Multi-Product Firms and Flexible Manufacturing in the Global Economy*

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April 8, 2009

Abstract

We present a new model of multi-product firms (MPFs) and flexible manufacturing, and explore its implications in partial and general oligopolistic equilibrium. Globalization affects the scale and scope (or intensive margin and *intra-firm* extensive margin) of MPFs through a competition effect and a demand effect. The model highlights a new source of gains from trade: productivity increases as firms become "leaner and meaner", concentrating on their core competence; but also a new source of losses from trade: product variety may fall. Our results also hold under free entry, which allows in addition for adjustment along the traditional *inter-firm* extensive margin.

Keywords: Flexible Manufacturing, General Oligopolistic Equilibrium (GOLE), International Trade, Multi-Product Firms, Product Diversity.

JEL Classification: F12, L13

*We thank the editor, Andrea Prat, and two anonymous referees, as well as Andy Bernard, David Greenstreet, Volker Grossmann, Victor Norman, Jim Rauch, Steve Redding, Nicolas Schmitt, and participants in various seminars, at ETSG 2005 in Dublin, and at the Winter 2005 Meetings of the NBER International Trade and Investment Program in Palo Alto for helpful comments. Carsten Eckel thanks the German Science Foundation (DFG grant number EC 216/4-1) for research support.

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1 Introduction

Multi-product firms are omnipresent in the modern world economy, especially in technologically advanced countries. Their importance is documented in a recent study of U.S. firms by Bernard, Redding and Schott (2006a).[1] This shows that multi-product firms are present in all industries; they account for less than half (41%) of the total number of firms but the vast bulk (91%) of total output; and they are very active in varying their product mix: 89% of multi-product firms do so on average every five years. Yet, despite this empirical importance, and despite the interest in trade as a source of increased product diversity, multi-product firms have received relatively little attention in the theory of international trade.

General equilibrium models of international trade typically rely on single-product firms only. In such a framework, intra-firm adjustments are limited to changes in the scale of production. Changes in diversity are linked exclusively to changes in the number of firms. In contrast to the theory of international trade, multi-product firms have received more attention in the field of industrial organization. (See, for example, Brander and Eaton (1984), Klemperer (1992), Ottaviano and Thisse (1999), Hallak (2000), Baldwin and Ottaviano (2001), Grossmann (2003), Johnson and Myatt (2003, 2006), Ju (2003), Allanson and Montagna (2005), and Baldwin and Gu (2005)). These studies have emphasized that, because of supply and demand linkages, intra-firm adjustments within multi-product firms are significantly different from adjustments via exit and entry. However, studies in industrial organization are commonly conducted in partial equilibrium, so that they cannot capture feedback effects through factor markets.[2] But given the omnipresence and empirical importance of multi-product firms across industries, these-general equilibrium effects can be significant and should be included in an analysis of multi-product firms in the global economy. In this paper, we develop a new model of multi-product firms that incorporates both supply and demand linkages and explore its implications in partial and general oligopolistic equilibrium. Our findings show that intra-firm adjustments imply quite different predictions regarding the impact of international trade on firm productivity, factor prices, and product diversity than traditional models of international trade.

The supply and demand linkages in our framework capture important differences between multi-product and single-product firms, which have been highlighted in the theory of industrial organization but largely neglected in the literature on international trade. First, in contrast to single-product firms, multi-product firms internalize demand linkages between the varieties they produce. This feature is called the "cannibalization effect" and is generally considered a defining feature of multi-product firms. The existence of a cannibalization effect requires that firms are large in their markets and behave like oligopolists. It gives rise to strategic interactions that are of particular importance for a firm's reaction

[1] This uses a longitudinal database derived from the U.S. Census of Manufactures with observations at five-yearly intervals between 1972 and 1997. Over 140,000 surviving firms are present in each census year. In this study a "product" is defined at the five-digit Standard Industry Classification (SIC) level.

[2] Ottaviano and Thisse (1999) allow for labour market equilibrium in their framework, but since they use quasi-linear preferences, they cannot address income effects. The same is true of Hallak (2000) and Baldwin and Gu (2005).

to changes in competition. Second, the varieties within a firm's product line are linked on the cost side through a flexible manufacturing technology (Milgrom and Roberts (1990), Eaton and Schmitt (1994), Norman and Thisse (1999), Eckel (2009)). Flexible manufacturing emphasizes the fact that firms typically possess a "core competence" in the production of a particular variety and that they are less efficient in the production of varieties outside their core competence. In our framework, this inefficiency translates into higher marginal labor requirements. Hence, flexible manufacturing allows firms to expand their product lines, but this expansion is subject to diseconomies of scope and creates cost heterogeneities between products. These cost heterogeneities are important for the general-equilibrium effects of changes in product ranges. The two types of linkages, cannibalization and flexible manufacturing, are the driving forces behind the intra-firm adjustments in our framework.

The nature of cost linkages and the existence of demand linkages and cannibalization distinguish our work from recent theoretical papers by Allanson and Montagna (2005), Bernard, Redding and Schott (2006b) and Nocke and Yeaple (2006). Allanson and Montagna assume both firm- and variety-specific fixed costs; Bernard, Redding and Schott assume that firm- and variety-specific costs are random and independent of each other; and Nocke and Yeaple assume that unit costs of all products are positively related to the range of products produced. Even more significantly, all three papers analyze multi-product firms in models of "large-group" monopolistic competition. In such a framework, demand linkages and strategic behaviour are excluded, making it impossible to address the issue of cannibalization.

This paper addresses the role of adjustment processes within multi-product firms and linkages with factor and goods markets in a global economy. In particular, we analyze how multi-product firms react to different globalization shocks (distinguishing between the competition and market-size effects of greater international market integration), how these intra-firm adjustments affect the demand for labour, and how induced changes in wages affect the optimal product range and the distribution of outputs within a firm's product range. In order to isolate adjustments within firms from adjustment via exit and entry, we focus for most of the paper on oligopolistic markets where barriers to entry are prohibitively high and the number of firms is exogenously given. However, we also extend our framework to allow for free entry, and show that our main predictions continue to hold. Our analysis provides plausible explanations for observable facts about multi-product firms and presents testable propositions with respect to the impact of economy-wide shocks on the scale and scope of multi-product firms.

Section 2 introduces our assumptions about individual agents, which generate a demand for differentiated products by consumers, and an endogenous choice of scale and scope by firms. Section 3 illustrates the determination of equilibrium in a single industry spread across a number of countries, and derives the paper's key results on the effects of globalization. Section 4 shows how these results are qualified and enriched when labour market adjustment in general equilibrium is allowed. Finally, Section 5 explores the robustness of our results when the model is extended to allow for free entry, heterogeneous firms, and partial trade liberalization. Proofs of all results are presented in the Appendix.

2 Consumers and Firms

We assume that the world economy consists of a finite number of countries, all with fully integrated goods markets but no international factor mobility. There is a continuum of identical industries, each of which has an oligopolistic market structure. We begin in this section by considering the behaviour of consumers and multi-product firms in a single industry. This introduces the two key features of the model: demand for differentiated products on the one hand, and a flexible manufacturing technology on the other.

2.1 Preferences and Demand

Our specification of preferences combines the continuum-quadratic approach to symmetric horizontal product differentiation of Ottaviano, Tabuchi and Thisse (2002) with the absence of an outside (or "numéraire") good as in Neary (2002). Each consumer maximizes a two-tier utility function that depends on their consumption levels $q(i)$, $i \in [1, N]$, where N is the measure of differentiated goods produced in each industry z, and z varies over the interval $[0, 1]$. The upper tier is an additive function of a continuum of sub-utility functions, each corresponding to one industry:

$$U[u\{z\}] = \int_0^1 u\{z\}\, dz. \tag{1}$$

As for the lower tier, each sub-utility function is quadratic:

$$u\{z\} = a\int_0^N q(i)\, di - \frac{1}{2}b\left[(1-e)\int_0^N q(i)^2\, di + e\left\{\int_0^N q(i)\, di\right\}^2\right]. \tag{2}$$

The parameters a, b and e are assumed to be non-negative and identical for all consumers: a denotes the consumer's maximum willingness to pay, while e is an inverse measure of product differentiation, assumed to lie strictly between zero and one. If $e = 1$, the goods are homogeneous (perfect substitutes) so that demand depends on aggregate output only. By contrast, $e = 0$ describes the monopoly case where the demand for each good is completely independent of other goods. We rule out these two extreme cases in order to focus on competition between firms producing differentiated products.

Consumers maximize utility as given by (1) and (2) subject to the budget constraint

$$\int_0^1 \int_0^N p(i)\, q(i)\, di dz \leq I, \tag{3}$$

where $p(i)$ is the price of variety i and I denotes individual income. This leads to the following individual inverse demand functions:

$$\lambda p(i) = a - b\left[(1-e)\,q(i) + e\int_0^N q(i)\,di\right]. \tag{4}$$

The parameter λ is the Lagrange multiplier, which denotes the consumer's marginal utility of income.

To move from individual to aggregate demands, we assume that there are L consumers located in each of k identical countries, all with identical preferences. In addition, we assume that the goods markets of all countries are completely integrated in a single world market and free trade prevails, so the price of a given variety is the same everywhere. Hence the market demand for a particular variety i in any industry, $x(i)$, facing a firm in any country consists of demand from all consumers, $kLq(i)$. This allows (4) to be rewritten as the inverse world market demand function for good i:

$$p(i) = a' - b'\left[(1-e)\,x(i) + eY\right]. \tag{5}$$

where $a' \equiv a/\lambda$, $b' \equiv b/\lambda kL$, and $Y \equiv \int_0^N x(i)\,di$ denotes the output of the entire industry. Note that the demand slope b' depends inversely on the total number of consumers in the world.

Because they depend on λ, the parameters a' and b' are endogenously determined in general equilibrium. However, with a continuum of industries they are perceived as exogenous by individual firms. Hence firms are "large" in their own market but "small" in the economy as a whole, which permits a consistent analysis of oligopoly in general equilibrium. For convenience, and with no loss in generality, we normalize by setting λ equal to one, so in general equilibrium all nominal variables should be interpreted as relative to the marginal utility of income. (See Neary (2002) for details.)

2.2 Costs and Technology of Multi-Product Firms

As explained in the introduction, the technology of multi-product firms is characterized by a core competence and flexible manufacturing. This is illustrated in Figure 1, where $c_j(i)$ denotes the marginal cost which a typical firm j incurs to produce good i.[3] We assume the marginal cost is constant with respect to the quantity produced, but varies across varieties. It is lowest for the core competence variety, which uses the firm's most efficient production process. We set a firm's core competence at $i = 0$ with $c_j(0) = c_j^0$. In addition to producing its core competence variety, the firm can add new products to its product line via flexible manufacturing, which describes its ability to produce additional varieties with only a minimum of adaptation. However, some adaptation is necessary, so each addition to the product line incurs a higher marginal production cost but leaves the marginal production costs of existing prod-

[3] Consumers are indifferent about which firm produces which varieties, so the subscript j was not needed in the previous sub-section. We use it here since in Section 2.3 we consider the behaviour of a single firm playing a Cournot game against other firms. Later (except in Section 5.2), we concentrate on symmetric equilibria, so we can omit it again.

ucts unchanged.[4] The marginal production cost of variety i is therefore a strictly increasing function of the mass of products produced: $\frac{\partial c_j(i)}{\partial i} > 0$, as shown. In general we do not need to impose any further restrictions on the $c_j(i)$ function, though some results are strengthened in the linear case, $c_j(i) = c_j^0 + \gamma i$, and the diagrams assume this for ease of interpretation.

Each multi-product firm produces a mass of products which is denoted by δ_j. Profits for a multi-product firm j are then given by

$$\Pi_j = \int_0^{\delta_j} [p_j(i) - c_j(i)]\, x_j(i)\, di - F, \tag{6}$$

where the fixed cost F is independent of both scale and scope.

2.3 Optimal Scale and Scope

We assume that firms play a single-stage Cournot game. Hence they simultaneously choose the quantity produced of each good and the mass of products produced, assuming that rival firms do not change their scale or scope. The first-order condition with respect to the scale of production of a particular good i is given by

$$\frac{\partial \Pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b' [(1-e)\, x_j(i) + e X_j] = 0, \tag{7}$$

where $X_j \equiv \int_0^{\delta_j} x_j(i)\, di$ denotes the firm's aggregate output.[5] Equation (7) shows that the optimal profit margin on good i is proportional to a weighted average of the output of that good and the firm's total output, where the weights depend on the substitution parameter e. Eliminating the price from equations (5) and (7) gives the output of a single variety:

$$x_j(i) = \frac{a' - c_j(i) - b'e\,(X_j + Y)}{2b'(1-e)}. \tag{8}$$

As always in Cournot competition, industry output has a negative effect on equilibrium output, reflecting the effect of greater competition from rival firms. In addition, equation (8) shows that the firm's total output X_j has a further negative effect on the output of each variety, reflecting the cannibalization effect discussed in the introduction. Because a larger output of one variety tends to lower the prices that consumers are willing to pay for all other varieties, a multi-product firm has an additional incentive to restrict its output of each variety beyond the familiar own-price effect. The effect is illustrated in Figure 2. Because of the cannibalization effect, the marginal revenue curve is lower than it would be for a single-product firm, so other things equal a multi-product firm produces less of every good.

[4] By contrast, Bernard, Redding and Schott (2006b) assume that each firm has a firm-specific and a variety-specific productivity draw, all of which are independent of each other; while Nocke and Yeaple (2006) assume that all products have the same marginal cost, and an expansion in a firm's product range raises the marginal costs of *all* its products.

[5] The second-order condition is easily verified: $\frac{\partial^2 \Pi_j}{\partial x_j(i)^2} = \frac{\partial p_j(i)}{\partial x_j(i)} - b'(1-e) - b'e \frac{\partial X_j}{\partial x_j(i)} < 0$.

Equation (8) also shows that, given its total output, a firm produces less of each variety the further it is from its core competence: $x_j(i)$ is decreasing in $c_j(i)$. Given the symmetric structure of demand, this means that it must charge higher prices for products that are further from its core competence, as can be seen by solving (5) and (7) for the price of each variety:

$$p_j(i) = \frac{1}{2}\left[a' + c_j(i) - b'e(Y - X_j)\right]. \tag{9}$$

This heterogeneity of prices charged across varieties is in contrast with models of multi-product firms where economies of scope arise from fixed costs, or where producing more varieties raises marginal costs for all varieties, as in Nocke and Yeaple (2006). However, in our model not all of the higher costs are passed on to consumers. Some (in fact, exactly half, because demand is linear) are absorbed by the firm itself in the form of lower profit margins on varieties that are further from its core competence:

$$p_j(i) - c_j(i) = \frac{1}{2}\left[a' - c_j(i) - b'e(Y - X_j)\right] \tag{10}$$

Note also that, by contrast with the output equation (8), the competition and cannibalization effects have opposite signs in (9) and (10). More competition reduces the prices which firms can charge in Cournot markets, but this is partly (though not fully) offset by the cannibalization effect, which encourages multi-product firms to charge higher prices on all their varieties, and also allows them to earn higher margins.

Consider next the firm's choice of product line. Multi-product firms add new products as long as marginal profits are positive. The first-order condition with respect to the scope of production is then:[6]

$$\frac{\partial \Pi_j}{\partial \delta_j} = \left[p_j(\delta_j) - c_j(\delta_j)\right] x_j(\delta_j) = 0. \tag{11}$$

From the first-order condition for scale, equation (7), the profit on the marginal variety $p_j(\delta_j) - c_j(\delta_j)$ cannot be zero. Equation (11) therefore implies that profit-maximizing multi-product firms choose their product range so that the output of the marginal variety is zero: $x_j(\delta_j) = 0$. Combining this with equation (8), the first-order condition with respect to scope can also be expressed as

$$c_j(\delta_j) = a' - b'e(X_j + Y). \tag{12}$$

The determination of the profit-maximizing product range is illustrated in Figure 1. Starting from its core competence variety, the firm adds new varieties up to the point where the marginal cost of producing the marginal variety equals the marginal revenue at zero output. To drive sales to zero, the price charged

[6] As $\frac{\partial c_j(\delta_j)}{\partial \delta_j} > 0$ and, thus, $\frac{\partial x_j(\delta_j)}{\partial \delta_j} = -\frac{1}{2b'(1-e)} \frac{\partial c_j(\delta_j)}{\partial \delta_j} < 0$, the second-order condition is easily verified: $\frac{\partial^2 \pi_j}{\partial \delta_j^2} = \left[p_j(\delta_j) - c_j(\delta_j)\right] \frac{\partial x_j(\delta_j)}{\partial \delta_j} < 0$.

on its marginal variety is the highest of all its varieties, equal from (9) to $p_j(\delta_j) = a' - b'eY$. However, it earns the lowest profit margin on its marginal variety, though as already noted it is strictly positive, equal from (10) and (12) to $b'eX_j$.

2.4 Productivity of Multi-Product Firms

Our assumptions about technology imply a direct relationship between a firm's scope of production and its productivity. Assume that labour is the only factor of production, and that the labour market is economy-wide and perfectly competitive. The unit cost of producing each variety can then be decomposed into a technological component, denoted $\gamma(i)$, and a factor-cost component equal to the wage w: $c(i) = w\gamma(i)$. (From now on we omit the firm subscript j.) Here $\gamma(i)$ measures the labour input needed to produce a unit of output of variety i.

Suppose now that the firm is subject to a shock to any exogenous variable, other than the level of technology (i.e., the function $\{\gamma(i)\}$). How do we measure the resulting change in the firm's productivity? Measuring total labour input is straightforward: ignoring fixed costs, it equals the integral of the labour requirements of each variety times its output:

$$l = \int_0^\delta \gamma(i)\, x(i)\, di \tag{13}$$

To measure the change in labour productivity ("LP") in response to an increase in some exogenous variable θ, we subtract the log-change in total labour input l from a Divisia index of the changes in outputs of different varieties, aggregated using weights $h(i)$ to be discussed below:

$$\frac{d\ln LP}{d\ln\theta} = \frac{\int_0^\delta h(i)\, \frac{dx(i)}{d\ln\theta} di}{\int_0^\delta h(i)\, x(i)\, di} - \frac{d\ln l}{d\ln\theta} \tag{14}$$

We first want to show that the effects on productivity of changes in any exogenous variable depend only on their effects on product scope δ. To see this, combine the first-order conditions for scale and scope, equations (8) and (12) respectively, to express the output of each variety in terms of the difference between its own marginal cost and that of the marginal variety:

$$x(i) = \frac{w\left[\gamma(\delta) - \gamma(i)\right]}{2b'(1-e)}. \tag{15}$$

Next, substituting from (15) for outputs $x(i)$ into the variable labour demand from (13) and evaluating the integral yields:

$$l = \frac{w\beta(\delta)}{2b'(1-e)}, \quad \text{where} \quad \beta(\delta) \equiv \int_0^\delta \gamma(i)\left[\gamma(\delta) - \gamma(i)\right] di \tag{16}$$

Here $\beta(\delta)$ is the technological component of the firm's variable demand for labour. Comparing (15) and (16), it is clear that both equal some function of technology and scope δ, multiplied by a common factor $\frac{w}{2b'(1-e)}$. It follows that the change in productivity in (14) depends only on the effect of product scope on productivity times the effect of the exogenous shock on the firm's equilibrium scope: $\frac{d \ln LP}{d \ln \theta} = \frac{\partial \ln LP}{\partial \ln \delta} \frac{d \ln \delta}{d \ln \theta}$.[7]

Consider next the choice of the weights $h(i)$ in the Divisia index. Note first that if the weights are marginal costs, $h(i) = c(i)$, which is equivalent to input requirements, $h(i) = \gamma(i)$, then the log-change in output is identical to that in employment from (13), and so productivity is independent of firm scope:

$$\left. \frac{\partial \ln LP}{\partial \ln \delta} \right|_{h(i)=\gamma(i)} = \frac{\int_0^\delta \gamma(i) \frac{\partial x(i)}{\partial \ln \delta} di}{\int_0^\delta \gamma(i) x(i) di} - \frac{\partial \ln l}{\partial \ln \delta} = 0 \tag{17}$$

This reflects the fact that the technology available to the firm is unaffected by the shock. However, the techniques in use are definitely affected, so an alternative measure of productivity change which takes account of this seems preferable. We consider two different sets of weights, $h(i) = 1$ and $h(i) = p(i)$. Weighting by prices as in the latter gives the most satisfactory measure of output and productivity change, whereas focusing simply on the unweighted sum of outputs X as in the former gives better intuition.

The key result for the response of productivity to changes in firm scope can be summarized as follows:

Proposition 1 *With given technology, any shock which raises the product range of a multi-product firm: (a) reduces productivity as measured in (14) when output is a simple aggregate,* $h(i) = 1$*; (b) reduces it but by less when output changes are price-weighted,* $h(i) = p(i)$*; and (c) leaves it unchanged when output changes are marginal-cost-weighted,* $h(i) = \gamma(i)$*.*

Proofs for parts (a) and (b) are given in the Appendix, but the underlying intuition can be explained as follows. Varieties further from the firm's core competence have higher labour requirements and hence lower productivity. Therefore, when the outputs of all varieties are weighted equally as in (a), an expansion in the firm's product range (even allowing for an optimal reallocation between existing varieties) must lower measured productivity. Weighting by prices as in (b) increases the importance attributed to marginal varieties, recalling from (9) that prices are higher the further the firm moves away from its core competence. Hence the change in measured productivity when output changes are price-weighted as in (b) must be algebraically greater than when they are uniformly weighted as in (a). However, equation (9) also showed that prices rise less quickly than marginal costs $w\gamma(i)$. Hence, measured productivity must fall by more (or rise by less) when output changes are price-weighted than when they are cost-weighted as in (c). But we have already seen from (17) that the change in output is exactly matched by the change in labour input in case (c). Hence it follows that measured productivity

[7] To see this more formally, any exogenous shock has both a direct and an indirect effect on productivity, the latter operating through firm scope: $\frac{d \ln LP}{d \ln \theta} = \frac{\partial \ln LP}{\partial \ln \theta} + \frac{\partial \ln LP}{\partial \ln \delta} \frac{d \ln \delta}{d \ln \theta}$. Write the common factor in (15) and (16) as $\psi(\theta) \equiv \frac{w}{2b'(1-e)}$. Then the log change in labour input for given scope equals the elasticity of ψ (which may be zero): $\frac{\partial \ln l}{\partial \ln \theta} = \frac{\theta \psi'}{\psi}$. But this is also the value of the weighted sum of output changes in (14) for given scope. Hence $\frac{\partial \ln LP}{\partial \ln \theta}$ is zero, as required.

must fall when δ rises in case (b).

As already noted, the intuition is more straightforward when output changes are uniformly weighted. A further advantage of uniform weights is that we can obtain an explicit expression for the level as well as the change in productivity. Integrating (15) over the entire mass of products produced yields an expression for total output:

$$X = \frac{w\alpha(\delta)}{2b'(1-e)}, \quad \text{where} \quad \alpha(\delta) \equiv \delta\gamma(\delta) - \int_0^\delta \gamma(i)\, di > 0 \tag{18}$$

The term $\alpha(\delta)$ is the technological component of total output and can be interpreted as a measure of the total cost savings from flexible manufacturing. It is represented by the shaded region in Figure 1. Comparison with (16) shows that, like the outputs of individual varieties $x(i)$, total output X depends on other non-technological influences in exactly the same way as does labour input l. Hence the firm's labour productivity, when output is measured by X, depends only on technology and on the product range δ:

$$\frac{X}{l} = \frac{\alpha(\delta)}{\beta(\delta)} = \left[\mu'_\gamma - \frac{\delta\sigma^2_\gamma}{\alpha(\delta)}\right]^{-1}. \tag{19}$$

where $\mu'_\gamma \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)\, di$ and $\sigma^2_\gamma \equiv \frac{1}{\delta}\int_0^\delta \left[\gamma(i) - \mu'_\gamma\right]^2 di$ are, respectively, the mean and variance of the distribution of labour requirements across all the varieties produced by the firm in equilibrium. The second expression for l in (19) follows by substituting for $\beta(\delta)$ from (49) in the Appendix. It provides another perspective on the gains from flexible manufacturing: a multi-product firm requires less labour than it would if it produced all its output using the average labour requirement of all its varieties, $l < \mu'_\gamma X$, because it produces relatively more output of varieties closer to its core competence. Moreover, the labour saved is greater the higher the variance of the distribution of labour requirements across varieties.

Note finally that all these results for productivity follow only from our assumptions about preferences and technology. However, this is as far as we can go without examining in more detail how firms interact. In the next section we turn to consider how equilibrium is determined in an industry made up of multi-product firms.

3 Industry Equilibrium

3.1 Determination of Equilibrium

We consider the case of a symmetric Cournot oligopoly, so we can continue to suppress the firm subscript j. Since we wish to focus on intra-firm adjustments as opposed to adjustments via exit and entry, we assume until Section 5.1 that there is an exogenously given number of multi-product firms m in each of the k countries. Industry output is then given by:

$$Y = kmX. \tag{20}$$

In industry equilibrium, the first-order condition for scope, equation (12), can therefore be rewritten as follows:

$$w\gamma(\delta) = a' - e(1 + km)b'X. \tag{21}$$

This implies a negative relationship between the output of each firm and the optimal choice of product range, as illustrated by the downward-sloping curve labelled "Scope: $\delta(X)$" in Figure 3. This comes from two sources, which can be explained with reference to the expression for the output of a single variety (8). First, even in the case of a monopoly firm (i.e., when km equals one), the desire to avoid cannibalizing other varieties induces the firm to produce less of each existing variety as its total output increases. Since the output of the marginal variety, $x(\delta)$, is already zero and so cannot be reduced further, this implies that the optimal product range δ should itself be reduced. Second, this effect is accentuated when the firm faces competition (so km exceeds one) and all firms expand their output symmetrically. Increases in rival output clearly reduce the optimal product range of every firm.

Equation (21) gives one relationship between δ and X. To derive a second, we integrate over the equations for individual outputs (8):

$$X = \frac{\left(a' - w\mu'_\gamma\right)\delta}{\Delta_1 b'} \quad \text{where:} \quad \Delta_1 \equiv 2(1-e) + e\delta(1+km) > 0 \tag{22}$$

This expression implies that a rise in δ initially raises total output, but once δ reaches its optimal level, further increases in product range reduce total output. This can be seen by differentiating (22) with respect to δ :

$$\frac{d\ln X}{d\ln\delta} = \frac{a' - w\gamma(\delta) - e(1+km)b'X}{a' - w\mu'_\gamma} \tag{23}$$

where the numerator of the right-hand side is the first-order condition for scope from (21), and equals zero when δ is at its optimal level. The relationship is shown by the curve labelled "Scale: $X(\delta)$" in Figure 3.

Clearly, the symmetric industry equilibrium must be at the intersection of the two curves in Figure 3, where the equilibrium conditions for scope and scale, equations (21) and (22), are both satisfied.[8] Note that this occurs at the maximum of $X(\delta)$. We can now illustrate how changes in exogenous variables perturb the equilibrium by considering their effects on this diagram and on the profile of outputs of

[8] The equilibrium is unique and stable, as the determinant of the coefficient matrix, equation (58) in the Appendix, is always positive.

individual varieties in Figure 4. The latter is equation (8) specialized to the case of symmetric equilibria:

$$x(i) = \frac{a' - w\gamma(i) - e(1+km)\,b'X}{2b'(1-e)} \tag{24}$$

Explicit expressions for all effects are given in the Appendix.

3.2 The Effects of Globalization

Our primary interest is in the effects of globalization, interpreted as an increase in the number of countries participating in the global economy. Such a shock operates through two distinct channels, and it is helpful to consider them separately. On the one hand, globalization means that existing firms face larger markets, as the number of consumers in the world economy expands: this effect of an increase in k is the same as that of an increase in L, the number of worker/consumers in each country. On the other hand, globalization means that existing firms are exposed to more competition from new firms on world markets: this effect of an increase in k is the same as that of an increase in m, the number of firms in each country. The net effect of an increase in k is the sum of these market-size and competition effects, so we consider them in turn.[9]

A positive market-size effect induced by an increase in L reduces (in absolute value) the slope b' of the demand function for each variety, recalling that $b' \equiv \frac{b}{\lambda kL}$. Hence, in Figure 2 the demand curve pivots counter-clockwise, and so does the marginal revenue curve. If aggregate firm and industry output, X and Y, remained constant, the fall in b' would also lead to an outward shift of the demand and marginal revenue curves. However, since an increase in market size boosts the output of all varieties and all firms, X and Y also rise, and so the intercepts of the two curves are unaffected. The outcome is an equi-proportionate increase in the output of all varieties already produced, but no change in the number of varieties. For the marginal variety, the cost curve $w\gamma(\delta)$ continues to intersect the marginal revenue curve at zero output. This can be seen more formally by inspecting the first-order conditions for scope and scale, equations (21) and (22): b' always appears multiplied by X, so a fall in b' is accommodated by an equal proportionate rise in total output X and no change in δ. In Figure 3, both equilibrium loci shift rightwards by an equal amount, while in Figure 4, the output schedule pivots clockwise around the initial marginal variety δ. Summarizing:

Proposition 2 *The market-size effect of an increase in k (which equals the total effect of an increase in L) is an equi-proportionate increase in the output of each variety and of total output, but no change in firm scope.*

The competition effect induced by an increase in m has very different effects. Now the demand

[9] Formally, the equations in the Appendix show that, in both partial and general equilibrium when the number of firms is exogenous, the proportional effects of an increase in k, $d\ln k$, equal those of an increase in L, $d\ln L$, plus those of an increase in m, $d\ln m$.

curve intercept for every variety shifts downwards by the same absolute amount, while their slopes are unaffected. The output of every variety therefore falls by the same absolute amount, and so in Figure 4 the output profile shifts uniformly downwards. With output of every variety falling, total output X must also fall.[10] However, X falls less than proportionally to m, so industry output $Y = kmX$ rises as a result of the entry of new firms: $\frac{d \ln Y}{d \ln m} = 1 + \frac{d \ln X}{d \ln m} > 0$. In addition, the uniform absolute fall in outputs means that in relative terms greater competition hits harder those varieties produced at higher cost, which implies that marginal varieties become unprofitable and so δ falls. In Figure 3, both equilibrium loci shift leftwards, but $X(\delta)$ shifts by less.[11] Summarizing:

Proposition 3 *The competition effect of an increase in k (which equals the total effect of an increase in m) is a uniform absolute fall in the output of each variety, coupled with falls in both total firm output and firm scope, but a rise in industry output.*

Having considered separately the market-size and competition effects, we can combine them to get the full effect of an increase in the number of countries in the world economy. Now both the slope and the intercept of each demand curve are affected, the former falling in absolute value as market size expands and the latter shifting downwards as competition intensifies. From equation (59) in the Appendix, the full expression for the change in output is:

$$\frac{d \ln X}{d \ln k} = 1 - \frac{e\delta km}{\Delta_1} \tag{25}$$

The terms on the right-hand side correspond respectively to the market-size effect, which encourages an equal proportionate increase in output, and the competition effect, which encourages a partially but not fully offsetting reduction. (Recall that $\Delta_1 - e\delta km = \Delta_0 > 0$.) In Figure 3, both equilibrium loci shift rightwards. Recalling that the number of varieties produced δ does not benefit from the market-size effect of a rise in k, whereas total output X does, it follows that the first-order condition for scale $X(\delta)$ shifts rightwards by more than the first-order condition for scope $\delta(X)$.[12] The net effect is an increase in output but a fall in the number of varieties, as shown by the dashed loci.

These divergent responses of X and δ imply non-uniform changes in the output profile. From equation

[10] From equation (61) in the Appendix, the absolute change in output of each variety is $\frac{dx(i)}{d \ln m} = -\frac{ekmX}{\Delta_1}$, which is independent of i; while equation (59) shows that the change in total output is the corresponding integral: $\frac{dX}{d \ln m} = -\frac{e\delta kmX}{\Delta_1}$.

[11] The proportionate fall in total output X exceeds that in scope δ if and only if $\alpha(\delta)$ has an elasticity greater than one: $\frac{d \ln X}{d \ln m} / \frac{d \ln \delta}{d \ln m} = \frac{\delta \alpha_\delta}{\alpha(\delta)}$. A sufficient condition for this is that $\gamma(\delta)$ has an elasticity greater than one, since $\frac{\delta \alpha_\delta}{\alpha(\delta)} = \frac{\gamma(\delta)}{\gamma(\delta) - \mu'_\gamma} \frac{\delta \gamma_\delta}{\gamma(\delta)}$.

[12] From equation (58) in the Appendix, the rightward shift in the first-order condition for scale $X(\delta)$ is $\left.\frac{d \ln X}{d \ln k}\right|_{X(\delta)} = \frac{\Delta_0}{\Delta_1}$; while that in the first-order condition for scope $\delta(X)$ is $\left.\frac{d \ln X}{d \ln k}\right|_{\delta(X)} = \frac{1}{1+km}$. The ratio of the former to the latter equals $1 + \frac{2(1-e)km}{\Delta_1}$ which is greater than one.

(61) in the Appendix, the change in the output of each variety equals:[13]

$$\frac{d\ln x\left(i\right)}{d\ln k}=1-\frac{ekm\alpha\left(\delta\right)}{\Delta_1\left[\gamma\left(\delta\right)-\gamma\left(i\right)\right]}=\frac{\Delta_0}{\Delta_1}+\left(1-\frac{\Delta_0}{\Delta_1}\right)\frac{\mu'_{\gamma}-\gamma\left(i\right)}{\gamma\left(\delta\right)-\gamma\left(i\right)} \tag{26}$$

where $\Delta_0 \equiv 2\left(1-e\right)+e\delta<\Delta_1$. The first expression on the right-hand side gives a decomposition of the total change into market-size and competition effects, similar to that for total output in (25). The second rewrites this as a weighted average of a uniform proportionate increase and a change which depends on the difference between the labour requirement for variety i, $\gamma\left(i\right)$, and the average labour requirement, μ'_{γ}. For marginal varieties, with labour requirements greater than the mean and very close to $\gamma\left(\delta\right)$, the second term is negative and dominates. Hence, matching the fall in firm scope, less is produced of varieties with relatively high costs. However, for all varieties with costs equal to or below average (i.e., with $\gamma\left(i\right)<\mu'_{\gamma}$), output rises. Hence the output profile pivots in a clockwise manner as shown in Figure 4. Solving (26) for $\tilde{\gamma}^{PE}$, the labour requirement of the threshold variety whose output is unchanged in partial equilibrium when k changes, it equals a weighted average of the labour requirements of the marginal and the average varieties:[14]

$$\tilde{\gamma}^{PE}=\frac{\Delta_0}{\Delta_1}\gamma\left(\delta\right)+\left(1-\frac{\Delta_0}{\Delta_1}\right)\mu'_{\gamma}. \tag{27}$$

All varieties with labour requirements less than $\tilde{\gamma}^{PE}$ (including all those with labour requirements less than average) expand, while those close to the marginal variety contract.

Summarizing:

Proposition 4 *The total effect of an increase in k is a rise in total output coupled with a fall in scope. Relatively high-cost varieties are discontinued or produced in lower volumes, whereas more is produced of all varieties with average costs or lower.*

The interpretation is clear: globalization encourages multi-product firms to become "leaner and meaner", pruning their product lines to focus on their core competencies. Although the number of firms is exogenous, so there is no change in the familiar inter-firm extensive margin, the endogenous response of firm scope introduces a new margin, the "intra-firm extensive margin", which implies a fall in the number of varieties per firm. In addition, combining Propositions 1 to 4, it also implies a rise in firm productivity:

Corollary *Firm productivity is unaffected by the market-size effect, but rises with the competition effect of an increase in k.*

[13] These are easier to interpret when expressed in terms of proportional changes $d\ln x\left(i\right)$. Of course, when applied to the marginal variety, for which $x\left(\delta\right)=0$, they must be reexpressed in terms of absolute changes $dx\left(i\right)$.

[14] In the linear case, where $\gamma\left(i\right)=\gamma_0+\gamma i$, the threshold variety is: $\tilde{\imath}=\frac{1}{2}\delta\left(1+\frac{\Delta_0}{\Delta_1}\right)$.

3.3 Globalization and Product Variety

We have seen that the number of varieties per firm falls with globalization, but of course the number of firms rises. However, the reduction in firm scope may dominate, implying a reduction in the total range of products available to consumers. To see this, note that the total number of varieties produced in symmetric equilibrium is given by $N = km\delta$. This is unaffected by the market-size effect. However, the competition effect of globalization has conflicting effects, raising the number of firms but lowering the number of varieties:

$$\frac{d\ln N}{d\ln k} = 1 + \frac{d\ln\delta}{d\ln k} = 1 - \frac{e\delta km}{\Delta_1}\frac{\alpha(\delta)}{\delta\alpha_\delta} \tag{28}$$

The expression $\frac{e\delta km}{\Delta_1}$ is the competition effect on total output, which as we have already seen is less than one. Hence a necessary condition for product diversity to fall is that $\frac{\delta\alpha_\delta}{\alpha(\delta)}$, the elasticity of the "cost savings from flexible manufacturing", is less than one, which implies that technology is highly flexible at the margin (since $\alpha_\delta = \delta\gamma_\delta$). This condition cannot hold, for example, if the cost function has constant curvature (of which a linear cost function is a special case), so in that case the fall in δ cannot reduce total diversity.[15] However, we cannot in general rule out the possibility that the effect of globalization in encouraging incumbent firms to prune their product lines may dominate the direct effect of the entry of new firms, so that the total number of varieties produced in the world may fall.[16] Summarizing:

Proposition 5 *In partial equilibrium, an increase in the number of countries cannot lower the total number of varieties if the function relating costs to varieties has constant curvature, but it may do so if technology is sufficiently flexible.*

This result shows the importance of taking the "intra-firm extensive margin" into account when trying to understand the effects of opening up markets to international trade: product diversity can move in the opposite direction to the number of firms once we allow for intra-firm adjustment.

4 General Equilibrium

The previous section considered the adjustment of an oligopolistic industry made up of multi-product firms to the market-size and competition effects of increased globalization. However, the analysis was unavoidably partial, since no consideration was given to the response of wages. In this section we first

[15] The constant-curvature cost function $\gamma(i) = \gamma_0 + \frac{\gamma}{\eta}i^\eta$ implies that the elasticity of cost savings, $\frac{\delta\alpha_\delta}{\alpha(\delta)}$, equals $\eta + 1$. This is less than one if and only if $\eta < 0$, which is inconsistent with $\gamma(i) > 0$ for all i. Hence, diversity cannot fall if the cost function has constant curvature, and in particular if it is linear ($\eta = 1$).

[16] An example which yields the desired result is: $\gamma(i) = 1 - (\gamma i + 1)^{-2}$. This function is increasing and concave in i for $\gamma > 0$, and the implied elasticity of cost savings is: $\frac{\delta\alpha_\delta}{\alpha(\delta)} = \frac{2}{\gamma\delta+1}$. Even with a strictly concave cost function such as this, it is necessary to check that there exist values of the exogenous variables which imply a fall in product variety, bearing in mind that δ itself is endogenous. The following values of the exogenous variables satisfy these requirements: $e = 0.5$, $a'/w = 2.25$, $m = 2$, $k = 3$, $\gamma = 1.5$. Then, $\delta = 2.0$, $\frac{\delta\alpha_\delta}{\alpha(\delta)} = 0.5$, and $d\ln\delta/d\ln k = -1.5$, implying that total product diversity N must fall.

examine the effects of exogenous wage changes on the equilibrium and then show how wages and outputs are simultaneously determined in general equilibrium.

4.1 Wage Effects on Scale and Scope

It is immediately apparent from inspection of the equilibrium conditions for firm scope and scale, equations (21) and (22), that an increase in the wage rate causes both curves to shift to the left in Figure 3. Hence, not surprisingly, total output X must fall as costs rise. This in turn implies that the relationship between X and the wage w is always decreasing, which is shown in Figure 5 (drawn in $\{w, X\}$ space) by the downward-sloping Industry Equilibrium locus labelled "IE: $X(w)$".

To determine what happens to firm scope δ, it is helpful to consider the effect on the profile of outputs. From equation (24), it can be seen that the direct effect of wages reduces the output of a given variety by more the greater its unit labour requirement, $\gamma(i)$. Hence the profile of outputs in Figure 4 is pushed inwards in an asymmetric fashion and becomes steeper, with the output of marginal varieties falling by more than those close to the firm's core competence. Potentially offsetting this is the effect of reduced competition, as other firms reduce their outputs, which in itself encourages a uniform absolute expansion of all varieties. We have already seen that total firm output must fall, so this asymmetric response across varieties implies that, at the very least, the outputs of marginal varieties must fall and so firm scope δ itself must fall. Hence the equilibrium condition for scope must shift leftwards by more than that for scale, to give a new equilibrium in Figure 3 exhibiting falls in both X and δ. Recalling Proposition 1, the fall in δ also implies that firm productivity must rise: although total output falls, it must do so by less than total labour input as the increase in wages encourages firms to prune marginal varieties and concentrate on their core competence.

The preceding discussion raises the possibility that the output of core varieties may actually rise, even though both X and δ must fall. To explore this, consider the expression for the change in individual outputs, from equation (61) in the Appendix:

$$\frac{d\ln x(i)}{d\ln w} = \frac{-2(1-e)\gamma(i) + e\delta(1+km)\left[\mu'_\gamma - \gamma(i)\right]}{\Delta_1\left[\gamma(\delta) - \gamma(i)\right]} \tag{29}$$

It is clear that all varieties with unit cost greater than average ($\gamma(i) > \mu'_\gamma$) must fall when the wage rises. However, for low values of i the expression is of indeterminate sign, and so it is possible that their output may rise. The condition for $x(0)$ to rise is:

$$\gamma(0) < \frac{e\delta(1+km)}{\Delta_1}\mu'_\gamma \tag{30}$$

which is more likely to hold the further is the cost of the core-competence variety from that of the average variety and the greater the number of firms.[17]

To summarize the results so far:

Proposition 6 *An exogenous increase in the wage leads all firms to reduce both their total output and their product range. However, the outputs of varieties with below-average costs may increase.*

4.2 Simultaneous Determination of Wages and Outputs

To close the model we need to specify how the wage is determined in general equilibrium. We assume that all households supply one unit of labour inelastically, so within each country the total labour supply equals L. The wage must adjust to ensure that this equals the total demand for labour, obtained by integrating across all sectors, firms and varieties, and including a labour requirement f to cover the fixed costs $F = wf$ of operating each firm. (As discussed in the introduction, we do not assume that there are fixed costs of adding an additional variety, nor of serving additional markets.) The labour-market equilibrium condition can therefore be written as follows:

$$L = m\,(l + f) = m\left[\int_0^\delta \gamma\,(i)\,x\,(i)\,di + f\right] \tag{31}$$

To proceed further, we substitute for $x\,(i)$ from equation (8), and evaluate the integral to obtain:

$$\frac{L}{m} = \frac{[a' - e\,(1 + km)\,b'X]\,\delta\mu'_\gamma - w\delta\mu''_\gamma}{2b'\,(1 - e)} + f \tag{32}$$

where $\mu''_\gamma \equiv \frac{1}{\delta}\int_0^\delta \gamma\,(i)^2\,di$ is the second moment around zero of the firm's equilibrium distribution of labour requirements. The left-hand side of (32) is the labour supply available to each firm, while the right-hand side is the typical firm's labour demand. The latter depends on δ among other variables, but, like the expression for aggregate output (22) discussed in the last section, it is independent of δ when firm scope is chosen optimally. To see this, differentiate the variable labour requirement from (31) with respect to δ: $\frac{\partial l}{\partial \delta} = \gamma\,(\delta)\,x\,(\delta)$, which equals zero from the first-order condition for firm scope, equation (11). Once again, this is an envelope result: for a given level of optimally-chosen total output, a small change in firm scope does not affect the aggregate demand for labour. Hence we can solve the model for X and w without considering δ explicitly, and we can illustrate the determination of equilibrium in $\{w, X\}$ space as in Figure 5.

From (32), it is clear that the labour-market equilibrium locus must be downward-sloping: an increase in output by all firms lowers their demand for labour because of the competition and cannibalization effects; restoring labour-market equilibrium requires a fall in the wage. It is also easy to show that its

[17] In the linear case, where $\gamma\,(i) = \gamma_0 + \gamma i$, the output of the core competence variety increases if and only if: $\frac{1}{2}e\,(1 + km)\,\gamma\delta^2 > 2\,(1 - e)\,\gamma_0$.

slope is less in absolute value than that of the IE locus.[18] This reflects a natural configuration: relative to the requirements for industry equilibrium, the labour market is more responsive to changes in the wage than in firm output. Hence the equilibrium is unique and stable with respect to an adjustment process whereby w and X vary in response to deviations from equilibrium in the labour and goods markets respectively.

4.3 Globalization in General Equilibrium

We can now deduce the effects of an expansion in the number of countries. The effect on the IE locus follows from the partial equilibrium results of the last section: at given wages, the competition effect tends to reduce equilibrium output, but this is more than offset by the market-size effect. Hence the IE locus shifts to the right as shown in Figure 6. If wages are unchanged, then the outcome is at point A, identical to that discussed in the last section.

However, the change in wages depends also on the shift in the LL locus.[19] This too can be broken into a positive market-size effect and a negative competition effect, and once again the former dominates, so the LL locus shifts to the right. As shown in the Appendix, output must rise in all cases. However, the change in the wage is ambiguous, since the relative impacts of the two effects cannot be determined a priori. From equation (65) in the Appendix, the change in wages is:[20]

$$\frac{\delta \Delta_2}{\alpha(\delta) \Delta_1} \frac{d \ln w}{d \ln k} = \frac{\beta(\delta)}{\alpha(\delta)} - \frac{e\delta k m}{\Delta_1} \mu'_\gamma \tag{33}$$

The intuition for this can be explained by recalling the effects of globalization on output per firm and per variety at constant wages from Section 3. Consider first the market-size effect. From (25) and (26), this encourages a uniform *proportionate* increase in the output of all varieties at initial wages. Hence this translates into a proportionate rise in labour demand equal to the firm's average labour requirement (the inverse of its productivity) $\beta(\delta)/\alpha(\beta)$. By contrast, the competition effect encourages a uniform *absolute* reduction in the output of each variety which yields a proportionate fall in total output of $-\frac{e\delta km}{\Delta_1}$. This translates into a reduction in labour demand equal to $-\frac{e\delta km}{\Delta_1}$ times the average of the firm's labour requirements across all its varieties, μ'_γ. Recalling equation (19), the firm's average labour requirement $\beta(\delta)/\alpha(\delta)$ is lower than μ'_γ; on the other hand, the market-size effect on output (equal to 1) dominates the competition effect (equal to $-\frac{e\delta km}{\Delta_1}$). Hence the net effect on the demand for labour at initial wages, and so the net effect on wages in general equilibrium, is indeterminate. In Figure 6, the LL locus shifts to the right, but the new equilibrium may be above or below point A as shown. A fall

[18] See the Appendix for a formal derivation.

[19] When differentiating the labour-market equilibrium condition (32), we hold the domestic labour supply L/m constant: we are interested only in the effects of changes in foreign L and m, because they illuminate the effects of an increase in k.

[20] The term $\Delta_2 \equiv 2(1-e)\mu''_\gamma + e\delta(1+km)\sigma^2_\gamma$ is the determinant of the system; the fact that it is positive ensures that the equilibrium is unique and stable.

in wages, implying a new equilibrium such as that at A', reinforces the increase in total output that we saw in partial equilibrium. However, a rise in wages leading to a point such as A'' offsets it, though it cannot do so fully as we have seen.

We can summarize the change in the wage rate as follows:

Proposition 7 *Globalization has an ambiguous effect on the wage rate, which is more likely to rise: (a) the greater is the market-size effect on total output relative to the competition effect; and (b) the closer is the firm's average labour requirement, $\beta(\delta)/\alpha(\delta)$, to the average of its labour requirements across all its varieties, μ'_{γ}, i.e., the lower is the variance of the firm's labour requirements across its varieties, σ^2_{γ}.*

Condition (b) is necessarily met in an otherwise-identical model with only single-product firms, since there is then no distinction between the labour requirements of the firm and of the good it produces.[21] Hence the possibility of a fall in wages arises specifically because of the heterogeneity of production techniques across the different varieties produced by multi-product firms.

Consider next the determination of firm scope. This is straightforward given our assumption of symmetry across sectors. The requirement that aggregate labour supply must equal labour demand fixes the firm's variable labour demand as given in (19). This implies from (51) that the expansion in firm scale induced by globalization must be matched by a contraction in firm scope, though for different reasons from those in partial equilibrium:

$$\frac{d\ln\delta}{d\ln k} = -\frac{\alpha(\delta)\beta(\delta)}{\delta^2\alpha_\delta\sigma^2_\gamma}\frac{d\ln X}{d\ln k} < 0 \tag{34}$$

Here the negative relationship between scale and scope is imposed by the aggregate resource constraint, whereas in partial equilibrium it arose because the competition effect squeezed the firm's higher-cost varieties by more. Comparing the two responses, the general-equilibrium fall in scope will be greater if and only if the wage rises in equilibrium, since a higher wage increases the cost penalty of producing varieties further from the firm's core competence, so encouraging a further contraction in the product range. As we have already seen in Section 2.4, this means in turn that higher wages induce a greater increase in firm productivity.

These changes in scale and scope have implications for the change in the output profile across varieties. Just as in partial equilibrium, the profile becomes steeper: the firm produces more of varieties closer to its core competence and less of those furthest away. In addition, it is clear from equation (29) that a wage increase accentuates this increased steepness whereas a wage fall attenuates it. As a result, the threshold variety whose output does not change is lower than in partial equilibrium if and only if the

[21] Derivations are available on request. In the case of a linear cost function, a necessary condition for a fall in wages is that the number of firms is greater than two, ensuring that the competition effect is sufficiently strong: $\frac{\delta\Delta_2}{\alpha(\delta)\Delta_1}\frac{d\ln w}{d\ln k} = [2(1-e)+e\delta]\gamma_0 + \frac{2}{3}(1-e)\gamma\delta + \frac{1}{6}e\delta^2\gamma(2-km)$.

wage rises:

$$\tilde{\gamma}^{PE} - \tilde{\gamma}^{GE} = \frac{\alpha(\delta)}{\delta\mu'_\gamma}\left[\frac{\beta(\delta)}{\alpha(\delta)} - \frac{e\delta km}{\Delta_1}\mu'_\gamma\right] = \frac{\Delta_2}{\mu'_\gamma\Delta_1}\frac{d\ln w}{d\ln k} \tag{35}$$

(See the Appendix for an explicit proof.) This implies that a higher wage induces the firm to reduce the output of more products, so increasing the tendency towards a leaner output profile.

Summarizing the effects of globalization in general equilibrium:

Proposition 8 *In general equilibrium, an increase in k raises total output and productivity and lowers firm scope. If and only if the wage rises, then, relative to partial equilibrium: output rises by less, productivity rises by more, scope falls by more, and the range of varieties which are produced in lower volumes is greater.*

Finally, the additional reduction in firm scope which a higher wage induces makes it more likely than in partial equilibrium that overall product diversity may fall as a result of globalization. In particular, it is now possible for product diversity to fall even if costs are linear in varieties, unlike in partial equilibrium. To compute the change in the total number of varieties, we totally differentiate $N = km\delta$ and use (34) to obtain:

$$\frac{d\ln N}{d\ln k} = 1 + \frac{d\ln\delta}{d\ln k} = 1 - \frac{\alpha(\delta)\beta(\delta)}{\delta^2\alpha_\delta\sigma_\gamma^2}\frac{d\ln X}{d\ln k} \tag{36}$$

The increase in k raises total output X less than proportionately, but as in Section 3.3 this could be offset if $\frac{\delta\alpha_\delta}{\alpha(\delta)}$, the elasticity of the cost savings from flexible manufacturing, is sufficiently low. Calculating the change in variety explicitly gives:

$$\frac{d\ln N}{d\ln k} = \frac{1}{\Delta'}\left[2(1-e)\left\{\mu''_\gamma - \frac{\beta(\delta)}{\delta\alpha_\delta}\gamma(\delta)\right\} + e\left\{\delta(1+km)\sigma_\gamma^2 - \alpha(\delta)\frac{\beta(\delta)}{\delta\alpha_\delta}\right\}\right] \tag{37}$$

which shows that both market-size and competition effects are dampened if the cost function is sufficiently flat at the optimum so that the number of varieties per firm falls by enough. This effect can dominate even if the cost function is linear.[22] Summarizing:

Proposition 9 *In general equilibrium, an increase in k may lower the total number of varieties irrespective of the curvature of the cost function; the change in the total number of varieties is smaller than in partial equilibrium if and only if the wage falls.*

5 Extensions

Having presented the properties of the core model in detail, we note in this section the implications of relaxing a number of simplifying assumptions, concentrating for ease of exposition on the industry

[22] With linear costs, the expression in brackets in (37) becomes: $2(1-e)\left[\frac{1}{2}\gamma_0^2 + \frac{1}{3}\gamma_0\gamma\delta + \frac{1}{6}\gamma^2\delta^2\right] + \frac{1}{12}e(km\gamma\delta - 3\gamma_0)$. Hence necessary conditions for diversity to fall are: $\frac{3e-4\gamma\delta}{12(1-e)} > \gamma_0 > \frac{1}{3}km\gamma\delta$.

equilibrium case. The working paper version of our paper, Eckel and Neary (2005), shows that the results are also robust to relaxing the assumption of symmetry, allowing for both inter-country asymmetries and for the coexistence of single- and multi-product firms.

5.1 Free Entry

The assumption of free entry sits uneasily with our model in which firms produce a large portfolio of differentiated varieties and engage in strategic interaction with each other.[23] Nevertheless it is desirable to check that our results are robust to relaxing the assumption that the number of firms in each country is fixed. This implies adding a third margin of adjustment, the "inter-firm extensive margin", to our model which so far has concentrated on the intensive margin and the "intra-firm extensive margin". We therefore reexamine the effects of globalization treating the number of firms per country, m, as a continuous variable.

To establish the effects of globalization on the incentives for entry or exit, we first consider its effects on profits for a given number of firms. Substituting from the first-order conditions for output (7) into expression (6) for profits, we find that operating profits π are proportional to a weighted average of the square of total output and the integral of squared outputs of all varieties:

$$\Pi = \pi - F \quad \text{where:} \quad \pi = b' \left[(1-e) \int_0^{\delta} x(i)^2 \, di + eX^2 \right] \tag{38}$$

To help with intuition in what follows, note that, for a given level of total output X, profits are greater the larger is the ratio of $\int_0^{\delta} x(i)^2 \, di$ to X^2. As we show in the Appendix, this ratio can be interpreted as an index of the *flexibility* of technology, which we denote by $\Phi \equiv \int_0^{\delta} x(i)^2 \, di / X^2$: high values of Φ imply that there is a relatively large variance in the cost savings on infra-marginal varieties due to flexible manufacturing.

Not surprisingly, profits rise when the market size expands, and fall when competition increases. However, the net effect of globalization is ambiguous:

$$\frac{d\pi}{d \ln k} = (1-e) b' X^2 \left[\Phi - \frac{e}{1-e} \frac{\Delta_0}{\Delta_1} (km - 1) \right] \tag{39}$$

This shows that the positive market-size effect is more likely to dominate the negative competition effect the greater the flexibility of technology, and it must do so when there is only one firm ($km = 1$). However, even in the case of linear costs, profits can fall if competition is sufficiently intense, due to a sufficiently high degree of substitutability e and a sufficiently large number of firms. Summarizing:

[23] The problem is not free entry by itself but free entry where we ignore the integer problem, as we do in this sub-section, in common with Bernard et al. (2006a and 2006b) and Nocke and Yeaple (2006), who model multi-product firms with free entry only. Strictly speaking, this implies that every firm is infinitesimal in size relative to the scale of the market and exerts no impact on its rivals.

Proposition 10 *Firm profits are increasing in L and decreasing in m, but depend ambiguously on k.*

The change in profits with fixed firm numbers determines in turn the endogenous response of firm numbers in the free-entry equilibrium.[24] Incumbent firms may be forced to exit or new firms may be encouraged to enter, and the change in firm output X is also ambiguous:[25]

$$\frac{d\ln X}{d\ln k} = \frac{(1-e)\,\delta}{2\,(1-e+e\delta)}\left[\frac{\Delta_0}{(1-e)\,\delta} - \Phi\right] \tag{40}$$

Combining (39) and (40), we can distinguish three cases, depending on the relative size of the flexibility index Φ:[26]

(i) Low flexibility, $\Phi < \frac{e}{1-e}\frac{\Delta_0}{\Delta_1}(km-1)$: For given total output X, profits in this case are relatively low on average, from (38), and are reduced by globalization at the margin, from (39). The exit of some incumbent firms means that survivors face less competition, so firm output rises by more than in the no-entry case.

(ii) Intermediate flexibility, $\frac{e}{1-e}\frac{\Delta_0}{\Delta_1}(km-1) < \Phi < \frac{\Delta_0}{(1-e)\delta}$: Now higher profits encourage more entry, but this is not sufficient to reduce firm output.

(iii) High flexibility, $\Phi > \frac{\Delta_0}{(1-e)\delta}$: Here the induced entry is sufficient to force a contraction of scale by incumbents.

Notwithstanding these ambiguities, a number of features are common to all three cases. First, as shown in the Appendix, although firm output may fall, the total output of all firms in each country, mX, must rise. Second, although the number of incumbent firms in each country may fall, the total number of firms in all countries, mk, must rise. Finally, this in turn means that, from Section 3, firm scope δ always falls, so firm productivity always rises. Summarizing:

Proposition 11 *With free entry, globalization may lead to exit of incumbent firms if flexibility is low and may reduce firm output if flexibility is high. However, in all cases it reduces firm scope and raises firm productivity, total national output, and the total number of firms in the global economy.*

Finally, to see the effect on product diversity, recall the expression for the total number of varieties:

$$N = km\delta \quad \rightarrow \quad \frac{d\ln N}{d\ln k} = 1 + \frac{d\ln m}{d\ln k} + \frac{d\ln\delta}{d\ln k} \tag{41}$$

[24] The argument is standard. Write the free-entry equilibrium condition as $\pi(m,\theta) = F$, where θ is any exogenous parameter, here either L or k. Totally differentiating yields: $\frac{dm}{d\theta} = -\frac{\pi_\theta}{\pi_m}$. The denominator π_m (which measures the competition effect on profits) must be negative in the neighbourhood of an equilibrium for local stability; and direct calculation (see equation (68) in the Appendix) shows that it is always negative, so equilibrium is unique. Hence the change in firm numbers in the free-entry equilibrium, $\frac{dm}{d\theta}$, has the same sign as the change in profits when firm numbers are given, π_θ.

[25] When the cost function has constant curvature, $\gamma(i) = \gamma_0 + \frac{\gamma}{\eta}i^\eta$, output must rise. To see this, substitute for the value of the flexibility index Φ implied by this cost function from equation (73) in the Appendix.

[26] Only three, because it is always true that $\frac{e}{1-e}\frac{\Delta_0}{\Delta_1}(km-1) < \frac{\Delta_0}{(1-e)\delta}$. The economic intuition is that, when the flexibility index equals the lower threshold, the profits of incumbent firms are unaffected by globalization. In that case, additional entry or exit, though free, does not occur, and firm output rises. Hence there is no case where both firm output and firm numbers fall.

In the free-entry case, the total change in δ can be decomposed into the impact effect of globalization without entry (i.e.,the effect considered in Section 3) and the induced effect of the rise in the number of firms per country:

$$\frac{d\ln\delta}{d\ln k}=\frac{\partial\ln\delta}{\partial\ln k}+\frac{\partial\ln\delta}{\partial\ln m}\frac{d\ln m}{d\ln k}=\frac{\partial\ln\delta}{\partial\ln k}\left(1+\frac{d\ln m}{d\ln k}\right) \tag{42}$$

The second equality follows because, as we saw in Section 3, there is no market-size effect on product scope, so the partial effects $\frac{\partial\ln\delta}{\partial\ln k}$ and $\frac{\partial\ln\delta}{\partial\ln m}$ are the same. Hence the total change in product diversity from (41) can be written as:

$$\frac{d\ln N}{d\ln k}=\left(1+\frac{\partial\ln\delta}{\partial\ln k}\right)\left(1+\frac{d\ln m}{d\ln k}\right) \tag{43}$$

The first expression in brackets is the total change in N without entry: as we saw in Section 3, this can be negative if technology is sufficiently flexible at the optimal variety. The second expression in brackets is the total increase in the number of firms in the world market following a unit increase in k: as we have seen in Proposition 11 this can be greater or less than one but must be positive. Hence we can conclude that the effect on diversity of changes in the inter-firm extensive margin induced by globalization cannot reverse the effect of changes in the intra-firm extensive margin. The total effect is qualitatively the same as in the no-entry case. Summarizing these results:

Proposition 12 *With free entry, the effect of globalization on total product variety is qualitatively identical to that in the no-entry case, but may be greater or less in absolute value.*

With appropriate qualifications therefore, the effects of globalization when entry is free are qualitatively the same as those in the no-entry case already considered.[27]

5.2 Heterogeneous Firms

Our presentation of the core model assumed for simplicity that firms were identical, but it is straightforward to show that the main results continue to hold when this assumption is relaxed. Returning to the case of no entry, we assume a given configuration of firms with arbitrary differences in their cost schedules $\gamma_j(i)$, subject to the boundary constraint that all produce at least some varieties in equilibrium.

Substituting (18) into (20) yields a single expression for industry output:

$$Y=\sum_{j=1}^{m}X_j \quad \text{where:} \quad X_j=\frac{w\alpha_j(\delta_j)}{2b'(1-e)}, \tag{44}$$

Clearly, industry output is increasing in the product range of every multi-product firm. Similarly, substituting (18) into (12) yields an expression for the product range of each multi-product firm as

[27] For ease of exposition this sub-section has presented results for the case of free entry in partial equilibrium only. Qualitatively identical results for changes in X, m and δ hold in general equilibrium: details on request.

a function of industry output:

$$w\gamma_j(\delta_j) + \frac{e}{2(1-e)} w\alpha_j(\delta_j) = a' - b'eY \tag{45}$$

Equations (44) and (45) comprise $m+1$ equations in δ_j and Y that can be solved for the industry equilibrium, with each firm's total output recoverable from (44). It is straightforward to show that the responses of each firm to changes in L, m and k are qualitatively identical to the average responses derived in Section 3.[28] (Details are available on request.) In addition, we can compare the responses of different multi-product firms. The relative responses of the product ranges of any two multi-product firms j and h to changes in L or k are given by:

$$\frac{d\delta_j}{d\delta_h} = \frac{\varphi_j \phi_j}{\varphi_h \phi_h} \tag{46}$$

where:

$$\varphi_j \equiv \frac{(2-e)\delta_j}{2(1-e)+e\delta_j} \qquad \text{and} \qquad \phi_j \equiv \left[\delta_j c_\delta^j(\delta_j)\right]^{-1} \tag{47}$$

Here φ_j is an increasing concave function of the product range δ_j, while ϕ_j is the inverse semi-elasticity of marginal cost, evaluated at the marginal variety, and so can be interpreted as a local measure of firm j's marginal flexibility of production (by contrast with the global measure Φ used in the last sub-section). Equation (46) shows that firms with longer product lines (for a given marginal flexibility) and with more flexible technology at the margin (for a given length of product line) tend to respond more to shocks. The former result is consistent with the empirical finding of Bernard, Redding and Schott (2006) that larger firms are more active in changing their product mix.

These results can be summarized as follows:

Proposition 13 *In partial equilibrium, an increase in competition reduces the product range δ_j of all multi-product firms and raises industry output Y. An increase in the size of the market also leads to an increase in industry output Y but leaves the product ranges δ_j unaffected. Multi-product firms with longer product lines and with more flexible technology tend to respond more to changes in market size and in the number of countries in the global economy.*

5.3 Partial Trade Liberalization

So far we have modelled the process of globalization as an all-or-nothing one, with previously autarkic economies joining a free-trading world. A natural question to ask is how the results are affected when we consider instead the more realistic case of a partial reduction of pre-existing non-prohibitive trade

[28] When firms are heterogeneous, we cannot simply differentiate with respect to m. However, inspection of equations (44) and (45) confirms that (provided all incumbent firms remain profitable) the entry of an additional firm in any country has a competition effect on incumbent firms with the same qualitative properties as in the homogeneous-firms case.

barriers. Exploring this extension in detail would take us too far afield, but it suffices to note briefly that the qualitative results of the paper continue to hold, subject to two qualifications.

The first qualification, familiar since Brander (1981), is that the relative importance of the competition and market-size effects differs depending on the initial height of trade costs. Suppose that the home and foreign markets are segmented, and that foreign sales incur specific tariffs. (Transport costs would have similar relative-price effects.) Hence the competition and market-size effects apply to home and foreign sales respectively. When the initial tariff is prohibitive, a small multilateral reduction in tariffs exposes domestic sales to a finite competition effect (since profit margins are squeezed on all units sold) whereas exports benefit from only an infinitesimal market-size effect (since exports are initially zero). Hence the negative competition effect dominates, so both firm scale and scope should contract, and profits fall, encouraging exit of some incumbent firms. By contrast, starting from free trade, the imposition of a small tariff benefits home sales by a small competition effect (since rivals' costs rise) but imposes a larger negative market-size effect on export sales (since the firm's own costs of serving foreign customers rise). Hence in this case too the negative effect dominates, now encouraging a fall in output though possibly a rise in scope, coupled with a fall in profits. The effects derived in the main body of the paper are the integral of the combined effects as trade costs are progressively lowered.

A second qualification arises from the special character of our model of multi–product firms. Not only the volume of sales but also the range of products sold will differ in general between markets. With obvious extensions, the model predicts that firms will sell a larger range of products in markets that have lower access costs and fewer competitors; while the volume of sales will be increasing in the size of the market, holding tariffs and numbers of competitors constant. For high trade costs only products close to a firm's core competence will be exported, while trade liberalization is likely to encourage firms to expand the range of products exported. Refining these predictions and confronting them with data is an obvious priority for future work.

6 Conclusion

In this paper we have developed a new model of multi-product firms which highlights the role of flexible manufacturing. In line with the increasing empirical evidence that adjustments in the range of products produced by firms are an important component of changes in output and exports, our model highlights this hitherto-neglected channel of adjustment. Our focus is on the intra-firm adjustments within multi-product firms and we find that economy-wide shocks can have a considerable impact on both scale and scope. In addition, our analysis shows that the general-equilibrium feedback effects, through changes in wages and income, are an important determinant of changes in product ranges.

Two predictions of our model stand out. First, the model highlights a new source of gains from trade, as within-firm adjustments generate a rise in productivity, even in the absence of entry and exit. Existing firms face pressure to become "leaner and meaner", contracting their product range in response to additional competition, while simultaneously expanding total output to avail of new foreign markets. Hence selection effects operate at the product level, with firms encouraged to focus on their "core competence" and drop marginal high-cost varieties. Second, the model draws attention to a new source of potential losses from trade liberalization, in the form of a fall in product diversity. Even though the number of firms in the global economy rises, each produces fewer products, and the latter effect may dominate if technology is sufficiently flexible.

Our results suggest that adjustment processes within multi-product firms are significantly different from adjustments through exit and entry. Standard trade theory based on single-product firms in monopolistic competition predicts that international market integration raises the real wages of all participating countries and unambiguously increases the choices available to consumers. While this outcome is still possible in our framework, our results show that other outcomes are also possible depending on the extent of competition and on the degree of flexibility in manufacturing. First, the change in the real wage depends on whether the impact of an increase in competition from abroad is accompanied by an increase in foreign demand, because the competition effect tends to lower the real wage while the demand effect tends to raise it. Second, if manufacturing technologies are highly flexible, multi-product firms respond to shocks more by altering their product range than their total output, which can lead to a fall in overall product diversity when new countries enter the world market.[29] These results are substantially different from the predictions of standard trade theory even though both sets of results are driven by the same forces, an increase in the number of firms and an increase in the size of the market. This difference in predictions underlines the importance of adjustments on the intra-firm extensive margin which our model highlights. At the same time, Section 5.1 shows that our model's predictions are robust to allowing for firm entry and exit, which adds adjustment on the inter-firm extensive margin, familiar from standard models of monopolistic competition, to the intra-firm adjustments of our model.

Furthermore, our look inside a firm's product range reveals new and testable insights into how infra-marginal products adjust. Because flexible manufacturing creates cost heterogeneities within firms, asymmetric adjustment processes are possible that differ significantly from adjustments via exit and entry. These processes arise even at the level of a single industry, and they are accentuated by changes in factor prices, underlining the importance of a general equilibrium approach.

Our framework can be extended in various directions. In Eckel and Neary (2006) we present an extension that analyzes the general-equilibrium feedback effects between asymmetric industries. This provides insights into how adjustments within multi-product firms can differ between industries and

[29] This is quite consistent with the findings of Broda and Weinstein (2006) that the diversity of *imports* has increased as a result of trade liberalization. Moreover, their study assumes CES preferences, which place a higher premium on diversity than quadratic preferences.

shows that industries which are not subject to direct foreign competition in their own markets are still affected by a competition effect through the labor market. We also allow for heterogeneous firms in our partial equilibrium analysis. Further extensions, to allow for heterogeneous firms in general equilibrium, and to consider how firms choose their degree of flexibility, seem well worth exploring in our framework.

Empirical evidence suggests that multi-product firms are an important feature of modern industries. Our results show that adjustment processes within multi-product firms differ substantially from adjustments via exit and entry and that globalization can be a driving force of these adjustment processes.

7 Appendix

7.1 Preliminaries

Equations (16) and (18) introduced shorthand terms $\beta(\delta)$ and $\alpha(\delta)$ for the technological components of the integrals of labour demand and output respectively, while equations (19) and (32) introduced the first and second central moments and the variance of the distribution of labour requirements across all the varieties produced by each firm in equilibrium. These terms can be related to each other as follows:

$$\alpha(\delta) = \delta\left[\gamma(\delta) - \mu'_\gamma\right] \tag{48}$$

$$\beta(\delta) = \delta\left[\gamma(\delta)\mu'_\gamma - \mu''_\gamma\right] = \alpha(\delta)\mu'_\gamma - \delta\sigma^2_\gamma = \frac{\alpha(\delta)\mu''_\gamma - \delta\gamma(\delta)\sigma^2_\gamma}{\mu'_\gamma} \tag{49}$$

Similarly for their derivatives:

$$\alpha_\delta = \delta\gamma_\delta \qquad \beta_\delta = \mu'_\gamma\alpha_\delta \tag{50}$$

7.2 Changes in Productivity: Proof of Proposition 1

Consider first the change in productivity when output changes are uniformly weighted. Using the expressions for $\beta(\delta)$, α_δ and β_δ from (49) and (50), the logarithmic change in measured productivity with respect to a change in firm scope is given by:

$$\left.\frac{\partial \ln LP}{\partial \ln \delta}\right|_{h(i)=1} = \frac{\partial \ln X}{\partial \ln \delta} - \frac{\partial \ln l}{\partial \ln \delta} = \frac{d\ln\alpha(\delta)}{d\ln\delta} - \frac{d\ln\beta(\delta)}{d\ln\delta} = \frac{\delta\alpha_\delta}{\alpha(\delta)} - \frac{\delta\beta_\delta}{\beta(\delta)} = -\frac{\delta^2\alpha_\delta\sigma^2_\gamma}{\alpha(\delta)\beta(\delta)} \tag{51}$$

Since the variance σ^2_γ must be positive, it follows that productivity is decreasing in δ as claimed.

Consider next the change in productivity when output changes are price-weighted:

$$\left.\frac{\partial \ln LP}{\partial \ln \delta}\right|_{h(i)=p(i)} = \frac{\int_0^\delta p(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta p(i)x(i)\,di} - \frac{\partial \ln l}{\partial \ln \delta} \tag{52}$$

As we have already seen, the change in labour input is: $\frac{\partial \ln l}{\partial \ln \delta} = \frac{\delta\beta_\delta}{\beta(\delta)} = \delta\frac{\mu'_\gamma\alpha_\delta}{\beta(\delta)}$. Turning to the change in output, we first write the value of output of a single variety as a function of its own and the threshold variety's unit labour requirements:

$$p(i)x(i) = w\left[\frac{1}{2}\{\gamma(i) + \gamma(\delta)\} + e\frac{\alpha(\delta)}{2(1-e)}\right]\frac{w}{2(1-e)b'}\left[\gamma(\delta) - \gamma(i)\right] \tag{53}$$

Integrating over all varieties produced by the firm and simplifying yields:

$$\int_0^\delta p(i)x(i)\,di = \frac{w^2}{4(1-e)^2 b'}H \quad \text{where:} \quad H \equiv (1-e)\{\alpha(\delta)\gamma(\delta) + \beta(\delta)\} + e\alpha(\delta)^2 \tag{54}$$

Similarly, the numerator of the weighted output change can be written as:

$$\int_0^\delta p(i)\frac{\partial x(i)}{\partial \ln \delta}di = \frac{w^2}{4(1-e)^2 b'}\left[(1-e)\left\{\mu'_\gamma + \gamma(\delta)\right\} + e\alpha(\delta)\right]\delta\alpha_\delta \tag{55}$$

Combining these results, we see that:

$$\left.\frac{\partial \ln LP}{\partial \ln \delta}\right|_{h(i)=p(i)} = \frac{(1-e)\left\{\mu'_\gamma + \gamma(\delta)\right\} + e\alpha(\delta)}{H}\delta\alpha_\delta - \delta\frac{\mu'_\gamma\alpha_\delta}{\beta(\delta)} \tag{56}$$

Simplifying, this becomes:

$$\left.\frac{\partial \ln LP}{\partial \ln \delta}\right|_{h(i)=p(i)} = -\frac{\delta^2\alpha_\delta}{\beta(\delta)H}\left[(1-e)\gamma(\delta) + e\alpha(\delta)\right]\sigma_\gamma^2 < 0 \tag{57}$$

This is negative, but, as can easily be checked, it is less so than the change in the fixed-weight productivity index given in (51), which confirms the results stated in Proposition 1.

7.3 Industry Equilibrium: Comparative Statics

Totally differentiating the equilibrium conditions for scope and scale, equations (21) and (22), with the results written as a matrix equation, gives:

$$\begin{bmatrix} \Delta_1 & 0 \\ e(1+km) & \frac{2(1-e)\delta\gamma_\delta}{\alpha(\delta)} \end{bmatrix}\begin{bmatrix} d\ln X \\ d\ln \delta \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ e(1+km) \end{bmatrix} d\ln L \tag{58}$$
$$- \begin{bmatrix} \delta \\ 1 \end{bmatrix} ekmd\ln m + \begin{bmatrix} \Delta_0 \\ e \end{bmatrix} d\ln k - \begin{bmatrix} \delta\mu'_\gamma \\ \gamma(\delta) \end{bmatrix}\frac{2(1-e)}{\alpha(\delta)}d\ln w$$

where Δ_1 and Δ_0 are defined in (22) and (26), with $\Delta_1 > \Delta_0 > 0$. The solutions are as follows:

$$d\ln X = d\ln L - \frac{e\delta km}{\Delta_1}d\ln m + \frac{\Delta_0}{\Delta_1}d\ln k - \frac{2(1-e)\delta\mu'_\gamma}{\Delta_1\alpha(\delta)}d\ln w \tag{59}$$

and

$$d\ln\delta = -\frac{e\delta km\alpha(\delta)}{\Delta_1\delta\alpha_\delta}(d\ln m + d\ln k) - \frac{2(1-e)\delta\mu'_\gamma + \Delta_1\alpha(\delta)}{\Delta_1\delta\alpha_\delta}d\ln w \tag{60}$$

Note that $\frac{d\ln X}{d\ln m} = -\left(1 - \frac{\Delta_0}{\Delta_1}\right) > -1$, so $\frac{d\ln Y}{d\ln m} = 1 - \frac{d\ln X}{d\ln m} > 0$ as noted in the text; and $\frac{d\ln\delta}{d\ln L} = 0$. We can also combine the total differential of the expression for $x(i)$ in (8) with (59) to obtain:

$$d\ln x(i) = d\ln L - \frac{ekm\alpha(\delta)}{\Delta_1[\gamma(\delta)-\gamma(i)]}d\ln m + \left[\frac{\Delta_0}{\Delta_1} + \left(1-\frac{\Delta_0}{\Delta_1}\right)\frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta)-\gamma(i)}\right]d\ln k \tag{61}$$
$$-\frac{2(1-e)\gamma(i) - e\delta(1+km)\left[\mu'_\gamma - \gamma(i)\right]}{\Delta_1[\gamma(\delta)-\gamma(i)]}d\ln w$$

7.4 General Equilibrium: Comparative Statics

Combining the total differential of the first-order condition for scale, equation (22), as in (59), and that of the labour-market equilibrium condition (32), and writing the results as a matrix equation gives:

$$\begin{bmatrix} \Delta_1 & \mu'_\gamma \\ e\delta(1+km)\mu'_\gamma & \mu''_\gamma \end{bmatrix} \begin{bmatrix} d\ln X \\ \frac{2(1-e)\delta}{\alpha(\delta)} d\ln w \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ 2(1-e)\frac{\beta(\delta)}{\alpha(\delta)} + e\delta(1+km)\mu'_\gamma \end{bmatrix} d\ln L \qquad (62)$$
$$- \begin{bmatrix} 1 \\ \mu'_\gamma \end{bmatrix} e\delta km d\ln m + \begin{bmatrix} \Delta_0 \\ 2(1-e)\frac{\beta(\delta)}{\alpha(\delta)} + e\delta\mu'_\gamma \end{bmatrix} d\ln k$$

(Recall that $e\delta km = \Delta_1 - \Delta_0$, and that L/m in the labour-market equilibrium condition is held constant, as explained in footnote 19.) The determinant of the coefficient matrix, denoted by Δ_2, is positive:

$$\Delta_2 \equiv 2(1-e)\mu''_\gamma + e\delta(1+km)\sigma^2_\gamma = 2(1-e)\left(\mu'_\gamma\right)^2 + \Delta_1\sigma^2_\gamma > 0 \qquad (63)$$

This is also proportional to the difference in slope between the LL and IE loci in Figure 5, implying the configuration of the loci discussed in the text.

The solutions are as follows:

$$\frac{\Delta_2}{\delta\sigma^2_\gamma} d\ln X = \left[2(1-e)\frac{\gamma(\delta)}{\alpha(\delta)} + e(1+km)\right] d\ln L - ekmd\ln m \qquad (64)$$
$$+ \left[2(1-e)\frac{\gamma(\delta)}{\alpha(\delta)} + e\right] d\ln k$$

$$\delta\Delta_2 d\ln w = \Delta_1\beta(\delta)\,d\ln L - e\delta km\alpha(\delta)\,\mu'_\gamma d\ln m + \left[\Delta_0\beta(\delta) - e\delta^2 km\sigma^2_\gamma\right] d\ln k \qquad (65)$$

Note that $\frac{d\ln X}{d\ln L} > 1$ and $0 < \frac{d\ln X}{d\ln k} < 1$. Also, $\frac{d\ln X}{d\ln m}$ must be less than -1, so we can be sure that $\frac{d\ln Y}{d\ln m}$ is positive, just like in partial equilibrium.

Consider next the changes in individual varieties. The easiest way to derive these is to combine (15) with (18) and totally differentiate:

$$x(i) = \frac{\gamma(\delta) - \gamma(i)}{\alpha(\delta)} X \quad \rightarrow \quad d\ln x(i) = d\ln X - \frac{\delta\alpha_\delta}{\alpha(\delta)} \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)} d\ln\delta \qquad (66)$$

Substituting from (34) for $d\ln\delta$ and from (64) for $d\ln X$ gives:

$$\frac{d\ln x(i)}{d\ln k} = \left[1 + \frac{\beta(\delta)}{\delta\sigma^2_\gamma} \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)}\right] \frac{d\ln X}{d\ln k} = \frac{2(1-e)\gamma(\delta) + e\alpha(\delta)}{\Delta_2} \frac{\mu''_\gamma - \mu'_\gamma\gamma(i)}{\gamma(\delta) - \gamma(i)} \qquad (67)$$

Hence the threshold variety whose output is unchanged is given by: $\tilde{\gamma}^{GE} = \frac{\mu''_\gamma}{\mu'_\gamma}$. Subtracting this from the corresponding expression in partial equilibrium from (27) gives equation (35).

7.5 Free Entry

To determine the comparative statics of profits, we differentiate equation (6), using the first-order condition for individual outputs, equation (24), to substitute for $x(i)$. This gives:

$$d\pi = -e(km-1)b'X^2 d\ln X + \left[\pi + e(km-1)b'X^2\right] d\ln L - ekmb'X^2 d\ln m + \left[(1-e)b'\int_0^\delta x(i)^2\, di\right] d\ln k \tag{68}$$

The coefficient of $d\ln X$ is negative, except in the monopoly case ($km = 1$): a symmetric increase in size by all firms lowers profits as the negative effect of greater competition from rivals dominates the positive effect of an increase in own sales. The other coefficients show that, at given output, the positive market-size effect dominates the negative competition effect, implying that a rise in k raises profits. Note that, for given X, profits are independent of δ by the envelope theorem.

Now, substitute from (59) for $d\ln X$ to give the full effects of exogenous shocks on profits when the number of firms is exogenous:

$$d\ln\pi = d\ln L - 2ekm\frac{b'X^2}{\pi}\frac{1-e+e\delta}{\Delta_1}d\ln m + (1-e)\frac{b'X^2}{\pi}\left[\Phi - \frac{e}{1-e}\frac{\Delta_0}{\Delta_1}(km-1)\right]d\ln k \tag{69}$$

This can alternatively be written in a form which brings out the role of Φ relative to its lower bound $(\frac{1}{\delta})$, using (38) to express π in terms of Φ,

$$d\ln\pi = d\ln L - \frac{2e\delta km}{\Delta_1}\frac{(1-e)\frac{1}{\delta}+e}{(1-e)\Phi+e}d\ln m + \left[1 - \frac{2e\delta km}{\Delta_1}\frac{(1-e)\frac{1}{\delta}+e}{(1-e)\Phi+e}\right]d\ln k \tag{70}$$

Setting $d\ln\pi$ equal to zero and moving the term in $d\ln m$ to the left-hand side gives the effects of changes in L and k on equilibrium firm numbers when entry is free. Similarly, by eliminating $d\ln m$ from (59) and (68), we can solve for the change in output with free entry, given by equation (40). (Note that $\frac{d\ln X}{d\ln L} = \frac{d\ln X}{d\ln k}$ with free entry.) Finally, direct calculations show that the changes in national output and global firm numbers are both positive:

$$\frac{d\ln(mX)}{d\ln k} = \Delta_0\frac{(1-e)\Phi+e}{2ekm(1-e+e\delta)} > 0 \quad \text{and} \quad \frac{d\ln(km)}{d\ln k} = \Delta_1\frac{(1-e)\Phi+e}{2ekm(1-e+e\delta)} > 0 \tag{71}$$

The ratio of these two terms is simply Δ_0/Δ_1, which from (59) is $d\ln X/d\ln k$ when m is fixed.

7.6 The Index of Technology Flexibility

As noted in the text, the ratio of the integral of squared outputs to the square of total output can be interpreted as an index of the flexibility of technology. It is closely related to the coefficient of variation of outputs, $\sigma_x^2/(\mu'_x)^2$, which in turn equals the coefficient of variation of the cost savings from flexible manufacturing, $\gamma(\delta)-\gamma(i)$, measured by the variance of $\gamma(i)$ deflated by the square of the average cost saving:[30]

$$\frac{\int_0^\delta x(i)^2\,di}{X^2} = \frac{1}{\delta}\left[1 + \frac{\sigma_x^2}{(\mu_x')^2}\right] = \frac{1}{\delta}\left[1 + \frac{\sigma_\gamma^2}{\left\{\gamma(\delta) - \mu_\gamma'\right\}^2}\right] \tag{72}$$

For example, suppose that the cost function takes the form $\gamma(i) = \gamma_0 + \frac{\gamma}{\eta} i^\eta$, with constant curvature $\frac{i\gamma_{ii}}{\gamma_i}$ equal to $\eta - 1$. Then the flexibility index becomes:

$$\frac{\int_0^\delta x(i)^2\,di}{X^2} = \frac{1}{\delta}\left[1 + \frac{1}{2\eta + 1}\right] = \frac{1}{\delta}\frac{2(\eta + 1)}{2\eta + 1} \tag{73}$$

which is decreasing in η. This cost function is convex for $\eta > 1$, and technology is least flexible, resembling that for a single-product firm (i.e., with $\gamma(i)$ close to vertical) for large η, as the flexibility index approaches its lower bound of $\frac{1}{\delta}$. By contrast, for $\eta < 1$, the cost function is concave and technology is more flexible, with the flexibility index approaching its upper bound of $\frac{2}{\delta}$.

[30] To prove this, we use equations (19) and (21). As already noted, the total cost savings from flexible manufacturing is $\alpha(\delta)$, so from (48) the average cost saving is $\alpha(\delta)/\delta = \gamma(\delta) - \mu_\gamma'$.

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Figure 1: Core Competence and Flexible Manufacturing: The Profit-Maximizing Product Range

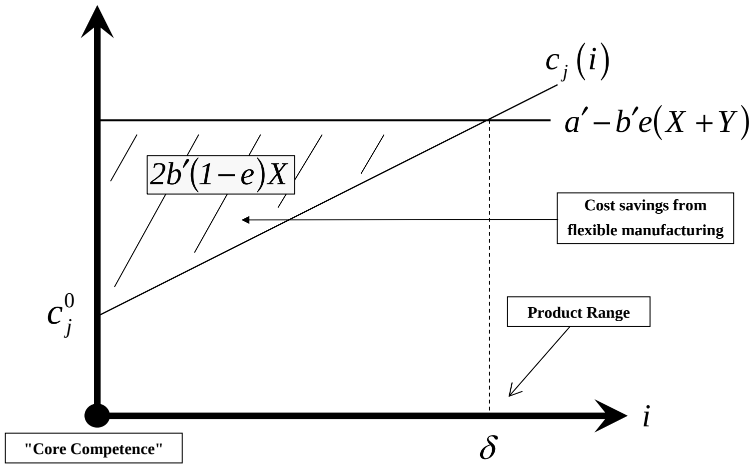

Figure 2: The Scale of Production and the Cannibalization Effect

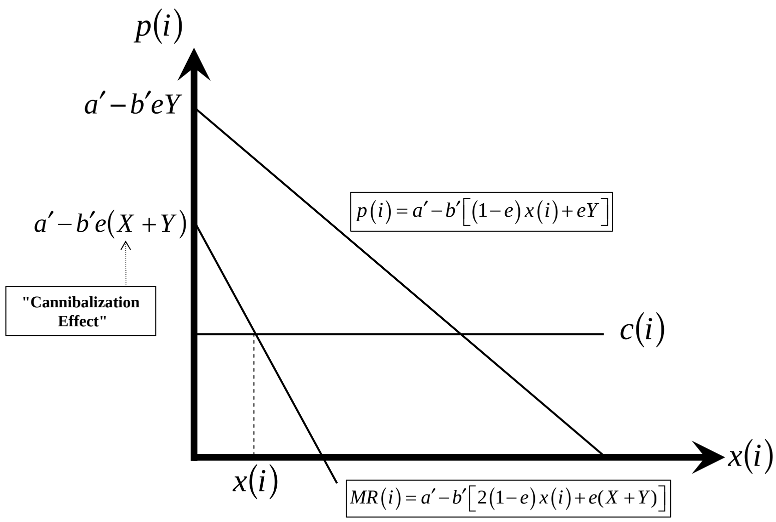

Figure 3: Industry Equilibrium and the Effects of Globalization

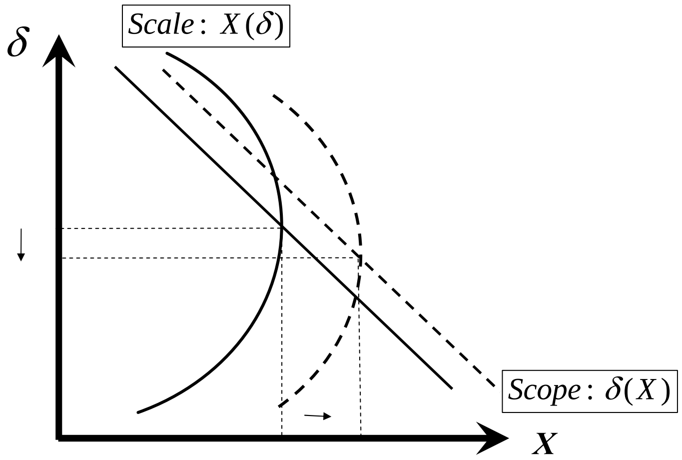

Figure 4: The Profile of Outputs in Partial Equilibrium

[The arrows indicate the responses to an increase in *k*]

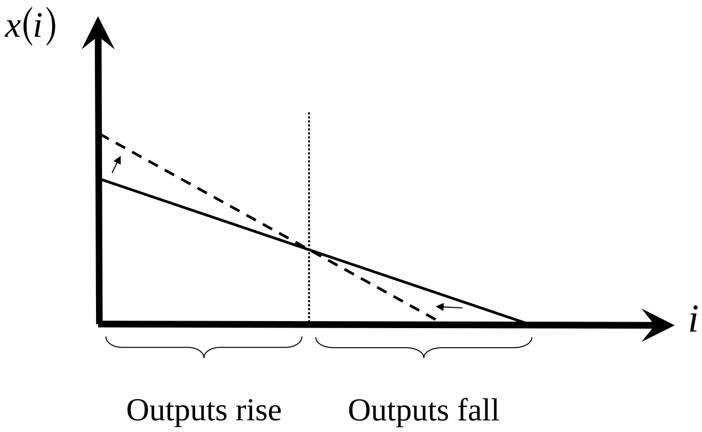

Figure 5: General Equilibrium

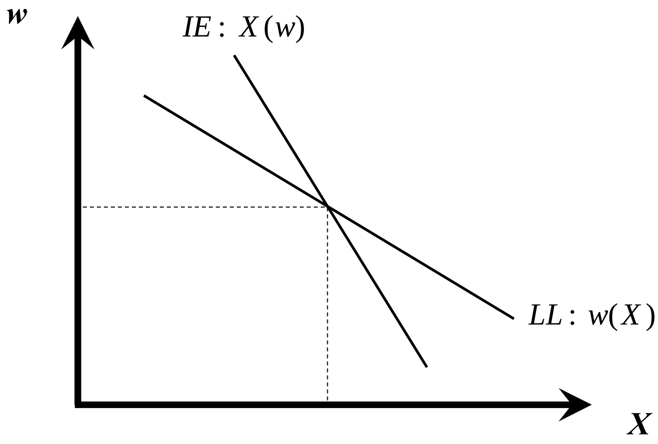

Figure 6: Effects of Globalization in General Equilibrium