2467 | - | - | - | - | - | - | - | - |
| bias | - | 3.7163 | 3.7071 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij}Y_w}{Y_iY_j}$ | | | | | | | | | | | |
| mean | 15.6348 | 15.7586 | 15.8197 | 15.7720 | 13.7475 | 14.6863 | 14.7388 | 20.9057 | 21.2467 | 21.2083 | 16.9485 |
| std. | 67.7946 | 68.2929 | 68.4372 | 68.4578 | 62.1077 | 63.0199 | 63.1647 | 81.6642 | 82.1467 | 73.8767 | 61.3019 |
| bias | - | 1.8482 | 1.8396 | 2.5982 | 3.7948 | 19.0619 | 19.0723 | 22.7052 | 22.8996 | 26.4978 | 24.1339 |
| $EV_i$ | | | | | | | | | | | |
| mean | 1.7030 | 1.7011 | 1.6972 | 1.7038 | 1.6365 | - | - | - | - | - | - |
| std. | 9.1959 | 9.1915 | 9.1818 | 9.1918 | 9.0527 | - | - | - | - | - | - |
| bias | - | 0.2234 | 0.2213 | 0.1483 | 0.2054 | - | - | - | - | - | - |
| | | | | | 20-country-world, $\sigma = 5$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -8 | -8.0107 | -8.0087 | -8.0107 | -7.6167 | -8.0107 | -8.0087 | -7.8510 | -7.7691 | -7.7905 | -7.2060 |
| std. | - | 0.1558 | 0.1559 | 0.2570 | 0.2333 | 0.1558 | 0.1559 | 0.2712 | 0.2235 | 0.2655 | 0.3237 |
| bias | - | 1.5518 | 1.5545 | 2.5415 | 4.8903 | 1.5518 | 1.5545 | 3.0960 | 3.2839 | 3.5705 | 9.9623 |
| $\sigma$ | | | | | | | | | | | |
| mean | 5 | 5.0092 | 5.0078 | - | - | - | - | - | - | - | - |
| std. | - | 0.1249 | 0.1248 | - | - | - | - | - | - | - | - |
| bias | - | 1.8199 | 1.8232 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij}Y_w}{Y_iY_j}$ | | | | | | | | | | | |
| mean | 23.3920 | 23.4703 | 23.4569 | 23.4934 | 21.2079 | 19.0991 | 19.0883 | 26.1341 | 25.4988 | 24.3130 | 20.7552 |
| std. | 90.5346 | 90.8372 | 90.7944 | 90.9789 | 83.7935 | 80.1990 | 80.1646 | 96.5163 | 94.8488 | 84.9751 | 75.3124 |
| bias | - | 1.1651 | 1.1674 | 1.8146 | 3.3970 | 17.9046 | 17.9053 | 14.7770 | 14.2352 | 22.2170 | 21.7635 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.8988 | 0.8984 | 0.8986 | 0.8997 | 0.8486 | - | - | - | - | - | - |
| std. | 7.2242 | 7.2194 | 7.2202 | 7.2294 | 7.0372 | - | - | - | - | - | - |
| bias | - | 0.0788 | 0.0789 | 0.0903 | 0.1692 | - | - | - | - | - | - |
| | | | | | 40-country-world, $\sigma = 5$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -8 | -8.0029 | -8.0026 | -8.0106 | -7.8565 | -8.0029 | -8.0026 | -7.7537 | -7.7207 | -7.7949 | -7.4301 |
| std. | - | 0.0764 | 0.0776 | 0.1359 | 0.1305 | 0.0764 | 0.0776 | 0.1944 | 0.2150 | 0.1205 | 0.1547 |
| bias | - | 0.7633 | 0.7733 | 1.3514 | 2.0033 | 0.7633 | 0.7733 | 3.2803 | 3.5994 | 2.5981 | 7.1239 |
| $\sigma$ | | | | | | | | | | | |
| mean | 5 | 5.0008 | 5.0003 | - | - | - | - | - | - | - | - |
| std. | - | 0.0780 | 0.0789 | - | - | - | - | - | - | - | - |
| bias | - | 1.0884 | 1.1039 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij}Y_w}{Y_iY_j}$ | | | | | | | | | | | |
| mean | 23.8155 | 23.8488 | 23.8473 | 23.8913 | 22.944 | 19.2797 | 19.2779 | 25.0291 | 24.7589 | 18.9762 | 17.066 |
| std. | 89.7503 | 89.8462 | 89.841 | 90.0318 | 87.0564 | 78.7351 | 78.731 | 92.7942 | 91.6696 | 78.7336 | 73.0796 |
| bias | - | 0.71597 | 0.726 | 0.9867 | 1.4566 | 20.6474 | 20.6487 | 11.3734 | 11.2878 | 23.504 | 23.5699 |
| $EV_i$ | | | | | | | | | | | |
| mean | -0.0880 | -0.0906 | -0.0909 | -0.0878 | -0.0941 | - | - | - | - | - | - |
| std. | 7.7841 | 7.7875 | 7.7880 | 7.7914 | 7.6844 | - | - | - | - | - | - |
| bias | - | 0.0658 | 0.0668 | 0.0587 | 0.0878 | - | - | - | - | - | - |

Table 2: Monte Carlo results for model parameters, predicted trade flow changes and predicted welfare changes due to changing trade frictions for asymmetric trade barriers

| Estimates | True | Suggested model uncorr. | A-vW uncorr. | Asymmetric MR terms uncorr. | BV-OLS 1 uncorr. | BV-OLS 2 uncorr. | OLS uncorr. |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | | | | 10-country-world, $\sigma = 5$ | | | |
| $(1-\sigma)\rho$ | | | | | | | |
| mean | -8 | -8.0109 | -7.5044 | -7.5044 | -8.0109 | -7.7438 | -7.3640 |
| std. | - | 0.3367 | 0.7253 | 0.7253 | 0.3367 | 0.5553 | 0.6564 |
| bias | - | 3.3200 | 8.2477 | 8.2477 | 3.3200 | 6.0877 | 9.1272 |
| $\sigma$ | | | | | | | |
| mean | 5 | 5.0023 | - | - | - | - | - |
| std. | - | 0.1568 | - | - | - | - | - |
| bias | - | 0.4920 | - | - | - | - | - |
| $\Delta \frac{X_{ij} y_w}{y_i y_j}$ | | | | | | | |
| mean | 22.3160 | 22.4589 | 24.1205 | 215.4520 | 15.3214 | 31.2530 | 13.3109 |
| std. | 86.8816 | 87.5805 | 94.3536 | 238.2345 | 70.2269 | 104.7547 | 69.6579 |
| bias | - | 2.9881 | 71.9292 | 214.0877 | 27.0730 | 19.6068 | 32.7325 |
| $EV_i$ | | | | | | | |
| mean | -0.5725 | -0.6139 | -0.2673 | 1.2564 | - | - | - |
| std. | 8.1963 | 8.3581 | 11.0434 | 27.4305 | - | - | - |
| bias | - | 0.2318 | 4.1761 | 19.4047 | - | - | - |
| | | | | 20-country-world, $\sigma = 5$ | | | |
| $(1-\sigma)\rho$ | | | | | | | |
| mean | -8 | -8.0046 | -7.4694 | -7.4694 | -8.0046 | -7.6565 | -7.6610 |
| std. | - | 0.1661 | 0.3396 | 0.3396 | 0.1661 | 0.3692 | 0.3096 |
| bias | - | 1.6546 | 6.8180 | 6.8180 | 1.6546 | 5.0570 | 4.8475 |
| $\sigma$ | | | | | | | |
| mean | 5 | 5.0022 | - | - | - | - | - |
| std. | - | 0.0938 | - | - | - | - | - |
| bias | - | 1.4822 | - | - | - | - | - |
| $\Delta \frac{X_{ij} y_w}{y_i y_j}$ | | | | | | | |
| mean | 26.8187 | 26.8728 | 20.4185 | 245.8379 | 18.4036 | 32.1303 | 16.6670 |
| std. | 96.5832 | 96.7879 | 85.7540 | 248.5817 | 75.4258 | 106.7482 | 75.6696 |
| bias | - | 1.3659 | 82.2296 | 245.4074 | 30.7439 | 16.1918 | 33.5216 |
| $EV_i$ | | | | | | | |
| mean | -0.4462 | -0.4681 | 0.4835 | 4.2408 | - | - | - |
| std. | 7.6639 | 7.5996 | 10.4687 | 27.1873 | - | - | - |
| bias | - | 0.0523 | 4.3126 | 19.5352 | - | - | - |
| | | | | 40-country-world, $\sigma = 5$ | | | |
| $(1-\sigma)\rho$ | | | | | | | |
| mean | -8 | -8.0018 | -7.6168 | -7.6168 | -8.0018 | -7.7064 | -7.7387 |
| std. | - | 0.0774 | 0.1813 | 0.1813 | 0.0774 | 0.3294 | 0.1164 |
| bias | - | 0.7717 | 4.8089 | 4.8089 | 0.7717 | 3.9873 | 3.2858 |
| $\sigma$ | | | | | | | |
| mean | 5 | 5.0011 | - | - | - | - | - |
| std. | - | 0.0476 | - | - | - | - | - |
| bias | - | 0.7560 | - | - | - | - | - |
| $\Delta \frac{X_{ij} y_w}{y_i y_j}$ | | | | | | | |
| mean | 22.1344 | 22.1512 | 25.1593 | 268.1988 | 19.3311 | 22.7766 | 18.6412 |
| std. | 89.3596 | 89.4200 | 88.2318 | 253.0506 | 79.2633 | 90.3994 | 78.6322 |
| bias | - | 0.5848 | 81.5282 | 264.1394 | 21.9378 | 10.9780 | 25.0804 |
| $EV_i$ | | | | | | | |
| mean | 0.1282 | 0.1278 | -0.4662 | 3.3475 | - | - | - |
| std. | 5.7528 | 5.7523 | 7.7350 | 29.3336 | - | - | - |
| bias | - | 0.0200 | 3.8352 | 22.5496 | - | - | - |

Table 3: Estimation results for the A-vW data-set

| (1) | A-vW (2) | Fixed Effects (3) | Suggested model (4) |
|---|---|---|---|
| Parameters | | | |
| $(1-\sigma)\rho$ | -0.788 | -1.252 | -1.252 |
| | (0.032) | (0.037) | (0.037) |
| $(1-\sigma)\ln b_{US,CA}$ | -1.646 | -1.551 | -1.551 |
| | (0.077) | (0.059) | (0.059) |
| $\sigma$ | – | – | 11.892 |
| $R^2$ | 0.435 | 0.664 | 0.664 |
| $\sigma^2$ | 1.062 | 0.841 | 0.841 |
| **Trade effects of border barrier abolition** | | | |
| Overall | | | |
| mean | 42.806 | 61.388 | 72.157 |
| min | -82.823 | -71.685 | -64.820 |
| max | 211.487 | 277.512 | 293.530 |
| std | 79.300 | 99.307 | 112.366 |
| Intra-US trade | | | |
| mean | 5.924 | 10.430 | 58.541 |
| min | -51.746 | -35.178 | -20.036 |
| max | 208.110 | 274.571 | 292.052 |
| std | 39.599 | 52.280 | 103.419 |
| Intra-CA trade | | | |
| mean | 19.055 | 51.503 | 67.459 |
| min | -82.823 | -71.685 | -63.599 |
| max | 209.755 | 263.328 | 293.530 |
| std | 102.968 | 116.319 | 136.396 |
| Inter trade | | | |
| mean | 98.533 | 134.879 | 92.108 |
| min | -82.823 | -71.685 | -64.820 |
| max | 211.487 | 277.512 | 293.530 |
| std | 84.758 | 101.426 | 117.374 |
| **Welfare effects of border barrier abolition (equivalent variation)** | | | |
| Overall | 10.984 | 6.660 | 1.847 |
| US | 1.428 | 1.179 | 0.541 |
| CA | 39.654 | 23.105 | 5.765 |
| **Average of $P^{(1-\sigma)}$** | | | |
| With border barrier (BB) | | | |
| US | 0.773 | 0.530 | 0.530 |
| | (0.015) | (0.020) | (0.020) |
| CA | 2.451 | 1.787 | 1.787 |
| | (0.060) | (0.062) | (0.062) |
| Borderless trade (NB) | | | |
| US | 0.754 | 0.519 | 0.518 |
| | (0.015) | (0.019) | (0.019) |
| CA | 1.179 | 1.136 | 1.147 |
| | (0.004) | (0.013) | (0.013) |
| Ratio (BB/NB) | | | |
| US | 1.025 | 1.022 | 1.022 |
| | (0.001) | (0.001) | (0.001) |
| CA | 2.079 | 1.573 | 1.558 |
| | (0.057) | (0.037) | (0.036) |
| **Impact of border barriers on bilateral trade** | | | |
| Ratio (BB/NB) | | | |
| US-US | 1.050 | 1.044 | 1.045 |
| | (0.002) | (0.002) | (0.002) |
| CA-CA | 4.321 | 2.475 | 2.428 |
| | (0.237) | (0.115) | (0.113) |
| US-CA | 0.411 | 0.341 | 0.322 |
| | (0.023) | (0.018) | (0.017) |
| Due to bilateral resistance | | | |
| US-US | 1.000 | 1.000 | 1.000 |
| | (0.000) | (0.000) | (0.000) |
| CA-CA | 1.000 | 1.000 | 1.000 |
| | (0.000) | (0.000) | (0.000) |
| US-CA | 0.193 | 0.212 | 0.202 |
| | (0.015) | (0.012) | (0.012) |
| Due to multilateral resistance | | | |
| US-US | 1.050 | 1.044 | 1.045 |
| | (0.002) | (0.002) | (0.002) |
| CA-CA | 4.321 | 2.475 | 2.428 |

Table 3 continued on next page

Table 3 continued from previous page

| (1) | A-vW (2) | Fixed Effects (3) | Suggested model (4) |
|---|---|---|---|
| | (0.237) | (0.115) | (0.113) |
| US-CA | 2.130 | 1.608 | 1.592 |
| | (0.060) | (0.039) | (0.038) |

Impact of border barriers on intranational trade relative to international trade

| | | | |
|---|---|---|---|
| Theoretically consistent estimate | | | |
| US | 2.557 | 3.061 | 3.249 |
| | (0.146) | (0.159) | (0.162) |
| CA | 10.524 | 7.258 | 7.551 |
| | (1.048) | (0.526) | (0.520) |
| McCallum parameter implied by theory | | | |
| US | 1.635 | 1.398 | 1.469 |
| | (0.103) | (0.074) | (0.074) |
| CA | 16.455 | 15.899 | 16.705 |
| | (1.485) | (1.042) | (1.042) |

Table 4: Estimation results for the A-vW data-set with asymmetric border barriers

| (1) | A-vW (2) | Fixed Effects (3) | Suggested model (4) |
|---|---|---|---|
| Parameters | | | |
| $(1-\sigma)\rho$ | -0.792 | -1.252 | -1.252 |
| | (0.032) | (0.037) | (0.037) |
| $(1-\sigma)\ln b_{US}$ | -1.378 | -0.470 | -0.470 |
| | (0.086) | (0.046) | (0.046) |
| $(1-\sigma)\ln b_{CA}$ | -1.808 | -0.825 | -0.825 |
| | (0.105) | (0.047) | (0.047) |
| $\sigma$ | − | − | 6.340 |
| $R^2$ | 0.444 | 0.664 | 0.664 |
| $\sigma^2$ | 1.053 | 0.841 | 0.841 |
| **Trade effects of border barrier abolition** | | | |
| Overall | | | |
| mean | 45.794 | 81.561 | 16.097 |
| min | -79.421 | -8.429 | -45.156 |
| max | 281.862 | 221.258 | 143.273 |
| std | 88.427 | 54.886 | 44.586 |
| Intra-US trade | | | |
| mean | 2.164 | 55.093 | -0.081 |
| min | -48.986 | 26.220 | -11.254 |
| max | 145.580 | 124.129 | 19.469 |
| std | 27.978 | 13.579 | 5.808 |
| Intra-CA trade | | | |
| mean | 47.351 | 88.898 | 30.282 |
| min | -79.421 | -8.429 | -44.184 |
| max | 279.564 | 220.568 | 143.273 |
| std | 130.059 | 85.701 | 74.233 |
| Inter trade | | | |
| mean | 107.186 | 117.829 | 36.782 |
| min | -79.421 | -8.429 | -45.1556 |
| max | 281.862 | 221.258 | 142.782 |
| std | 99.553 | 63.499 | 58.153 |
| **Welfare effects of border barrier abolition (equivalent variation)** | | | |
| Overall | 9.978 | 2.970 | 1.740 |
| US | 1.543 | 0.795 | 0.767 |
| CA | 35.281 | 9.493 | 4.660 |

Table 5: Estimation results for the GTAP data-set

| (1) | Suggested model (2) |
|---|---|
| Parameters | |
| $-\sigma\kappa$ | -1.865 |
| | (0.063) |
| $(1-\sigma)\rho$ | -1.056 |
| | (0.029) |
| $(1-\sigma)\ln b_{\text{comlang}}$ | 0.442 |
| | (0.079) |
| $(1-\sigma)\ln b_{\text{contig}}$ | 0.658 |
| | (0.135) |
| $(1-\sigma)\ln b_{\text{colony}}$ | 0.416 |
| | (0.125) |
| $(1-\sigma)\ln b_{\text{comcol}}$ | 0.284 |
| | (0.157) |
| $\sigma$ | 7.380 |
| $R^2$ | 0.861 |
| $\sigma^2$ | 1.089 |
| **Trade effects of world-wide abolition of import duties** | |
| Overall | |
| mean | 167.594 |
| min | -94.702 |
| max | 78921.000 |
| std | 2293.100 |
| **Welfare effects of world-wide abolition of import duties (equivalent variation)** | |
| Overall | 6.611 |

Table A1: Monte Carlo results for model parameters, predicted trade flow changes and predicted welfare changes due to changing trade frictions in the case of a $\sigma = 3$

| Estimates | True | Suggested model | | A-vW | | BV-OLS 1 | | BV-OLS 2 | | OLS | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | 10-country-world, $\sigma = 3$ | | | | | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -4 | -4.0136 | -4.0194 | -4.0119 | -4.0035 | -4.0136 | -4.0194 | -3.9321 | -4.0462 | -3.7571 | -3.2041 |
| std. | - | 0.1698 | 0.1690 | 0.3167 | 0.2881 | 0.1698 | 0.1690 | 0.3235 | 0.3411 | 0.2799 | 0.3539 |
| bias | - | 3.3738 | 3.3803 | 6.2824 | 5.8070 | 3.3738 | 3.3803 | 6.5394 | 7.0723 | 7.5737 | 19.9271 |
| $\sigma$ | | | | | | | | | | | |
| mean | 3 | 3.0080 | 3.0129 | - | - | - | - | - | - | - | - |
| std. | - | 0.1153 | 0.1153 | - | - | - | - | - | - | - | - |
| bias | - | 3.0183 | 3.0335 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 6.6662 | 6.7489 | 6.7416 | 6.6014 | 6.8860 | 3.5120 | 3.5190 | 8.2709 | 9.3040 | 3.2435 | 2.3500 |
| std. | 36.9948 | 37.2688 | 37.2623 | 37.1356 | 38.1079 | 27.7128 | 27.7392 | 41.0057 | 44.4533 | 27.8818 | 23.5375 |
| bias | - | 0.9934 | 0.9887 | 1.6633 | 1.5481 | 15.2644 | 15.2651 | 4.4485 | 5.3880 | 17.8267 | 17.6405 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.2351 | 0.2304 | 0.2317 | -0.0611 | -0.0875 | - | - | - | - | - | - |
| std. | 14.1707 | 14.1729 | 14.1630 | 7.4562 | 7.5313 | - | - | - | - | - | - |
| bias | - | 0.1355 | 0.1352 | 4.3464 | 4.3028 | - | - | - | - | - | - |
| | 20-country-world, $\sigma = 3$ | | | | | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -4 | -4.0034 | -4.0043 | -4.0033 | -3.8925 | -4.0034 | -4.0043 | -3.9528 | -3.9859 | -3.7878 | -3.1911 |
| std. | - | 0.0787 | 0.0785 | 0.1926 | 0.1841 | 0.0787 | 0.0785 | 0.2024 | 0.1871 | 0.1631 | 0.2800 |
| bias | - | 1.5676 | 1.5565 | 3.7673 | 4.3525 | 1.5676 | 1.5565 | 4.0657 | 3.7943 | 5.5451 | 20.2220 |
| $\sigma$ | | | | | | | | | | | |
| mean | 3 | 3.0032 | 3.0044 | - | - | - | - | - | - | - | - |
| std. | - | 0.0926 | 0.0919 | - | - | - | - | - | - | - | - |
| bias | - | 2.1690 | 2.1483 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 5.4797 | 5.4923 | 5.4933 | 5.4408 | 5.1722 | 4.4173 | 4.4194 | 6.0417 | 6.0668 | 3.7566 | 2.5930 |
| std. | 35.3275 | 35.3767 | 35.3852 | 35.4044 | 34.3500 | 31.3472 | 31.3551 | 36.6611 | 36.8558 | 31.1362 | 25.9515 |
| bias | - | 0.4772 | 0.4750 | 1.0460 | 1.1888 | 11.3259 | 11.3260 | 3.7866 | 3.7163 | 13.1648 | 13.4517 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.1196 | 0.1184 | 0.1187 | -0.0231 | -0.0290 | - | - | - | - | - | - |
| std. | 9.8557 | 9.8611 | 9.8581 | 5.2396 | 5.1146 | - | - | - | - | - | - |
| bias | - | 0.1346 | 0.1339 | 3.4358 | 3.5203 | - | - | - | - | - | - |
| | 40-country-world, $\sigma = 3$ | | | | | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -4 | -4.0021 | -4.0017 | -4.0011 | -3.9242 | -4.0021 | -4.0017 | -3.9462 | -3.9459 | -3.8842 | -3.5452 |
| std. | - | 0.0380 | 0.0382 | 0.0868 | 0.0997 | 0.0380 | 0.0382 | 0.0975 | 0.0972 | 0.0694 | 0.1030 |
| bias | - | 0.7595 | 0.7626 | 1.7105 | 2.6967 | 0.7595 | 0.7626 | 2.1731 | 2.2230 | 2.9590 | 11.3697 |
| $\sigma$ | | | | | | | | | | | |
| mean | 3 | 3.0016 | 3.0013 | - | - | - | - | - | - | - | - |
| std. | - | 0.0286 | 0.0288 | - | - | - | - | - | - | - | - |
| bias | - | 0.7539 | 0.7585 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 4.7364 | 4.7431 | 4.7416 | 4.6834 | 4.5225 | 4.4345 | 4.4335 | 4.7299 | 4.7390 | 4.5261 | 3.7749 |
| std. | 33.7559 | 33.7794 | 33.7748 | 33.7530 | 33.0860 | 31.4914 | 31.4874 | 34.1411 | 34.1706 | 31.2879 | 28.2547 |
| bias | - | 0.2050 | 0.2058 | 0.4662 | 0.7302 | 9.0524 | 9.0524 | 2.2595 | 2.2712 | 9.8183 | 9.9459 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.4752 | 0.4748 | 0.4749 | 0.1975 | 0.1907 | - | - | - | - | - | - |
| std. | 7.0784 | 7.0774 | 7.0777 | 3.7986 | 3.7388 | - | - | - | - | - | - |
| bias | - | 0.0190 | 0.0191 | 2.5146 | 2.5626 | - | - | - | - | - | - |

Table A2: Monte Carlo results for model parameters, predicted trade flow changes and predicted welfare changes due to changing trade frictions in the case of a $\sigma = 10$

| Estimates | True | Suggested model | | A-vW | | BV-OLS 1 | | BV-OLS 2 | | OLS | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. | uncorr. | corr. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | | | | | 10-country-world, $\sigma = 10$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -18 | -18.1151 | -18.0950 | -18.0302 | -17.3963 | -18.1151 | -18.0950 | -16.4232 | -16.3901 | -18.0739 | -17.4395 |
| std. | - | 0.7788 | 0.7756 | 0.8402 | 0.7863 | 0.7788 | 0.7756 | 2.6003 | 2.5866 | 1.0715 | 1.1069 |
| bias | - | 3.4823 | 3.4579 | 3.7112 | 4.4491 | 3.4823 | 3.4579 | 13.1853 | 13.1519 | 4.7878 | 5.4869 |
| $\sigma$ | | | | | | | | | | | |
| mean | 10 | 10.0318 | 10.0207 | - | - | - | - | - | - | - | - |
| std. | - | 0.5865 | 0.5889 | - | - | - | - | - | - | - | - |
| bias | - | 4.1089 | 4.0843 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 123.3248 | 127.7351 | 127.5996 | 125.6036 | 113.0115 | 91.1245 | 90.9670 | 227.5535 | 233.1863 | 129.9514 | 116.2394 |
| std. | 469.3185 | 494.9210 | 495.4916 | 485.1469 | 432.9379 | 293.4474 | 293.1617 | 1128.8546 | 1204.7017 | 521.3890 | 453.9210 |
| bias | - | 17.5295 | 17.4202 | 15.3387 | 16.9658 | 74.4546 | 74.4805 | 214.5082 | 219.7298 | 92.7117 | 85.5440 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.1390 | 0.1034 | 0.1056 | 0.9072 | 0.9780 | - | - | - | - | - | - |
| std. | 7.0641 | 7.0674 | 7.0776 | 15.5048 | 15.6467 | - | - | - | - | - | - |
| bias | - | 0.3054 | 0.3043 | 5.4158 | 5.6047 | - | - | - | - | - | - |
| | | | | | 20-country-world, $\sigma = 10$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -18 | -18.0253 | -18.0327 | -18.0037 | -17.5787 | -18.0253 | -18.0327 | -15.6098 | -15.5545 | -17.6570 | -17.0613 |
| std. | - | 0.3721 | 0.3759 | 0.4886 | 0.5454 | 0.3721 | 0.3759 | 2.1116 | 2.0400 | 0.6485 | 0.6821 |
| bias | - | 1.6554 | 1.6777 | 2.1710 | 3.1475 | 1.6554 | 1.6777 | 14.1517 | 13.9282 | 3.3918 | 5.6584 |
| $\sigma$ | | | | | | | | | | | |
| mean | 10 | 10.0001 | 9.9912 | - | - | - | - | - | - | - | - |
| std. | - | 0.7965 | 0.7933 | - | - | - | - | - | - | - | - |
| bias | - | 4.3285 | 4.3509 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 137.9529 | 139.7014 | 139.9970 | 139.2621 | 129.7217 | 129.4379 | 129.5752 | 136.3740 | 133.9121 | 139.7465 | 125.7076 |
| std. | 509.0532 | 516.9189 | 517.5073 | 516.0786 | 476.7905 | 471.1198 | 471.2312 | 531.6067 | 509.3490 | 536.1079 | 474.6907 |
| bias | - | 11.2513 | 11.3483 | 10.0604 | 13.7876 | 80.4346 | 80.4387 | 116.1598 | 114.3426 | 79.3370 | 76.5853 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.5568 | 0.5251 | 0.5120 | 1.6953 | 1.6870 | - | - | - | - | - | - |
| std. | 6.5455 | 7.0969 | 6.6141 | 14.3039 | 14.3381 | - | - | - | - | - | - |
| bias | - | 0.2931 | 0.2868 | 5.5801 | 5.6378 | - | - | - | - | - | - |
| | | | | | 40-country-world, $\sigma = 10$ | | | | | | |
| $(1-\sigma)\rho$ | | | | | | | | | | | |
| mean | -18 | -18.0035 | -18.0024 | -17.7132 | -17.4493 | -18.0035 | -18.0024 | -15.4103 | -15.4223 | -17.5999 | -17.2310 |
| std. | - | 0.1831 | 0.1838 | 0.3260 | 0.3568 | 0.1831 | 0.1838 | 0.8177 | 0.8364 | 0.3311 | 0.3615 |
| bias | - | 0.8119 | 0.8141 | 1.9529 | 3.1321 | 0.8119 | 0.8141 | 14.3872 | 14.3203 | 2.3740 | 4.2745 |
| $\sigma$ | | | | | | | | | | | |
| mean | 10 | 9.9974 | 10.0029 | - | - | - | - | - | - | - | - |
| std. | - | 1.0969 | 1.0989 | - | - | - | - | - | - | - | - |
| bias | - | 5.5566 | 5.5380 | - | - | - | - | - | - | - | - |
| $\Delta \frac{X_{ij} Y_w}{Y_i Y_j}$ | | | | | | | | | | | |
| mean | 192.7730 | 196.6792 | 196.5124 | 185.2525 | 177.1900 | 154.7657 | 154.7352 | 177.6109 | 177.6029 | 160.0134 | 149.9590 |
| std. | 714.2798 | 736.2606 | 735.6959 | 686.9376 | 652.4061 | 574.5780 | 574.4365 | 787.0798 | 792.4892 | 574.2420 | 532.3568 |
| bias | - | 19.2990 | 19.1996 | 11.3912 | 17.6107 | 112.1997 | 112.2014 | 146.8040 | 146.9326 | 111.2852 | 111.3648 |
| $EV_i$ | | | | | | | | | | | |
| mean | 0.2995 | 0.1826 | 0.1867 | 1.2536 | 1.2613 | - | - | - | - | - | - |
| std. | 7.4031 | 7.6870 | 7.6855 | 16.2158 | 16.1786 | - | - | - | - | - | - |
| bias | - | 0.5404 | 0.5367 | 6.5144 | 6.5101 | - | - | - | - | - | - |