INTERNATIONAL TRADE IN GENERAL OLIGOPOLISTIC EQUILIBRIUM*,†

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Abstract

This paper presents a new model of oligopoly in general equilibrium and explores its implications for positive and normative aspects of international trade. Using a "continuum-quadratic" specification of preferences, the model allows for consistent aggregation over a continuum of sectors, each of which is characterised by Cournot competition between a small number of home and foreign firms. I explore the model's implications for the gains from trade, for the distribution of income between wages and profits, and for production and trade patterns in a two-country world, and show how competitive advantage interacts with comparative advantage to determine resource allocation.

Keywords: Comparative and competitive advantage; continuum-quadratic preferences; GOLE (General Oligopolistic Equilibrium); market integration.

JEL Classification: F10, F12

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1 Introduction

International markets are typically characterized by firms which are relatively large in the markets in which they compete. What are the implications of this undeniable fact for trade patterns, the gains from trade, and the effects of trade policy on income distribution? These are some of the questions with which this paper is concerned.

Of course, these questions are not new. For almost thirty years, they have been extensively addressed in the literature on the "new trade theory", which has contributed enormously to our understanding of international markets. However, this literature really consists of two distinct strands which have relatively little in common with each other. On the one hand, general equilibrium models of monopolistic competition have been applied to mostly positive questions of trade and location; on the other hand, partial equilibrium models of oligopoly have been applied to mostly normative questions of "strategic" policy choice.[1] While both approaches have proved extremely fruitful, they suffer from some limitations. Models of monopolistic competition allow for increasing returns to scale and product differentiation in general equilibrium. However, since they assume that firms are atomistic and do not engage in strategic behaviour, they represent little advance in descriptive realism over models of perfect competition. Oligopoly models by contrast allow for a wide range of sophisticated strategic interactions between firms. However, since they typically ignore interactions between markets, and especially between goods and factor markets, they cannot deal with many of the classic questions of international trade theory.[2]

This paper aims to advance the unfinished part of the new trade theory revolution, by developing a framework which should also have applications in other fields: a tractable but consistent model of oligopoly in general equilibrium. Previous attempts at this goal have foundered on one of a number of related problems.[3] The essence of any oligopoly model is that firms have significant power in their own market, and that they exploit this market power strategically. But many attempts to model this formally have assumed that firms which are large in their own market are also large players in the economy as a whole, which opens a Pandora's Box of technical difficulties. For example, if firms are large in the economy, they influence the factor prices they face, in which case they should exercise this monopsony market power strategically. Moreover, if firms

[1] Helpman and Krugman (1985) and (1989) give classic overviews of these two strands, respectively. Of course, the two strands overlap to some extent. For example, Chapter 5 of Helpman and Krugman (1985) presents some models of oligopoly in general equilibrium, though without addressing the problems discussed below.

[2] Naturally, this brief summary fails to do justice to an enormous literature. One conspicuous exception is the work of Brander (1981), subsequently extended by Brander and Krugman (1983), Weinstein (1992) and Yomogida (2008). Though confined to partial equilibrium, this shows that oligopolistic competition in segmented markets provides a distinct motive for "cross-hauling" or two-way trade. With hindsight, the papers by Dixit and Grossman (1986) and Neary (1994) can be seen as attempts to provide rudimentary general-equilibrium foundations for open-economy oligopoly models, by endogenising factor supplies and the government budget respectively. The model of the present paper can be viewed as a general-equilibrium generalisation of Brander's, though for simplicity only integrated markets are considered.

[3] Probably the best-known papers in this area are Gabszewicz and Vial (1972) and Roberts and Sonnenschein (1977). For detailed references and further discussion, see Neary (2003).

are large in the economy, they also influence national income, so they should take this too into account in their behaviour. The resulting income effects often imply reaction functions which are extremely badly behaved, so that existence of equilibrium cannot be guaranteed even in the simplest models.[4] A more subtle difficulty is that, since firm owners influence the prices of their own outputs, they prefer lower rather than higher prices, the more they consume these goods. As a result, profit maximization leads to different outcomes depending on the tastes of profit-income recipients. Hence the apparent paradox that the properties of the model become sensitive to the choice of numéraire.[5]

Earlier writers have circumvented these difficulties either by ignoring them, or by explicitly modelling the simultaneous exercise of monopoly and monopsony power, or by assuming that firms maximize utility rather than profits. None of these approaches has met with wide approval. In this paper I adopt a different approach. I assume that the economy consists of a large number, strictly a continuum, of sectors, each with a small number of firms. Factors are intersectorally mobile, so factor prices are determined at the economy-wide level. This makes it possible to model firms as having market power in their own industry but not in the economy as a whole. They behave strategically against their local rivals but take income, prices in other sectors, and factor prices as given. Profits are earned in equilibrium, but they are distributed to consumers in a lump-sum fashion. Hence the difficulties faced by other models of oligopoly disappear.

Three technical building blocks are required to implement this approach. First, we need a specification of demand which is tractable at the sectoral level but also allows consistent aggregation over different sectors. Section 2 introduces a specification which meets these requirements. Second, we need to understand the implications of oligopolistic competition between firms located in different countries, which differ in their cost structures. Even in partial equilibrium this requires considering the effects of market integration on production patterns. Section 3 extends the theory of oligopoly in open economies to consider these issues. Third, we need to link goods and factor markets in a consistent way. A natural framework in which to do this is the Ricardian continuum model of Samuelson (1964). Each one of a continuum of sectors is assumed to have different costs at home and abroad. Whereas previous writers have explored this model under competitive assumptions,[6] Section 4 shows how it can form the basis for a model of general oligopolistic equilibrium. The remainder of the paper explores the model's implications for production and trade patterns in a two-country world.

[4] See Roberts and Sonnenschein (1977).

[5] See Gabszewicz and Vial (1972).

[6] See Dornbusch, Fischer and Samuelson (1977 and 1980), Wilson (1980) and Matsuyama (2000) for extensions of the continuum model under perfect competition, and Romalis (2004) for an application to monopolistic competition.

2 Continuum-Quadratic Preferences

Utility is defined as an additive function of a continuum of goods defined on the unit interval, with each sub-utility function quadratic:

$$U\left[\{x(z)\}\right] = \int_0^1 u\left[x(z)\right] dz \qquad \text{where:} \qquad u\left[x(z)\right] = ax(z) - \frac{1}{2}bx(z)^2 \tag{1}$$

This implies that utility is increasing in the mean μ_1^x of consumption levels, and decreasing in their "uncentred variance", or second moment, μ_2^x:

$$U\left[\{x(z)\}\right] = a\mu_1^x - \frac{1}{2}b\mu_2^x \tag{2}$$

I assume a single representative consumer in each country, who maximizes (1) subject to the budget constraint:

$$\int_0^1 p(z)\, x(z)\, dz \leq I \tag{3}$$

where I is aggregate income. It is straightforward to calculate the inverse and direct demand functions for each good:

$$p(z) = \frac{1}{\lambda}\left[a - bx(z)\right] \qquad \text{and} \qquad x(z) = \frac{1}{b}\left[a - \lambda p(z)\right] \tag{4}$$

where λ is the marginal utility of income, the Lagrange multiplier attached to the budget constraint.

Previous applications of quadratic utility functions similar to (1) have been of two kinds. On the one hand, they have been extensively applied in international trade, industrial organization, and other fields to partial equilibrium issues, in which λ can be treated as fixed.[7] Formally, this is justified by adding an extra good x_0 to (1), so the utility function becomes quasi-linear: $U\left[x_0, \{x(z)\}\right] = x_0 + \int_0^1 u\left[x(z)\right] dz$. The marginal utility of income is then unity. On the other hand, quadratic preferences without the contrivance of quasi-linearity have been widely used in stochastic consumption theory (where they rationalize Euler equations that are linear in consumption) and finance (where they provide a distribution-free rationalization for the capital asset pricing model).[8] However, the focus and approach in these literatures are very different from mine. For example, in an intertemporal choice context, there is a natural sequencing of periods, so it makes sense to combine the demand (i.e., consumption) functions for adjacent periods and to eliminate λ.

In the model of this paper, there is no natural association between markets from the consumer's perspective, so λ cannot be eliminated. Moreover, with no quasi-linear term in (1), the value of λ is not constant. Instead, it depends on prices and income I, which in general equilibrium depend in turn on the underlying determinants: tastes, technology and market structure. To solve for λ, multiply each direct demand function

[7] See, for example, Dixit (1980), Vives (1985) and Ottaviano, Tabuchi and Thisse (2002).
[8] See, for example, Hall (1978), Deaton (1992) and Cochrane (2001).

by the corresponding price and add over all goods, using (3), to obtain:

$$\lambda\left[\{p(z)\}, I\right] = \frac{a\mu_1^p - bI}{\mu_2^p} \tag{5}$$

The effects of prices on λ are summarized by two price indices, μ_1^p and μ_2^p, which are the first and second moments of the distribution of prices:

$$\mu_1^p \equiv \int_0^1 p(z)\,dz \qquad \text{and} \qquad \mu_2^p \equiv \int_0^1 p(z)^2\,dz \tag{6}$$

Hence, a rise in income, a rise in the uncentred variance of prices, or a fall in the mean of prices, all reduce λ and so shift the demand function for each good outwards.

It is instructive to compare the "continuum-quadratic" preferences in (1) with the constant-elasticity-of-substitution specification of Dixit and Stiglitz (1977), widely used in models of monopolistic competition in general equilibrium:

$$U\left[\{x(z)\}\right] = \left[\int_0^1 x(z)^\theta\,dz\right]^{\frac{1}{\theta}}, \qquad 0 < \theta < 1 \tag{7}$$

The implied inverse and direct demand functions can be written as follows:[9]

$$p(z) = \frac{\theta}{\lambda}x(z)^{-\frac{1}{\eta}} \qquad \text{and} \qquad x(z) = \left[\frac{\lambda p(z)}{\theta}\right]^{-\eta} \tag{8}$$

(where the elasticity of demand η equals $(1-\theta)^{-1}$ and is also the elasticity of substitution between every pair of goods). Just as in (5), the marginal utility of income λ is a function of prices and income, except that now the former are aggregated into a single index, the true cost-of-living index P, which is a "mean of order $1-\eta$" of the individual prices:[10]

$$\lambda\left[\{p(z)\}, I\right] = \frac{\theta}{P^\theta I^{1-\theta}} \qquad \text{where:} \qquad P \equiv \left[\int_0^1 p(z)^{1-\eta}\,dz\right]^{\frac{1}{1-\eta}} \tag{9}$$

In this case too, λ is endogenous in general equilibrium, except when (7) is a sub-utility function in a quasi-linear specification, as in Spence (1976), so λ equals one.

The preference systems (1) and (7) share a key feature which makes them ideally suited to studying imperfect competition in general equilibrium: they imply perceived demand functions which take conveniently

[9] This way of writing the demands implied by Dixit-Stiglitz preferences facilitates comparison with (4). In practice, it is usually easier to write them directly in terms of prices and income, both deflated by P: $x(z) = \{p(z)/P\}^{-\eta}I/P$. Note that λ is the shadow price of the budget constraint when the Lagrangian is written in terms of U^θ. If written in terms of U, then λ is simply P^{-1}, and the demand functions do not yield such a neat parallel with (4).

[10] See Diewert (1993). The utility function in (7) is a mean of order θ, while the first two moments of the price distribution, μ_1^p and μ_2^p, are a mean of order 1 and a squared mean of order 2 respectively.

simple forms. In both cases the marginal utility of income serves as a "sufficient statistic" for the rest of the economy in each sector.[11] From the continuum assumption, individual firms are infinitesimally small, and so it is fully rational for them to treat λ as fixed.

In other respects, the two preference systems are very different. The iso-elastic perceived demand functions implied by (7) have proved their usefulness in the enormous literature on monopolistic competition in general equilibrium. However, iso-elastic demand functions in oligopoly are less attractive. In Cournot competition, they imply that outputs are strategic complements for many parameter values, and reaction functions may be non-monotonic.[12] By contrast, the linear perceived demand functions implied by (1) ensure that outputs are always strategic substitutes and that reaction functions are always well-behaved.

A potential drawback of linear demand functions is that the consumer may reach satiation if income is high enough or prices are low enough.[13] However, this is more than offset by the fact that demands for high-price goods may fall to zero. By contrast, with Dixit-Stiglitz preferences, the consumer always demands all goods even if some are much more expensive than others. This matters in the context of trying to explain intra-industry trade. Dixit-Stiglitz preferences come close to assuming that intra-industry trade will take place, whereas with continuum-quadratic preferences its occurrence is endogenous.[14]

A final difference between the two demand systems is that Dixit-Stiglitz preferences are homothetic, whereas continuum-quadratic are not.[15] This might suggest that continuum-quadratic demands cannot be aggregated across countries. However, Appendix 1 proves:[16]

Proposition 1 *Continuum-quadratic preferences are a sub-class of the Gorman polar form.*

The Gorman (1961) polar form, also known as quasi-homothetic preferences, implies that all income-consumption curves are linear (though not necessarily through the origin) so tastes are homothetic at the margin. Crucially, it allows for consistent aggregation over individuals, or, in a trade context, countries, with different incomes, provided the parameter b is the same for all.[17] In particular, in a two-country world, if the foreign country's preferences are represented by (1), with a^* instead of a, and if free trade prevails so

[11] This property holds for any additively separable specification of preferences. See Browning, Deaton and Irish (1985).

[12] Bandyopadhyay (1997) illustrates the complexities of even the simplest Cournot duopoly model when demands are iso-elastic.

[13] The satiation point is a/b. To see this, add $-a^2/2b$ to (1), to rewrite $u(x)$ as $-b(a/b-x)^2/2$.

[14] In this paper I allow for inter-sectoral trade only. At the macro level, this can be described as intra-industry trade, just as it is in the literature which uses Dixit-Stiglitz preferences. The model can easily be extended to allow for intra-industry trade at the micro (i.e., sectoral) level, either by assuming that international markets are segmented, as in Brander (1981), or by extending the sub-utility functions in (1) to allow for product differentiation within each sector, as in Vives (1985) or Neary (2002).

[15] Datta and Dixon (2000, 2001) develop a different way of rationalising linear demand functions, which is consistent with homothetic preferences but not with a continuum of goods.

[16] After this was written, I found essentially the same result in Hansen and Sargent (2002). However, like the references in footnote 8, their interest in stochastic intertemporal choice leads to a totally different focus from mine.

[17] It also rationalises my use of a single representative consume to characterise demands in each country. Disaggregation leads to essentially the same results, provided all consumers have Gorman polar form preferences, with the same b but possibly different values of a.

prices are the same in both countries, then world demands are:

$$\overline{x}(z) \equiv x(z) + x^*(z) = \frac{1}{b}\left[\overline{a} - \overline{\lambda} p(z)\right] \tag{10}$$

where $\overline{a} \equiv a + a^*$ is the world demand intercept, and $\overline{\lambda} \equiv \lambda + \lambda^*$ is the world marginal utility of income. World demands depend on total world income $\overline{I} \equiv I + I^*$, where I^* is foreign income, but not on its distribution between countries or between wages and profits.

Proposition 1 facilitates the normative as well as the positive applications of the model. We can substitute from the direct demand functions in (4) into the direct utility function (1) to obtain the indirect utility function, which (ignoring constants) equals minus the squared marginal utility of income times the second moment of prices:

$$\widetilde{U} = -\lambda^2 \mu_2^p \tag{11}$$

This is the most convenient way of evaluating consumer welfare in many applications. However, an alternative welfare index has a more natural interpretation. Appendix 1 shows that welfare can also be measured by the Gorman polar form utility index defined as follows:

$$\widetilde{u} = \frac{I - \frac{a}{b}\mu_1^p}{\left(\mu_2^p\right)^{\frac{1}{2}}} \tag{12}$$

Utility (which in this form is always negative) equals the shortfall of nominal income I below the "bliss" level of consumption, a/b, valued at the average price, all in turn deflated by the square root of the uncentred second moment of prices. This utility index is similar to minus the marginal utility of income in (5), except that it is homogeneous of degree zero (rather than of degree one) in nominal variables. Note also that, when written in this way, the ratios of total and marginal utilities of income between countries are the same: $\widetilde{u}/\widetilde{u}^* = \lambda/\lambda^*$.

3 Specialisation Patterns in an International Oligopoly

Turning from demand to supply, consider the determination of equilibrium in a single international oligopolistic industry, indexed by z. I assume that firms are Cournot competitors, choosing their outputs on the assumption that their rivals will keep theirs fixed. I also assume that there are barriers facing new firms, so oligopoly rents are not eroded by entry. Of course, some incumbent firms may choose to produce zero output in equilibrium, effectively dropping out of the market if they cannot make positive profits.

Assume that international markets are fully integrated and there are no transport costs or other barriers to international trade, so the same price prevails at home and abroad. Hence the inverse demand function is obtained by inverting (10), and, from the perspective of firms in the industry, can be written as: $p(z) = a' - b'\overline{x}(z)$. Here a' and b' are parameters, taken as given by firms but determined endogenously in general equilibrium: in free-trade equilibrium they equal $\overline{a}/\overline{\lambda}$ and $b/\overline{\lambda}$ respectively. We assume a given number n of home firms, all of which have the same marginal cost $c(z)$, so all home firms have the same equilibrium output, denoted by $y(z)$. Similarly, there is a given number n^* of foreign firms, all with the same marginal cost $c^*(z)$ and the same equilibrium output $y^*(z)$. Market clearing implies that total sales to both home and foreign consumers equal the sum of total production by home and foreign firms: $\bar{x}(z) = ny(z) + n^*y^*(z)$.

Regime:	H $[n > 0,\ n^* = 0]$	HF $[n > 0,\ n^* > 0]$	F $[n = 0,\ n^* > 0]$
y	$\frac{a'-c}{b'(n+1)}$	$\frac{a'-(n^*+1)c+n^*c^*}{b'(n+n^*+1)}$	0
y^*	0	$\frac{a'-(n+1)c^*+nc}{b'(n+n^*+1)}$	$\frac{a'-c^*}{b'(n^*+1)}$
$\bar{x} = \bar{y} = ny + n^*y^*$	$n\frac{a'-c}{b'(n+1)}$	$\frac{\bar{n}a'-nc-n^*c^*}{b'(n+n^*+1)}$	$n^*\frac{a'-c^*}{b'(n^*+1)}$
$p = a' - b'\bar{x}$	$\frac{a'+nc}{n+1}$	$\frac{a'+nc+n^*c^*}{n+n^*+1}$	$\frac{a'+n^*c^*}{n^*+1}$
$x = \frac{1}{b}(a - \lambda p)$	$\frac{(n+1)a-\lambda\left(a'+nc\right)}{b(n+1)}$	$\frac{(n+n^*+1)a-\lambda\left(a'+nc+n^*c^*\right)}{b(n+n^*+1)}$	$\frac{(n^*+1)a-\lambda\left(a'+n^*c^*\right)}{b(n^*+1)}$
$m = x - ny$	$-\frac{(n+1)a^*-\lambda^*\left(a'+nc\right)}{b(n+1)}$	$\frac{\left(n^*+\frac{\lambda^*}{\bar{\lambda}}\right)\left(a+n\bar{\lambda}c\right)-\left(n+\frac{\lambda}{\bar{\lambda}}\right)\left(a^*+n^*\bar{\lambda}c^*\right)}{b(n+n^*+1)}$	x

Table 1: Equilibrium Outputs, Prices, Demands and Net Imports (m) in Different Oligopoly Regimes

Firms take wages and the marginal utility of income as given, but exercise market power in their own sector. Calculating equilibrium outputs and prices in this relatively simple oligopoly model is straightforward, and explicit expressions are given in Table 1. Inspecting the first row of this table, we see that home firms in sector z will produce positive output only in one of two circumstances: either $c(z)$ must be below a' when foreign firms are not active, or $c(z)$ must be below $[a' + n^*c^*(z)]/(n^* + 1)$ when they are active. Combining these conditions with similar restrictions on the foreign firms allows us to illustrate the possible equilibria in $\{c(z), c^*(z)\}$ space, for given n and n^*, as shown in Figure 1. The solid lines indicate the boundaries of the regions in which the market is served by firms from both countries (denoted "HF"), from one (denoted "H" and "F") or from none (denoted "O"). Region HF is of particular interest: it is a "cone of diversification" which, since costs are given, disappears in the competitive limit (as both n and n^* tend towards infinity).

Before proceeding, consider the effects of an increase in n, the number of firms in all home sectors, on the outputs of foreign firms. From the expressions for foreign output in the second row of Table 1, foreign output falls in all sectors. In addition, the locus separating the HF and H regions in Figure 1 shifts to the left, as foreign firms exit marginal sectors. In line with the Ricardian nature of the model, we can say that foreign production contracts at both the intensive and extensive margins. This perspective will prove useful in understanding the model's properties.

4 Linking Factor and Goods Markets

To embed the sectoral structure from the last section in general equilibrium, we need to specify how costs are determined. As explained in the Introduction, we assume a Ricardian cost structure. Each sector requires a fixed labour input per unit output, denoted $\alpha(z)$ and $\alpha^*(z)$ in the home and foreign countries respectively. Hence the unit costs in sector z are:

$$c(z) = w\alpha(z), \qquad c^*(z) = w^*\alpha^*(z) \tag{13}$$

Here w and w^* denote the wages in each country, which are common across sectors, and determined by the condition that labour demand and supply are equal.

What restrictions do we need to impose on the $\alpha(z)$ and $\alpha^*(z)$ functions? As in all applications of the continuum model, we make the mild technical restriction that they are continuous in z. In addition, we suppose (without loss of generality) that goods are ordered such that the home country is more efficient at producing goods with low values of z. In the diagrams below it is implicitly assumed that $\alpha(z)$ and $\alpha^*(z)$ are respectively increasing and decreasing in z, but this is far stronger than needed. Dornbusch, Fischer and Samuelson (1977) assume that the ratio $\alpha(z)/\alpha^*(z)$ is increasing in z, but this is not sufficient to ensure that the model is well behaved. The precise condition we need is:

Assumption 1 *When both home and foreign firms operate, $y(z)$ and $y^*(z)$ are respectively decreasing and increasing in z.*

Lemma A1 in Appendix 4 relates this to underlying parameters, and Lemma A2 shows that it reduces to the Dornbusch-Fischer-Samuelson assumption in the competitive limit.

The assumptions we have made ensure that, for given technology (which is exogenous) and wages (whose determination will be explained below), there is a functional relationship between the home and foreign unit costs in each sector. Moreover, Assumption 1 ensures that in Figure 1 the locus representing this relationship cuts each boundary of the HF region at most once; otherwise it can take any form. The dashed lines in Figure 1 illustrate some possible equilibrium configurations. For example, the line labelled SS illustrates an equilibrium in which both countries are partly specialized. In this case, there are two threshold sectors, $\widetilde{z}$ and $\widetilde{z}^*$, with $0 < \widetilde{z}^* < \widetilde{z} < 1$. All sectors for which z is less than $\widetilde{z}$ are competitive in the home country;

all sectors for which z is greater than $\widetilde{z}^*$ are competitive in the foreign country; and home and foreign firms coexist in sectors between $\widetilde{z}^*$ and $\widetilde{z}$.

Other equilibrium configurations are also illustrated in Figure 1. The line labelled DD denotes one in which both countries are fully diversified, producing all goods; that labelled SD denotes one in which the home country is partly specialized and the foreign country fully diversified; and that labelled SOS denotes one in which both countries are partly specialized but some goods are not produced in either. (The latter case is a curiosum and will not be considered further.) Hence it is possible for either z to equal one and/or z^* to equal zero. Which of these cases obtains is determined endogenously as part of the full general equilibrium of the model, to which we now turn. We begin by considering the determination of equilibrium in autarky.

5 General Oligopolistic Equilibrium: Autarky[18]

To close the model of a single isolated economy we need only one further condition: the home labour market must clear:

$$L = \int_0^1 \alpha(z)\, ny(z)\, dz \tag{14}$$

This equates the supply of labour L, assumed exogenous, to the aggregate demand for labour, which is the sum of labour demand from all sectors. The model is now easily solved. We can eliminate firm output $y(z)$ from (14) using the expression in the first row of Table 1, and then use the first equation in (13) to eliminate $c(z)$. Making these substitutions gives:

$$L = \frac{n}{b(n+1)} \int_0^1 \alpha(z)\left[a - \lambda w \alpha(z)\right] dz \tag{15}$$

This equation has only a single unknown, the product λw, which is the consumer's *real wage at the margin*. Changes in the units in which nominal magnitudes are measured lead to equal and opposite changes in the values of λ and w, but no change in λw.

Evaluating the integral in (15), we can solve for the equilibrium marginal real wage in autarky:

$$w_a \equiv (\lambda w)_a = \left(a\mu_1 - \frac{n+1}{n} bL\right)\frac{1}{\mu_2} \tag{16}$$

where μ_1 and μ_2 denote the first and second moments of the home technology distribution:

$$\mu_1 \equiv \int_0^1 \alpha(z)\, dz \qquad \text{and} \qquad \mu_2 \equiv \int_0^1 \alpha(z)^2\, dz \tag{17}$$

[18] The closed-economy case is also discussed in Neary (2003).

From (16), the wage in autarky is increasing in n and μ_1 and decreasing in L and μ_2.

To calculate welfare in autarky, recall from equation (11) that it depends inversely on the second moment of the price distribution. The latter can be calculated explicitly by using the Cournot equilibrium price formula from Table 1, and by evaluating μ_2^p from (6) to obtain:

$$\tilde{U}_a = -\left(\lambda^2 \mu_2^p\right)_a = -\frac{1}{(n+1)^2}\left(a^2 + 2an\mu_1 w_a + n^2 \mu_2 w_a^2\right) \tag{18}$$

So a higher wage raises the variance of prices and hence lowers welfare. Substituting from (16) for w_a, this can be expressed in terms of underlying parameters as follows (details are given in Appendix 2):

$$\tilde{U}_a = -\frac{a^2}{(n+1)^2}\frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2} \tag{19}$$

where σ^2 is the variance of the home technology distribution:

$$\sigma^2 \equiv \int_0^1 \left[\alpha(z) - \mu_1\right]^2 dz = \mu_2 - \mu_1^2 \tag{20}$$

Equation (19) shows that autarky welfare is increasing in n. This is a familiar competition effect, but it has to be qualified in general equilibrium: the competition effect is stronger the greater the variance of costs across sectors, σ^2, and it is zero if all sectors have identical costs ($\sigma^2 = 0$), the case called the "featureless economy" in Neary (2003). Increased competition in all sectors raises the aggregate demand for labour, but the general-equilibrium constraint of full employment means that output can only increase if labour is reallocated from less to more efficient sectors. When all sectors are identical, this channel is blocked off, and so the welfare costs of imperfect competition vanish: a result first pointed out by Lerner (1933-34).

Equation (19) also implies that a mean-preserving spread in the distribution of costs raises autarky welfare: $\tilde{U}_a$ is increasing in σ^2 for given μ_1. This reflects two conflicting effects. On the one hand, from (2), consumers dislike heterogeneous consumption levels, and hence from (11) they dislike heterogeneous prices, so a rise in σ^2 tends to reduce welfare at given wages. On the other hand, more heterogeneous technology across sectors implies from (16) a fall in the wage, which reduces the second moment of prices from (18), and hence tends to raise welfare. It can be checked that the second effect dominates, so welfare is increasing in σ^2.[19]

[19] The partial derivative of $\tilde{U}_a$ with respect to σ^2 is $\left[-\frac{a^2}{(n+1)^2}\mu_1^2 + (a\mu_1 - bL)^2\right]\frac{1}{\mu_2}$. This simplifies to $\frac{n(n+2)}{(n+1)^2}\left(a\mu_1 - \frac{n+1}{n+2}bL\right)\left(a\mu_1 - \frac{n+1}{n}bL\right)\frac{1}{\mu_2}$, which must be positive when the wage in (16) is positive.

6 Free Trade with Symmetry and Full Diversification

Consider next a free trade equilibrium in which both countries are fully diversified. Assume also that the countries are symmetric in the sense that they are the same size: $L = L^*$; have the same tastes: $a = a^* = \frac{1}{2}\bar{a}$; the same industrial structure: $n = n^*$; and the same technology moments: $\mu_1 = \mu_1^*$ and $\mu_2 = \mu_2^*$ (where μ_1^* and μ_2^* are defined analogously to the home moments in (17)). In equilibrium they therefore have the same marginal utility of income: $\lambda = \lambda^* = \frac{1}{2}\bar{\lambda}$; and the same wage: $w = w^*$.

Although the countries are symmetric, they are not necessarily identical. As we will see, a key role is played by the covariance of their technology distributions. The "uncentred" covariance γ is defined as:

$$\gamma \equiv \int_0^1 \alpha(z)\,\alpha^*(z)\,dz \tag{21}$$

while the true or "centred" covariance ω is defined as:

$$\omega \equiv \int_0^1 [\alpha(z) - \mu_1][\alpha^*(z) - \mu_1^*]\,dz = \gamma - \mu_1\mu_1^* \tag{22}$$

Using the standard property that $\mu_2 + \mu_2^* \geq 2\gamma$, so with symmetry $\mu_2 \geq \gamma$, we can define δ:

$$\delta \equiv \mu_2 - \gamma = \sigma^2 - \omega \tag{23}$$

as a measure of the *technological dissimilarity* between the two countries, or simply as a measure of *comparative advantage*. Only when δ attains its minimum value of zero, so comparative advantage is zero, are the two countries identical.

The labour-market equilibrium condition is identical to that in autarky, equation (14), except that the expression for output now comes from the central column of Table 1. This differs in two respects from the autarky case. First, home firms now face competition from foreign firms in all markets. Second, the size of the market has increased; this is reflected in the fact that the slope of the perceived inverse demand function, b', has fallen: it now equals $\frac{b}{\bar{\lambda}} = \frac{b}{2\lambda}$ instead of $\frac{b}{\lambda}$. Making these substitutions into (14), integrating and solving as in the previous section, we can derive the wage in both countries:

$$w_f \equiv (\lambda w)_f = \left(a\mu_1 - \frac{2n+1}{2n}bL\right)\frac{1}{\mu_2 + n\delta} \tag{24}$$

Comparing this with the autarky wage (16), there are two conflicting influences at work. On the one hand, the numerator of (24) is larger due to the market size effect, represented by the term $-\frac{1}{2n}$ in (24) in place of $-\frac{1}{n}$ in (16): doubling the market size raises the demand for labour and so the equilibrium wage in both countries. On the other hand, the competition effect tends both to reduce the numerator and raise the denominator of w_f in (24) relative to w_a in (16). Overall the market size effect dominates the change in the numerator: the term $-\frac{2n+1}{2n}$ in w_f exceeds the corresponding term $-\frac{n+1}{n}$ in w_a. However, the competition effect on the denominator is larger the greater the degree of comparative advantage: the higher is δ, the more free-trade output tends to be higher in sectors with relatively lower labour requirements (because low-cost home firms compete against high-cost foreign rivals) and conversely, so depressing the aggregate demand for labour and tending to reduce the wage. Overall, therefore, the change in the wage between autarky and free trade is indeterminate.

This comparison of wages is instructive in suggesting the change in incentives for labour usage as a result of moving to free trade. However, it has no direct implications for utility. Although w_f in (24) is the wage evaluated by the domestic marginal utility of income λ (not the world marginal utility of income $\bar{\lambda}$), it is still not directly comparable with w_a since these only measure real wages at the margin. We turn therefore to consider the gains from trade themselves.

6.1 Gains from Trade

As in the previous section, we measure aggregate welfare using the second moment of the price distribution. Evaluating this (using the Cournot equilibrium price formula from the central column of Table 1, with $w = w^*$) gives:

$$\tilde{U}_f = -\left(\lambda^2 \mu_2^p\right)_f = -\frac{1}{(2n+1)^2}\left[a^2 + 4an\mu_1 w_f + 2n^2\left(2\mu_2 - \delta\right) w_f^2\right] \tag{25}$$

There are gains from trade if and only if this expression is greater than the corresponding expression in autarky, given by (18). Comparing the two, there are three sources of difference. The first (reflected in differences in the denominators) is a direct competition effect: with more firms in all markets, prices tend to be bid down, reducing their variability and so raising welfare. The second difference (corresponding to the coefficient of the third term in brackets) reflects a direct comparative advantage or technological dissimilarity effect: the greater is δ, the more high-cost home firms tend to face low-cost foreign firms and vice versa, so tending to reduce price variability across sectors and raise welfare. Finally, the third difference arises from the difference in wages. If free-trade wages are exactly twice those in autarky, reflecting the doubling of the market size, then this effect does not arise. However, a free-trade wage which is more than twice that in autarky tends to raise prices relative to autarky and so works against gains from trade. Of course, we have seen in the previous sub-section that the difference in wages depends on the same factors, market size, competition, and comparative advantage, as the direct effects. Hence we need further analysis to determine the overall gains from trade.

To proceed, we first restate (25) in terms of underlying parameters (details in Appendix 2):

$$\tilde{U}_f = -\frac{a^2}{(2n+1)^2}\frac{2\sigma^2-\delta}{2\mu_2-\delta} - \left(a\mu_1\frac{2\mu_2}{2\mu_2-\delta} - bL\right)^2 \frac{2\mu_2-\delta}{2\left(\mu_2+n\delta\right)^2} \tag{26}$$

To prove that the gains from trade are always non-negative, we need to show that this cannot be less than the corresponding expression in autarky, (19). We first consider two special cases. The more extreme is the *featureless world*, where all sectors are identical at home and abroad. Formally, $\sigma^2 = \delta = 0$ and $\mu_2 = \mu_1^2$. In this case (19) and (26) are equal: $\tilde{U}_a = \tilde{U}_f = -(a - bL/\mu_1)^2$, and so there are no gains from trade. Summarizing:

Lemma 1: *In the featureless world where $\sigma^2 = \delta = 0$, welfare in autarky and in free trade are identical, and both are independent of the number of firms.*

This extends our earlier formalization of Lerner's insight: when the "degree of monopoly" is the same in all sectors, neither free trade nor competition policy has any scope for raising welfare.

Next, consider the case where the two countries are still identical, so $\delta = 0$, but sectors are heterogeneous: $\sigma^2 > 0$, implying that $\mu_2 = \gamma > \mu_1^2$. In Figure 1, the cost locus in this case coincides with a segment of the 45° line. Equation (26) now reduces to:

$$\tilde{U}_f = -\frac{a^2}{(2n+1)^2}\frac{\sigma^2}{\mu_2} - \frac{\left(a\mu_1 - bL\right)^2}{\mu_2} \tag{27}$$

The second term is identical to the corresponding term in the expression for autarky welfare (19), but the first term is unambiguously larger because the number of firms has risen, and the difference is increasing in σ^2. So:

Lemma 2: *When the two countries are identical, $\delta = 0$, but there is some heterogeneity across sectors, $\sigma^2 > 0$, there are unambiguous gains from trade due to the competition effect, and the extent of gains is increasing in σ^2.*

Finally, we need to show that the gains from trade are increasing in the degree of comparative advantage δ. Since welfare in autarky is independent of δ, we need only differentiate expression (25) for welfare in free trade:

$$\frac{\partial \tilde{U}_f}{\partial \delta} = \frac{2n}{(2n+1)^2}\left[nw_f^2 - 2\left\{a\mu_1 + n\left(2\mu_2-\delta\right)w_f\right\}\frac{\partial w_f}{\partial \delta}\right] > 0 \tag{28}$$

This shows that an increase in comparative advantage, reflecting greater technological dissimilarity between countries, has two effects. First, it raises free-trade welfare at given wages: as we already saw in discussing

(26), an increase in comparative advantage reduces the variability of prices across sectors and so raises welfare. Second, it raises welfare by reducing the wage: as we already saw in discussing (24), an increase in comparative advantage skews the pattern of output and hence the demand for labour towards more efficient, and hence less labour-intensive sectors. Formally, $\frac{\partial w_f}{\partial \delta} = -\frac{n w_f}{\mu_2 + n\delta} < 0$. Hence the total effect is unambiguous:

Lemma 3: *For given σ^2, the gains from trade are strictly increasing in the degree of comparative advantage δ.*

Combining these three lemmas, we can conclude:

Proposition 2 *The gains from trade are always positive, strictly so provided there is some heterogeneity in technology between sectors ($\sigma^2 > 0$), and increasingly so the greater is σ^2 and the greater the degree of comparative advantage δ.*

Lemma 3, which shows that international differences in technology increase the gains from trade, is not too surprising, though it should be stressed that all the analysis applies without any specialisation in production, which is the source of gains from trade in the competitive Ricardian model. Lemma 2 is the most novel part of the result, since it shows that the pro-competitive effects of trade can raise welfare even when countries are identical, both *ex ante* and *ex post.* By contrast, in trade models based on the Dixit-Stiglitz model, countries differ *ex post* since they produce different varieties of the monopolistic competitive good in free trade. Lemma 2 also highlights the importance of taking a general equilibrium perspective: the competition effect of opening up to trade is only effective if there is scope for allocation of labour between sectors.

6.2 The Volume of Trade

Next, we want to consider how the volume of trade is affected by the degree of competition. The level of net imports in a typical sector, $m(z)$, equals home demand less home production, $x(z) - ny(z)$. Using the results from Table 1, specialized to the symmetric fully diversified case, this equals:

$$m(z) = \frac{1}{2b} n w_f \left[\alpha(z) - \alpha^*(z)\right] \tag{29}$$

Thus net imports are positive if and only if home firms are less productive than foreign. In the symmetric case, trade patterns are determined solely by comparative advantage. Equation (29) also shows that, for given relative labour efficiencies, the volume of trade increases in proportion to the number of firms and to the wage rate. Totally differentiating (29):

$$\hat{m}(z) = \hat{n} + \hat{w}_f \tag{30}$$

where "hats" denote proportional changes. We have already seen that the wage rate may fall as the world economy becomes more competitive. However, it cannot fall sufficiently to lead to a contraction of trade. Substituting for the change in w_f from (24) into (30) yields:

$$\frac{\hat{m}(z)}{\hat{n}} = \frac{\mu_2 w_f + \frac{bL}{n}}{2a\mu_1 - \frac{2n+1}{n} bL} > 0 \tag{31}$$

So lower wages may dampen but not reverse the direct trade-expanding effect of higher n. Hence we can conclude:[20]

Proposition 3 *An increase in the number of firms raises the volume of imports in all sectors.*

Of course, since real income rises as the economy becomes more competitive, this result is not surprising. Of greater interest is whether trade rises faster than consumption. Totally differentiating the expression for $x(z)$ from Table 1, the proportional change in consumption equals:

$$\hat{x}(z) = \frac{1}{2n+1}\hat{n} - \frac{[\alpha(z) + \alpha^*(z)] w_f}{4a - [\alpha(z) + \alpha^*(z)] w_f}\hat{w}_f \tag{32}$$

Combining (30) and (32), the effect of an increase in the number of firms in all world markets on the share of imports in home consumption is:

$$\hat{m}(z) - \hat{x}(z) = \frac{2n}{2n+1}\hat{n} + \frac{4a}{4a - [\alpha(z) + \alpha^*(z)] w_f}\hat{w}_f \tag{33}$$

So the effect of an increase in competition on the import share in partial equilibrium (i.e., at constant wages) is unambiguously positive, but this could be offset if technology is sufficiently dissimilar that wages fall. Substituting for $\hat{w}_f$ gives an ambiguous result (detailed derivations are in Appendix 3):

$$\frac{\hat{m}(z) - \hat{x}(z)}{\hat{n}} \propto \frac{2n+1}{n}\left[2a + n\{\alpha(z) + \alpha^*(z)\} w_f\right] bL + 2an\left[2\mu_2 - \delta - \{\alpha(z) + \alpha^*(z)\}\mu_1\right] w_f \tag{34}$$

Only the second term can be negative, so this implies a sufficient condition for the import share to rise:

Proposition 4 *A sufficient condition for an increase in the number of firms to raise the share of imports in consumption in sector z is that the sector is not extremal in its technology, i.e., that $\alpha(z) + \alpha^*(z) \leq \frac{2\mu_2 - \delta}{\mu_1}$.*

[20] Ruffin (2003) independently derives this result in a model with two oligopolistic sectors.

The sufficient condition in this Proposition could be violated in some sectors, but it must hold on average in all sectors. So we can conclude:

Proposition 5 *An increase in the number of firms raises the average share (in absolute value) of net imports in consumption across all sectors.*

These results show clearly that oligopoly tends to reduce trade volumes. An obvious implication is the light this may throw on the "mystery of the missing trade" documented by Trefler (1995): real-world trade volumes are much less than the Heckscher-Ohlin model suggests they should be. Davis and Weinstein (2001) go some way to solving the mystery, while remaining in a competitive Heckscher-Ohlin framework. Propositions 4 and 5 suggest a different explanation for low import shares, and point towards testable hypotheses linking concentration levels and technology to trade volumes.

7 Changes in International Competitiveness

An alternative approach to examining the model's properties is to look at small perturbations around an arbitrary free-trade equilibrium. In this section I first illustrate the determination of equilibrium and then consider the effects of a particularly interesting exogenous shock: an increase in the number of firms in all home sectors. This can be interpreted as an improvement in the competitive advantage of the home country.

In the general asymmetric two-country model, there are four equilibrium conditions: a labour-market clearing condition and a condition for the threshold sector in each. Consider first the market for labour in the home country. In equilibrium the total labour supply L must equal aggregate labour demand, which in turn equals the sum of labour demand from those sectors labelled $z \in [0, \tilde{z}^*]$ in which home firms face no foreign competition ($n^* = 0$), and from those with $z \in [\tilde{z}^*, \tilde{z}]$ in which both home and foreign firms operate ($n^* > 0$):[21]

$$L = L^D(\underset{-}{w}, \underset{+}{w^*}; \underset{+}{n}) = \int_0^{\tilde{z}^*} n\alpha(z)\, y(z)|_{n^*=0}\, dz + \int_{\tilde{z}^*}^{\tilde{z}} n\alpha(z)\, y(z)|_{n^*>0}\, dz \tag{35}$$

Aggregate labour demand is summarized by the function L^D, where the signs below the arguments indicate the responsiveness of labour demand to changes in its determinants. These signs are justified formally in Appendix 4, and can be explained intuitively as follows. Note first that labour demand is unaffected by small changes in the threshold sectors $\tilde{z}$ and $\tilde{z}^*$. Changes in either of these thresholds implies entry or exit of extra firms (home firms in the case of $\tilde{z}$, foreign in the case of $\tilde{z}^*$) which are just at the margin of profitability and hence whose effect on aggregate labour demand can be ignored.[22]

[21] Since we consider only the free-trade equilibrium in this section, we dispense with the "f" subscript.

[22] So, for example, $\frac{\partial L^D}{\partial \tilde{z}} = n\alpha(\tilde{z})\, y(\tilde{z}) = 0$ since $y(\tilde{z}) = 0$.

Consider next the effects of an increase in the home wage w. At the intensive margin, this raises production costs for all active home firms and hence lowers their demand for labour. At the extensive margin, if home firms in some sector are just on the threshold of profitability (i.e., if $\tilde{z}$ is less than one), they will no longer be able to compete. Hence the margin of home specialisation changes: the threshold home sector $\tilde{z}$ falls and for this reason too home demand for labour falls, though for small changes this effect is negligible. The net effect is a fall in the total demand for labour at home.

Similar arguments show that home demand for labour is increasing in the foreign wage w^*. Hence the L locus, representing home labour-market equilibrium, must be upward-sloping in $\{w, w^*\}$ space. This is illustrated in Figure 2, though the slope of the locus is the same whichever specialisation regime obtains. Points above it correspond to excess supply of labour, which we would expect to put downward pressure on the home wage. Conversely, points below the L locus correspond to excess demand for labour, which we would expect to put upward pressure on the home wage. These tendencies are indicated by the vertical arrows in the diagram.

Analogous arguments apply to the foreign country, where the labour-market equilibrium condition is:

$$L^* = L^{*D}(\underset{+}{w}, \underset{-}{w^*}, \underset{-}{n}) = \int_{\tilde{z}}^{1} n^* \alpha^* (z) \, y^* (z)|_{n=0} \, dz + \int_{\tilde{z}^*}^{\tilde{z}} n^* \alpha^* (z) \, y^* (z)|_{n>0} \, dz \tag{36}$$

Now the responsiveness of labour demand to home and foreign wages is reversed, but the two continue to exert opposing effects and so the L^* locus is also upward sloping, with horizontal arrows indicating the direction of foreign wage adjustment. Like home labour demand, that in foreign is unaffected by small changes in the threshold sectors $\tilde{z}$ and $\tilde{z}^*$. Hence the two labour-market equilibrium conditions can be solved as an independent sub-system, and equilibrium wages are determined by the intersection of the two loci at point A. For this equilibrium to be stable, the L^* locus must be more steeply sloped than the L locus as shown.

We are now ready to consider the comparative statics properties of the model. At initial wages, an increase in the number of home firms in all sectors reduces output per firm but not by enough to offset the rise in the number of firms. (See Appendix 4 for details.) Hence home demand for labour increases, shifting upwards the L locus in the right-hand panel of Figure 3. Similar but opposite effects in the foreign country reduce labour demand there, shifting leftwards the L^* locus. Figure 3 illustrates the presumptive outcomes: equilibrium wages rise at home and fall abroad. Appendix 4 derives the exact conditions for these to occur, and proves the following:

Proposition 6 *A sufficient condition for an increase in n to raise the home country's relative wage, w/w^*, is that the own-effects of wages on labour demand dominate the cross-effects (as they must if the initial* *equilibrium is symmetric).*

Proposition 7 *A sufficient condition for an increase in n to raise the home wage w and lower the foreign wage w^* is that the own-effects of wages and of n on labour demand dominate the cross-effects (as they must if the initial equilibrium is symmetric).*

Finally, consider the effects of an increase in n on specialisation patterns. From the expressions for output, the threshold sectors in each country, $\widetilde{z}$ and $\widetilde{z}^*$ are defined by the following equations:

$$y\left(\widetilde{z}\right) \geq 0 \Longleftrightarrow \bar{a} - \left(n^* + 1\right) w\alpha\left(\widetilde{z}\right) + n^* w^* \alpha^*\left(\widetilde{z}\right) \geq 0, \qquad \widetilde{z} \leq 1 \tag{37}$$

$$y^*\left(\widetilde{z}^*\right) \geq 0 \Longleftrightarrow \bar{a} - \left(n + 1\right) w^* \alpha^*\left(\widetilde{z}^*\right) + nw\alpha\left(\widetilde{z}^*\right) \geq 0, \qquad \widetilde{z}^* \geq 0 \tag{38}$$

Each pair of inequalities in (37) and (38) is complementary slack. So, in (37) for example, if $y(\widetilde{z})$ is strictly positive, then $\widetilde{z}$ equals one: this is the case where home firms are profitable in all sectors, so the home country is fully diversified in equilibrium. By contrast, if $\widetilde{z}$ is strictly less than one, then $y\left(\widetilde{z}\right)$ is zero: this is the case where home firms in sectors with $z \geq \widetilde{z}$ are unprofitable, so the home country is partially specialized in equilibrium. We consider this case (so $\widetilde{z} < 1$ and $y\left(\widetilde{z}\right) = 0$) and totally differentiate equation (37) to obtain:

$$\frac{d\widetilde{z}}{dn} = \frac{\partial\widetilde{z}}{\partial w}\frac{dw}{dn} + \frac{\partial\widetilde{z}}{\partial w^*}\frac{dw^*}{dn} \tag{39}$$

Since wage changes affect the threshold sector in an unambiguous manner, we can again state a sufficient condition:

Proposition 8 *A sufficient condition for an increase in n to lower the home threshold sector is that the home wage rises and the foreign wage falls.*

This result is not expressed in terms of primitive parameters, but from Proposition 7 we can see that it will always hold if the initial equilibrium is symmetric. It has a striking implication. An improvement in the home country's competitive advantage raises output in all sectors at initial wages. However, the induced wage changes make marginal home sectors uncompetitive. Hence improved competitive advantage leads the home country to specialize more in accordance with comparative advantage, exiting some sectors as home wages rise.

8 Conclusion

This paper has developed a tractable but consistent model of oligopoly in general equilibrium; and used it to take a small step towards completing the "new trade theory" agenda of integrating international trade with industrial organisation. The step is a small one because the functional forms assumed are special, and because many simplifications are made in specifying agents' behaviour and the workings of goods and factor markets. Nevertheless, it is hopefully a step in the right direction. The model allows for consistent aggregation over a continuum of sectors, each of which is characterized by Cournot competition between home and foreign firms. The model makes explicit the links between goods and factor markets, and so is able to give a coherent yet tractable analysis of the effects of a variety of exogenous shocks.

The key idea in the paper is that oligopolistic firms should be modelled as large in their own markets but small in the economy as a whole. This perspective avoids at a stroke all the problems (of non-existence, ambiguity of profit maximization, sensitivity of the model's properties to the choice of numeraire, etc.) which have concerned writers such as Gabszewicz and Vial (1972) and Roberts and Sonnenschein (1977) who have tackled the problem of oligopoly in general equilibrium. Hopefully it thus opens up a rich vein of research, combining the insights of modern theories of industrial organisation with those of general equilibrium theory.

The paper's central idea could be operationalized in a great variety of ways. Here I have chosen to work with quadratic preferences on the demand side, and a Cournot-Ricardo (or Brander-Samuelson) specification of goods and factor markets. While the individual building blocks are familiar, the full model exhibits many novel properties and throws light on a number of substantive issues. I have shown that trade between economies which are identical both ex ante and ex post is welfare-improving because it enhances competition; that barriers to entry reduce trade volumes both absolutely and relative to total consumption, suggesting a plausible (and testable) explanation for Trefler's "mystery of the missing trade"; and that a rise in one country's competitive advantage is likely to raise its relative wage and lead it to specialize more in the direction of comparative advantage.

There are many obvious ways in which the approach adopted here could be extended. I have already explored in a simplified version of the model the implications for the effects of trade on income inequality of having more than one factor of production and of allowing firms to engage in entry-preventing behaviour. (See Neary (2002).) Other applications, such as the effects of trade on innovative behaviour or cross-border mergers, immediately suggest themselves. Overall I hope the model points towards a richer theory of imperfect competition in open economies than is possible in models where entry is never difficult and firm behaviour is never strategic, whether under perfectly or monopolistically competitive assumptions.

9 Appendix

9.1 Continuum-Quadratic Preferences and the Gorman Polar Form

To prove Proposition 1, first evaluate the indirect utility function by substituting the direct demand functions into (1) and use (5) to eliminate λ:

$$\begin{aligned}\widetilde{U}\left[\{p(z)\},I\right] &= \frac{1}{2b}\left[a^2-\lambda\{p(z),I\}^2\int_0^1 p(z)^2\,dz\right]\\ &= \frac{1}{2b}\left[a^2-\frac{\left(a\mu_1^p-bI\right)^2}{\mu_2^p}\right]\end{aligned} \tag{40}$$

This can be written in terms of an alternative utility index $\widetilde{u}$ as follows:

$$\widetilde{u}=-\frac{1}{b}\left(a^2-2b\widetilde{U}\right)^2=\frac{I-f(p)}{g(p)} \tag{41}$$

where the functions $f(p)$ and $g(p)$ are defined as:

$$f(p)\equiv\frac{a}{b}\mu_1^p \qquad \text{and} \qquad g(p)\equiv\mu_2^p \tag{42}$$

and both are homogeneous of degree one in prices. The right-hand side of (41) satisfies the restrictions of the Gorman polar form indirect utility function. (See Gorman (1961) and Blackorby, Boyce and Russell (1978).) All that remains is to check that $a^2-2b\widetilde{U}$ is positive, thus ensuring that $\widetilde{u}$ in (41) is well defined. This must be so since, from (40), $a^2-2b\widetilde{U}$ can be written as the integral of a square:

$$a^2-2b\widetilde{U}=\int_0^1\{\lambda p(z)\}^2\,dz>0 \tag{43}$$

This proves Proposition 1.

9.2 Proof of Proposition 2

We give the steps in deriving the free-trade level of welfare (26). Deriving the autarky level of welfare proceeds along similar lines: details are in Neary (2003). Rewrite the expression in brackets in (25) as a difference of squares and then substitute for w_f from (24):

$$
\begin{aligned}
-(2n+1)^2\tilde{U}_f &= a^2 + 4an\mu_1 w_f + 2n^2\left(2\mu_2-\delta\right)w_f^2 \\
&= a^2 + 2\left(2\mu_2-\delta\right)nw_f\left[nw_f + \frac{2a\mu_1}{2\mu_2-\delta}\right] \\
&= a^2 + 2\left(2\mu_2-\delta\right)\left[\left(nw_f+\frac{a\mu_1}{2\mu_2-\delta}\right)^2 - \left(\frac{a\mu_1}{2\mu_2-\delta}\right)^2\right] \\
&= a^2\left(1-\frac{2\mu_1^2}{2\mu_2-\delta}\right) + 2\left(2\mu_2-\delta\right)\left(nw_f+\frac{a\mu_1}{2\mu_2-\delta}\right)^2 \\
&= a^2\frac{2\sigma^2-\delta}{2\mu_2-\delta} + 2\left(2\mu_2-\delta\right)\left(\frac{an\mu_1-\frac{2n+1}{2}bL}{\mu_2+n\delta}+\frac{a\mu_1}{2\mu_2-\delta}\right)^2 \\
&= a^2\frac{2\sigma^2-\delta}{2\mu_2-\delta} + (2n+1)^2\left(a\mu_1\frac{2\mu_2}{2\mu_2-\delta}-bL\right)^2\frac{2\mu_2-\delta}{2\left(\mu_2+n\delta\right)^2}
\end{aligned}
\tag{44}
$$

This gives equation (26). The proofs of Lemmas 1, 2, and 3 follow by inspection.

9.3 Proof of Proposition 4

Substituting for $\hat{w}_f$ into (33), the numerator of the resulting expression is:

$$
\frac{2n+1}{n}\left[2a+n\left\{\alpha\left(z\right)+\alpha^*\left(z\right)\right\}w_f\right]bL + 2an\left[2\mu_2-\delta-\left\{\alpha\left(z\right)+\alpha^*\left(z\right)\right\}\mu_1\right]w_f
\tag{45}
$$

This gives the sufficient condition in Proposition 4.

9.4 Comparative Statics of Free Trade

When both countries are partly specialized, so $0 < \widetilde{z}^* < \widetilde{z} < 1$ and equations (37) and (38) which define the threshold sectors hold with equality, the total differential of the system is as follows:

$$
\begin{bmatrix}
L_w & L_{w^*} & 0 & 0 \\
L_w^* & L_{w^*}^* & 0 & 0 \\
-(n^*+1)\alpha\left(\widetilde{z}\right) & n^*\alpha^*\left(\widetilde{z}\right) & -H & 0 \\
-n\alpha\left(\widetilde{z}^*\right) & (n+1)\alpha^*\left(\widetilde{z}^*\right) & 0 & -H^*
\end{bmatrix}
\begin{bmatrix} dw \\ dw^* \\ d\widetilde{z} \\ d\widetilde{z}^* \end{bmatrix}
=
\begin{bmatrix} -L_n \\ -L_n^* \\ 0 \\ -J \end{bmatrix} dw
\tag{46}
$$

To explain the new symbols introduced in (46), consider first the derivatives of the home labour market equilibrium condition (35). Differentiating this with respect to home and foreign wages, and using the expressions in Table 1 to sign the individual terms, gives:

$$L_w = n \int_0^{\tilde{z}^*} \alpha(z) \left.\frac{\partial y(z)}{\partial w}\right|_{n^*=0} dz + n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \left.\frac{\partial y(z)}{\partial w}\right|_{n^*>0} dz < 0 \tag{47}$$

$$L_{w^*} = n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \left.\frac{\partial y(z)}{\partial w^*}\right|_{n^*>0} dz > 0 \tag{48}$$

So, home labour demand is decreasing in the home wage and increasing in the foreign wage. Next, differentiating with respect to the threshold sectors:

$$L_{\tilde{z}} = \alpha(\tilde{z})\, ny(\tilde{z}) = 0 \qquad \text{and} \qquad L_{\tilde{z}^*} = \alpha(\tilde{z}^*)\, \bar{x}(\tilde{z}^*) - \alpha^*(\tilde{z}^*)\, ny(\tilde{z}^*) = 0 \tag{49}$$

where the last result follows from (38) with $y^*(\tilde{z}^*) = 0$. So, for small changes, home labour demand is independent of both home and foreign threshold sectors. Finally, differentiating with respect to the number of home firms:

$$L_n = n \int_0^{\tilde{z}^*} \alpha(z) \left.\frac{\partial y(z)}{\partial n}\right|_{n^*=0} dz + n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \left[y(z) + n \left.\frac{\partial y(z)}{\partial n}\right|_{n^*>0} \right] dz \tag{50}$$

$$= \frac{1}{n+1} \int_0^{\tilde{z}^*} \alpha(z)\, y(z)|_{n^*=0}\, dz + \frac{n^*+1}{n+n^*+1} \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z)\, y(z)|_{n^*>0}\, dz > 0 \tag{51}$$

implying that home labour demand is increasing in the number of home firms in each sector.

The derivatives of the foreign labour demand schedule are derived similarly by totally differentiating (36). Foreign labour demand is decreasing in the foreign wage and increasing in the home wage. It is independent of both the home and foreign threshold sectors. Finally:

$$L_n^* = n^* \int_{\tilde{z}^*}^{\tilde{z}} \alpha^*(z) \left.\frac{\partial y^*(z)}{\partial n}\right|_{n>0} dz = -\frac{n^*}{n+n^*+1} \int_{\tilde{z}^*}^{\tilde{z}} \alpha^*(z)\, y^*(z)|_{n>0}\, dz > 0 \tag{52}$$

So foreign labour demand is decreasing in the number of home firms in each sector.

Solving the sub-system consisting of the first two equations in (46) and evaluating at a stable equilibrium gives:

$$\frac{dw}{dn} \propto -L_n L_{w^*}^* + L_n^* L_{w^*} \qquad \text{and} \qquad \frac{dw^*}{dn} \propto L_n L_w^* - L_n^* L_w \tag{53}$$

Converting to proportional changes and subtracting gives:

$$\frac{\hat{w} - \hat{w}^*}{\hat{n}} \propto L_n^* (wL_w + w^* L_w^*) - L_n (w^* L_{w^*}^* + wL_w^*) \tag{54}$$

The expressions in brackets are respectively negative and positive provided the own effects of wages dominate the cross effects in home and foreign labour demand respectively, which proves Proposition 7. If we assume in addition that the positive own effect of n on home labour demand dominates its negative cross effect, so $L_n + L_n^* > 0$, then w must rise and w^* must fall. This proves Proposition 6.

Consider next equations (37) and (38) which define the threshold sectors. Recalling Assumption 1 in the text (i.e., that $y(z)$ is increasing and $y^*(z)$ is decreasing in z), it is easy to confirm:

Lemma A1 *The conditions $dy(\tilde{z})/dz < 0$ and $dy^*(\tilde{z}^*)/dz > 0$, are equivalent to $H > 0$ and $H^* > 0$ respectively, where H and H^* are defined as follows:*

$$H \equiv (n^* + 1)\, w\alpha'(\tilde{z}) - n^* w^* \alpha^{*\prime}(\tilde{z}) \qquad \text{and} \qquad H^* \equiv nw\alpha'(\tilde{z}^*) - (n+1)\, w^* \alpha^{*\prime}(\tilde{z}^*) \tag{55}$$

When $y(\tilde{z})$ and $y^*(\tilde{z}^*)$ are zero, we can substitute from (37) and (38) to obtain:

$$H > 0 \Leftrightarrow \alpha^*(\tilde{z})\,\alpha'(\tilde{z}) > \alpha(\tilde{z})\,\alpha^{*\prime}(\tilde{z}) - \frac{\bar{a}}{n^* w^*} \qquad \text{and} \qquad H^* > 0 \Leftrightarrow \alpha^*(\tilde{z}^*)\,\alpha'(\tilde{z}^*) > \alpha(\tilde{z}^*)\,\alpha^{*\prime}(\tilde{z}^*) + \frac{\bar{a}}{nw} \tag{56}$$

Apart from the final terms, which are of order $1/n^*$ and $1/n$ respectively, these conditions are identical, except that they are evaluated at different points. Hence:

Lemma A2 *In the competitive limit, as n and n^* approach infinity, Assumption 1 collapses to $\alpha^*(z)\alpha'(z) > \alpha(z)\alpha^{*\prime}(z)$; i.e., $\alpha(z)/\alpha^*(z)$ is increasing in z.*

It is clear from (46) that Assumption 1 allows us to sign the partial effects of changes in wages on the threshold sectors:

Lemma A3 *Given Assumption 1, so H and H^* are positive, both $\tilde{z}$ and $\tilde{z}^*$ are decreasing in w and increasing in w^*.*

Finally, while the home threshold sector in (37) is independent of the number of home firms, (38) is related to it by the parameter J:

$$J \equiv w^* \alpha^*(\tilde{z}^*) - w\alpha(\tilde{z}) = \frac{\bar{a} - w\alpha(\tilde{z}^*)}{n+1} = y(\tilde{z}^*)\big|_{n=0} > 0 \tag{57}$$

Thus, at given wages, an increase in the number of home firms raises the threshold foreign sector: i.e., it reduces the number of sectors which are competitive in the foreign country.

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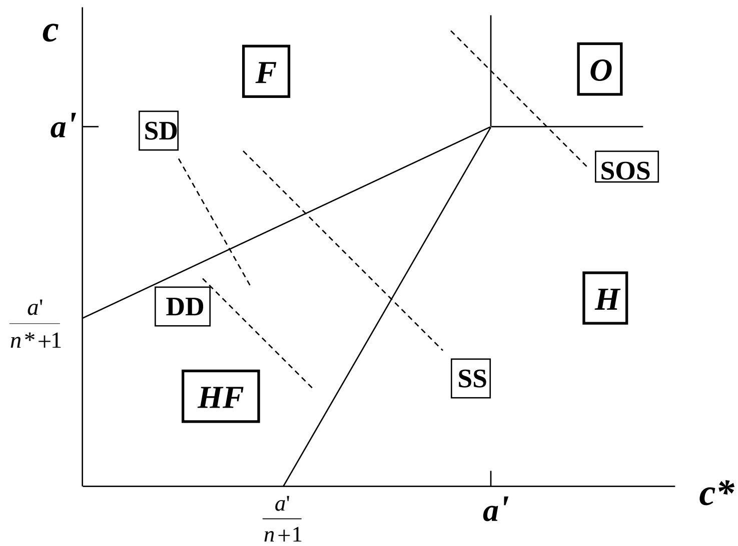

Fig. 1: Illustrative Equilibrium Configurations

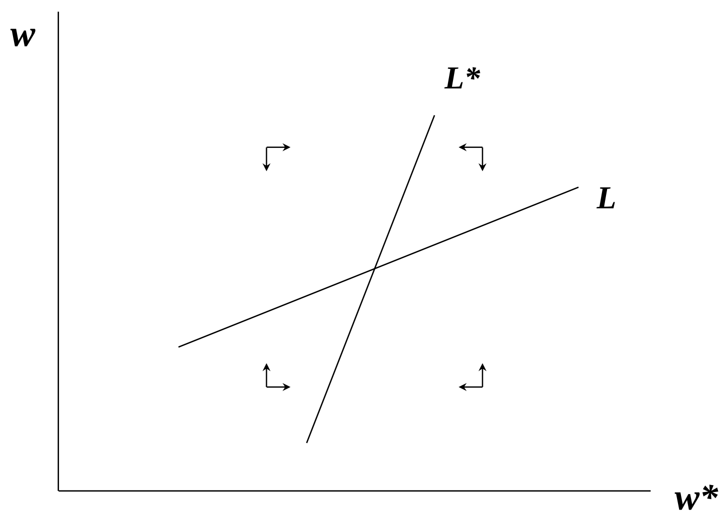

Fig. 2: Stability of Equilibrium

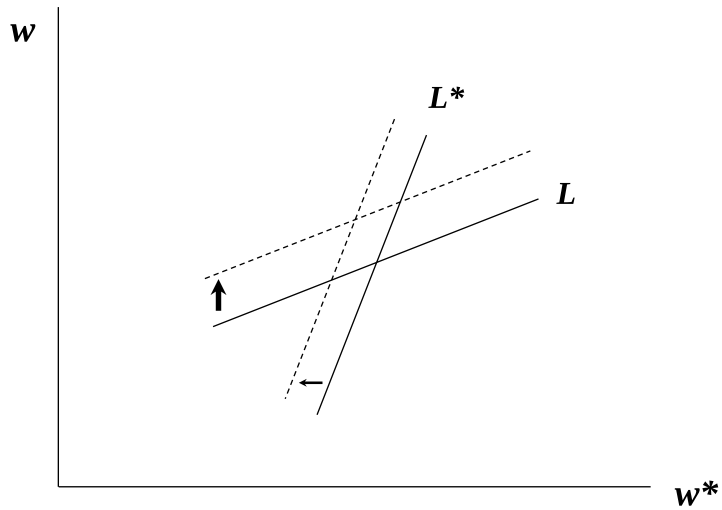

Fig. 3: Comparative versus Competitive Advantage:
Effects of an Increase in *n*