RATIONALISING THE PENN WORLD TABLE: TRUE MULTILATERAL INDICES FOR INTERNATIONAL COMPARISONS OF REAL INCOME

J. Peter Neary*

University College Dublin and CEPR

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ABSTRACT

Real incomes are routinely compared internationally using methods which "correct" for deviations from purchasing power parity. The most widely used of these is the Geary method which, though theoretically suspect, underlies the Penn World Table. This paper provides a theoretical foundation for the Geary method which I call the GAIA ("Geary-Allen International Accounts") System. I show that the Geary method is exact when preferences are non-homothetic Leontief and, more generally, gives a (possibly poor) approximation to the GAIA benchmark. An empirical application suggests that both it and other widely-used methods underestimate the degree of international inequality.

Keywords: GAIA ("Geary-Allen International Accounts") System; Geary Method; Index Numbers of Prices and Real Incomes; Penn World Table; PPP (Purchasing Power Parity); QUAIDS (Quadratic Almost Ideal Demand System).

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Address (2003-2004): Institute for the Study of Social Change, University College Dublin, Belfield, Dublin 4, Ireland, and CEPR, London; tel.: +353-1-716 4618/4625; e-mail: peter.neary@ucd.ie; home page: http://www.ucd.ie/~economic/staff/pneary/neary.htm.

RATIONALISING THE PENN WORLD TABLE: TRUE MULTILATERAL INDICES FOR INTERNATIONAL COMPARISONS OF REAL INCOME

How should we compare real incomes between countries? The question is important, since the demand for such comparisons is enormous. Apart from their intrinsic interest, they are essential for testing hypotheses about comparative growth performance. Indeed, such tests have themselves become a major growth industry in recent years, reflecting both the revival of interest in growth theory, and the relatively recent availability of comparative data on real incomes for a wide range of countries and years. The major source of these is the Penn World Table, which in turn originates from the U.N. International Comparison Project (ICP). (See Robert Summers and Alan Heston (1991) and Irving B. Kravis (1984) respectively.)

However, a paradox lies at the heart of the Penn World Table. To construct internationally comparable data on real incomes, it uses a method for computing "purchasing-power-parity-corrected" exchange rates due to R.C. Geary (1958). Though the method has many practical advantages, it has no theoretical foundation, and most index-number theorists have argued for alternative approaches. (See P.A. Samuelson and S. Swamy (1974), Douglas W. Caves et al. (1982a), and W. Erwin Diewert (1999).) The best-known of these is the "EKS" index, a multilateral extension of the Fisher "Ideal" index, which has been used by the OECD and by Eurostat (the European Union’s Statistical Office) to produce purchasing-power-parity-corrected real income data for their member countries.[1]

This paper reexamines the conceptual foundations for international comparisons of real incomes. I question the theoretical superiority of methods based on the Fisher index, by showing that their desirable properties for *bilateral* comparisons do not necessarily extend to the *multilateral* case, especially when tastes are not homothetic. More positively, I propose a new set of true indices for international comparisons which combine the desirable aggregation property of the Geary method with a firm foundation in economic theory. I then use this true index as a benchmark to compare the theoretical and empirical performance of the EKS and Geary indices, and explore the extent to which it provides a theoretical justification for the Penn World Table.

Section I sets up the problem and introduces the three empirical multilateral indices to be compared in the paper, the EKS index, the closely-related CCD index of Caves et al. (1982a), and the Geary index. Section II gives some general background to the theory of index numbers, while Section III reconsiders the results of A.A. Konüs and S.S. Byushgens (1926) and Diewert (1976) which provide a theoretical justification for Fisher-type indices. Section IV introduces my proposed true index, which I call the GAIA ("Geary-Allen International Accounts") index. Sections V and VI consider its theoretical properties, relate it to the Geary index, and draw on the theory of linear aggregation to explain the world prices which it implies. Section VII presents an empirical application. By estimating some members of the QUAIDS ("Quadratic Almost Ideal Demand System") family on a data set drawn from the International Comparison Project, I show how different assumptions about income responsiveness affect the performance of the EKS and Geary indices relative to the GAIA index. Finally, Section VIII gives a summary of the results and some general remarks on the conceptual foundations of international comparisons of real income when tastes are not homothetic.

I. Preliminaries

A. *The Problem*

Suppose that, for each of m countries, labelled $j = 1, \ldots m$, we have observations on the prices (expressed in national currencies) and quantities consumed (expressed in common units) of n commodities, labelled $i = 1, \ldots n$. Price and quantity vectors in country j are denoted $\boldsymbol{p}^j$ and $\boldsymbol{q}^j$, with typical elements p_{ij} and q_{ij}, respectively. Total expenditure or "income" in domestic prices is denoted $z_j=\boldsymbol{p}^j.\boldsymbol{q}^j$. Commodities are identical in quality worldwide but, because of transport costs, imperfect competition or other barriers to arbitrage, prices are not equalised internationally. Hence, official exchange rates are not appropriate for comparing real incomes between countries. What we seek is a set of index numbers Q_{jk} which give the real income of each country j relative to every other country k. In this section I introduce three such index numbers which, since they are observable but lack any explicit economic-theoretic underpinnings, I will call "empirical" indices.

B. *The EKS Index*

The simplest way of making multilateral comparisons of real incomes is the so-called "star" method, which revalues each country’s consumption vector by the prices of a single reference country. This amounts to constructing a set of Laspeyres quantity indices, with the reference country as base. But this method is clearly arbitrary since the results are sensitive to the choice of base country. Moreover, it leads to a well-known bias, the "Gerschenkron Effect": a country’s measured real income is usually higher the more the base country’s prices differ from its own.[2] In bilateral comparisons these problems are typically avoided by adopting some compromise between the base-weighted Laspeyres index and the current-weighted Paasche index, of which the most widely-used is their geometric mean, the Fisher "Ideal" index:

$$\ln Q_{jk}^{F} \quad = \quad \tfrac{1}{2} \left(\ln \frac{\boldsymbol{p}^{k}.\boldsymbol{q}^{j}}{\boldsymbol{p}^{k}.\boldsymbol{q}^{k}} + \ln \frac{\boldsymbol{p}^{j}.\boldsymbol{q}^{j}}{\boldsymbol{p}^{j}.\boldsymbol{q}^{k}} \right). \qquad \textbf{(1)}$$

The EKS index extends the Fisher index to the multilateral context. It equals the geometric mean of the ratios of all *m* bilateral Fisher indices, taking each country in turn as base:

$$\ln Q_{jk}^{EKS} \quad = \quad \frac{1}{m} \sum_{l=1}^{m} \left(\ln Q_{jl}^{F} - \ln Q_{kl}^{F} \right). \qquad \textbf{(2)}$$

This reduces to the Fisher index in the two-country case (i.e., when m=2), so it is indeed an appropriate multilateral generalisation of the Fisher index.

C. *The CCD Index*

Caves et al. (1982a) have proposed an alternative to the EKS index which resembles it in many respects, but whose theoretical properties are easier to elucidate (as we will see in Section III). Its starting point is the bilateral Törnqvist index, defined as:

$$\ln Q_{jk}^{T} \quad = \quad \tfrac{1}{2} \sum_{i} \left(\omega_{ij} + \omega_{ik} \right) \left(\ln q_{ij} - \ln q_{ik} \right), \qquad \textbf{(3)}$$

where ω_{ij} is the budget share of good *i* in country *j*. The CCD index extends the Törnqvist index to multilateral comparisons in the same way as the EKS index extends the Fisher index:

$$\ln Q_{jk}^{CCD} \quad = \quad \frac{1}{m} \sum_{l=1}^{m} \left(\ln Q_{jl}^{T} - \ln Q_{kl}^{T} \right). \qquad \textbf{(4)}$$

This too reduces to the Törnqvist index in the two-country case.

D. *The Geary Method*

The Geary method proceeds in a very different way. It first postulates the existence of "true" exchange rates ε and "world" prices π. The true exchange rates are Laspeyres price indices, which compare the world prices with the prices of each country in turn:[3]

$$\epsilon_j = \frac{\Sigma_i \, \pi_i \, q_{ij}}{\Sigma_i \, p_{ij} \, q_{ij}} \, , \qquad j = 1, \ldots \, m \, . \tag{5}$$

Put differently, each country's real income is the same, whether valued at world prices ($\Sigma_i \pi_{ij} q_{ij}$) or at domestic prices converted at the true exchange rates ($\varepsilon_j \, \Sigma_i \, p_{ij} \, q_{ij}$). As for the world prices themselves, they are implicitly defined by the requirement that total world spending on commodity *i* is the same whether valued at its world price ($\pi_i \, \Sigma_j \, q_{ij}$) or at domestic prices converted at the true exchange rates ($\Sigma_j \, \varepsilon_j \, p_{ij} \, q_{ij}$):

$$\pi_i = \frac{\Sigma_j \, \epsilon_j \, p_{ij} \, q_{ij}}{\Sigma_j \, q_{ij}} \, , \qquad i = 1, \ldots \, n \, . \tag{6}$$

Solving simultaneously for ε and π, we can calculate each country's income at world prices, $z_j^G = \varepsilon_j z_j = \Sigma_i \, \pi_i q_{ij}$, $\forall \; j$, and the implied real income indices, $Q_{jk}^G = z_j^G / z_k^G$, $\forall \; j,k$. Thus the Geary method is a star system with the hypothetical country whose prices are π as centre.

II. Criteria for Choosing between Index Numbers

How can we choose between different empirical real-income indices? Two distinct approaches can be taken to this problem. The "test" or "axiomatic" approach treats prices and quantities as independent variables and assesses the extent to which different empirical indices satisfy certain desirable, though not necessarily mutually consistent, properties. By contrast, the "economic" approach assumes that quantities arise from optimising behaviour, and explores how closely empirical indices approximate to some "true" (and usually unobservable) index based on economic theory.

Consider first the test approach. Among the many tests which have been proposed, three are especially relevant to multilateral cross-section comparisons:

1. *Transitivity* or *Circularity*: A satisfactory multilateral index number should provide a unique cardinal ranking of real incomes. Thus, country j's real income relative to country k should be the same whether the two are compared directly or via an arbitrary intermediate country l: $Q_{jk} = Q_{jl}.Q_{lk}$. A corollary is *Base-Country Invariance*: country j's real income relative to country k should not be sensitive to the choice of base country, so $Q_{jl}/Q_{kl} = Q_{jh}/Q_{kh}$. Clearly the star and Fisher indices do not satisfy circularity, whereas both multilateral systems do.

2. *Characteristicity* or *Independence of Irrelevant Countries*: Country j's real income relative to country k's should ideally be unaffected by changes in third countries. The EKS index exhibits this to a high degree, since it is the solution to the problem of finding a transitive index which minimises the sum of squared deviations from the Fisher indices. (See Laszlo Drechsler (1973).)

3. *Matrix Consistency*: Finally, the usefulness of a set of real income indices is much enhanced if they can be consistently disaggregated by commodity as well as by country. The EKS index fails this test, whereas the Geary system satisfies it. Summing (5) over countries or (6) over commodities yields the same expression for aggregate world expenditure: $\Sigma_j\, \varepsilon_j\, \Sigma_i\, p_{ij}q_{ij} = \Sigma_i\, \pi_i\, \Sigma_j\, q_{ij}$. It was primarily for this reason that the Geary system was used in the International Comparison Project and subsequently as the foundation for the Penn World Table.

The test approach is a useful starting point but it has no foundation in economic theory. By contrast, the economic approach postulates a reference consumer, usually characterised by either an expenditure function, $e(\boldsymbol{p},u) \equiv Min_{q} \{\boldsymbol{p}.\boldsymbol{q}: U(\boldsymbol{q})\geq u\}$, or a distance function, $d(\boldsymbol{q},u) \equiv Max_{\delta} \{\delta: U(\boldsymbol{q}/\delta)\geq u\}$, where $U(\boldsymbol{q})$ is the reference consumer's direct utility function.[4] In bilateral comparisons, three distinct true measures of real income have been proposed. The *Allen Quantity Index* equals the ratio of the reference consumer's minimum cost of attaining the two countries' optimal expenditures evaluated at a common price vector $\boldsymbol{p}^r$: $\ln Q^{A}_{jk} \equiv \ln e(\boldsymbol{p}^r,u_j) - \ln e(\boldsymbol{p}^r,u_k)$. A problem with the Allen index is that it is not in general consistent with the true price or cost-of-living index due to Konüs. An alternative index which meets this criterion by construction is the *Konüs Quantity Index*, which equals the ratio of actual expenditures divided by the Konüs price index, evaluated at a reference utility level u_r: $\ln Q^{K}_{jk} \equiv \ln(\boldsymbol{p}^j.\boldsymbol{q}^j/\boldsymbol{p}^k.\boldsymbol{q}^k) - \ln[e(\boldsymbol{p}^j,u_r)/e(\boldsymbol{p}^k,u_r)]$. Finally, a difficulty with both the Allen and Konüs indices is that they are not homogeneous of degree one in quantities. A third index which meets this desirable criterion is the *Malmquist Quantity Index*, which equals the ratio of the distances of the actual consumption vectors from the indifference surface corresponding to the reference utility level u_r: $\ln Q^{M}_{jk} \equiv \ln d(\boldsymbol{q}^j,u_r) - \ln d(\boldsymbol{q}^k,u_r)$.

If tastes are homothetic, all three true indices reduce to the ratio of utility levels, u_j/u_k (since the expenditure function and distance function become $e(\boldsymbol{p},u)=u\varepsilon(\boldsymbol{p})$ and $d(\boldsymbol{q},u)=U(\boldsymbol{q})/u$ respectively). Otherwise they differ among themselves and the value of each depends on the choice of reference consumer (i.e., the selection of $e(\boldsymbol{p},u)$ or $d(\boldsymbol{q},u)$) and of the reference price vector or utility level. This underlines the fact that with non-homothetic tastes there is no such thing as a *unique* true measure of real income.

Finally, what can be said about the different empirical indices introduced in the last section in the light of the economic approach to index numbers? As far as the Geary method is concerned, the consensus appears to be that it has no basis in economic theory. By contrast, the EKS and CCD indices have obtained considerable support from some theoretical results in the literature. (See, for example, Caves et al. (1982a).) In the next section I review these results and consider their relevance to multilateral comparisons.

III. Superlative Indices and Multilateral Comparisons of Real Income

One of the towering achievements of index number theory is the set of results due to Konüs and Byushgens (1926) and Diewert (1976), which show that the empirical indices of Fisher and Törnqvist equal some true index under particular specifications of preferences. These results have deservedly had an enormous influence on empirical research and official statistics. (Witness for example the "Divisia" − strictly speaking, Törnqvist − indices of the UK money supply published by the Bank of England.) However, these results have two drawbacks. First, they only give sufficient conditions for the empirical indices to equal a true index: necessary conditions seem unattainable. Second, they only apply to bilateral comparisons. In this section I give two new results for multilateral comparisons which nest those of Konüs-Byushgens and Diewert, and argue that they weaken the case for using the multilateral extensions of the Fisher and Törnqvist empirical indices.

The first result is a straightforward extension of that of Konüs and Byushgens:

Proposition 1: *The EKS index equals the ratio of utilities,* $Q_{jk}^{EKS}=u_j/u_k$, *when the utility function is a homogeneous quadratic:* $U(\boldsymbol{q})=(\boldsymbol{q}'\boldsymbol{A}\boldsymbol{q})^{1/2}$, $\boldsymbol{A}$ *symmetric.*

(The proof is in Appendix A.) At first sight, Proposition 1 appears to justify the EKS method, at least when tastes are homothetic. As Diewert (1976) has noted, the quadratic utility function is a *flexible* functional form, i.e., it provides a second-order approximation to an arbitrary twice-differentiable homothetic utility function. He argues strongly for the use of index numbers which he labels *superlative*, meaning that, given a flexible functional form, they equal some true index, and Proposition 1 shows that the EKS index is superlative.

The difficulty with Proposition 1 is that it goes too far. There are no other conditions under which either the Fisher or EKS indices is known to be superlative.[5] Hence, Proposition 1 implies that, as far as economic theory is concerned, there is nothing to be gained by using the EKS procedure over the bilateral Fisher index. (Of course, on practical grounds, the EKS index has the great advantage that, by construction, it always yields a transitive ranking of income levels, unlike the Fisher index. But this is a statistical property and says nothing about how well the EKS approximates an underlying transitive preference ordering when the utility function is not a homogeneous quadratic.) While the EKS equals the true index for the quadratic utility function, it is also redundant, since it equals the bilateral Fisher index in that case. Moreover, the fact that in practice the Fisher and EKS indices do not coincide (and the Fisher index tends not to yield transitive multilateral rankings) is itself evidence against quadratic preferences, implying that the only conditions known to justify the EKS index do not hold.

What if tastes are not homothetic? The quadratic utility function does not generalise to this case. However its logarithmic equivalent, the translog, does, in the form of the translog distance function:[6]

$$\ln d(\boldsymbol{q},u) \;=\; a_0 \;+\; \boldsymbol{a}'\ln\boldsymbol{q} \;+\; \tfrac{1}{2}(\ln\boldsymbol{q})'\boldsymbol{A}\ln\boldsymbol{q} \;+\; b_0\ln u \;+\; (\ln u)\boldsymbol{b}'\ln\boldsymbol{q} \;+\; \tfrac{1}{2}c_0(\ln u)^2\,. \qquad \textbf{(7)}$$

Diewert has shown that, when preferences are described by (7), the empirical Törnqvist index Q^T_{jk} equals the true Malmquist index Q^M_{jk}, evaluated at the geometric mean of the two countries' utilities. While enormously useful in a bilateral context, the difficulty with this result in the multilateral context is that the Malmquist index is evaluated at a particular utility level which is specific to the two countries being compared. This suggests that the CCD index, which as we have seen is the multilateral counterpart to the Törnqvist, aggregates in general over *m* inconsistent bilateral comparisons. This is confirmed by the next proposition:[7]

Proposition 2: The CCD index deviates systematically from the Malmquist index wherever the latter is evaluated, if the distance function is a general (non-homogeneous) translog.

The proof is in Appendix B, where the deviation of the CCD index from the Malmquist index evaluated at u^*, the geometric mean of *all m* countries' utility levels, is shown to equal:

$$\ln Q^{CCD}_{jk} - \ln Q^{M}_{jk}(u^*) \;=\; \tfrac{1}{2}(\ln u_j - \ln u^*)\boldsymbol{b}'(\ln\boldsymbol{q}^j - \ln\boldsymbol{q}^*) - \tfrac{1}{2}(\ln u_k - \ln u^*)\boldsymbol{b}'(\ln\boldsymbol{q}^k - \ln\boldsymbol{q}^*)\,. \qquad \textbf{(8)}$$

($\boldsymbol{q}^*$ is the geometric mean of all *m* countries' quantity vectors.) Of course, the CCD index is also biased, and in a less symmetric way, if Q^M_{jk} is evaluated at any other utility level.

Inspection of equation (8) shows that the bias of the CCD index vanishes only in a few special cases. One is when there are only two countries, which is the bilateral result of Diewert already mentioned. Another is when tastes are homothetic (i.e., $\boldsymbol{b}$=0), though the CCD index is then exactly equal to the Törnqvist and so, by analogy with Proposition 1, it is redundant. Finally, the bias is also zero when the two countries compared deviate symmetrically from average, although it should be noted that both the benchmark u^* and the bias in (8) vary with non-proportional transformations of the utility function.

In conclusion, the propositions in this section do not provide theoretical arguments against using the EKS and CCD indices in multilateral comparisons, but they undermine the case in favour of doing so. They are not conclusive because, like the Konüs-Byushgens-Diewert results themselves, they give only sufficient conditions for the various multilateral indices to equal some true index (or not, in the case of Proposition 2). Nevertheless, it is suggestive that, in all the cases where the Fisher or Törnqvist indices are known to have desirable properties, the corresponding multilateral indices are either redundant or systematically biased.

IV. The GAIA System

The propositions in the last section throw doubt on the claims that the EKS and CCD indices have a firm basis in economic theory when applied to multilateral comparisons. By contrast, the Geary method at least uses a consistent set of world prices to compare real incomes. However, like all fixed-weight indices, it does not make any allowance for substitution in consumption and so is likely to generate a Gerschenkron Effect bias. In this section I propose a true index which overcomes these drawbacks while preserving the spirit of the Geary method.

Given a reference consumer, the first step is to replace the Laspeyres formula in the Geary exchange rates (5) by their true equivalents, which I call Geary-Konüs exchange rates:

$$\mathbf{E}_j \quad = \quad \frac{e(\boldsymbol{\Pi}, u_j)}{e(\boldsymbol{p}^j, u_j)} \quad = \quad \frac{\Sigma_i \; \Pi_i q_{ij}^*}{\Sigma_i \; p_{ij} q_{ij}}, \qquad j = 1, \ldots \; m. \qquad \textbf{(9)}$$

Here the q_{ij}^* denote the "virtual" quantities which the reference consumer would choose if it had country *j*'s level of utility and faced the world prices Π. By Shephard's Lemma these equal the price derivatives of the expenditure function: $q_{ij}^* = e_i(\Pi, u_j)$, $\forall \; i,j$.

Comparing the prices of all countries with a common world price vector is unremarkable in itself. The next step is to require that the world prices satisfy aggregation conditions of the Geary type. They cannot do so in terms of actual quantities consumed.[8] However, they can in terms of virtual quantities. In place of (5), this leads to the following world prices:

$$\Pi_i \quad = \quad \frac{\Sigma_j \; \mathrm{E}_j p_{ij} q_{ij}}{\Sigma_j \; q_{ij}^*}, \qquad i = 1, \ldots \; n. \qquad \textbf{(10)}$$

The corresponding measures of real income can be expressed in three equivalent ways:

$$z_j^* \quad = \quad \mathrm{E}_j z_j \quad = \quad \sum_i^n \; \Pi_i q_{ij}^* \quad = \quad e(\boldsymbol{\Pi}, u_j), \qquad j = 1, \ldots \; m. \qquad \textbf{(11)}$$

These in turn imply Geary-Allen true indices of real income: $Q_{jk}^* = z_j^* / z_k^*$, $\forall \; j,k$. In words, z_j^*, the real income of country *j*, is the expenditure needed to give the reference consumer the same level of utility at world prices as it would attain at country *j*'s prices. Country *j*'s true exchange rate is the ratio of its income at world prices z_j^* to its income at domestic prices z_j. Finally, the world price of good *i* equates the value of world virtual consumption of good *i* with the sum of each country's actual spending on that good converted at the true exchange rates.

The advantages of this proposed system are that it combines the best features of the economic approach to index numbers and the Geary method. Like the former, it is firmly based on the microeconomic theory of the consumer and allows for inter-commodity substitution. Like the latter, it satisfies matrix consistency, albeit in terms of virtual rather than actual consumption levels: the q_{ij}^* can be consistently aggregated across countries and across commodities using the world prices and true exchange rates. Hence I use the acronym GAIA ("Geary-Allen International Accounts") for the system. Finally, the system presented here avoids the conflict between bilateral Allen and Konüs quantity indices noted in Section II: each exchange rate E_j is a Konüs true price index, while each real income index Q_{jk}^* is an Allen true quantity index.

V. Properties of the GAIA System

In this section I explore the theoretical properties of the GAIA system, and show how it reduces to the EKS and Geary indices in special cases. I first prove an existence result:

Proposition 3: Assume $q_{ij}>0$ and $p_{ij}>0$, $\forall$ i,j. Then there exists a solution to equations (9), (10) and (11) with all E_j, Π_i and z_j^* strictly positive.

Proof: Like their Geary counterparts (5) and (6), the $m+n$ equations (9) and (10) are linearly homogeneous in E and Π. Hence, we need to choose an appropriate normalisation. In this section, we set: $\Sigma_i \pi_i = \Sigma_i \Pi_i = 1$.

The first step is to prove that the Geary prices and real incomes are unique and strictly positive.[9] To do this, rewrite the equations for π and $\boldsymbol{z}^G$ from Section I in matrix notation:

$$(a) \quad \boldsymbol{\pi} = \hat{\boldsymbol{q}}^{-1}\boldsymbol{W}\boldsymbol{z}^{G} \qquad \text{and} \qquad (b) \quad \boldsymbol{z}^{G} = \boldsymbol{Q}'\boldsymbol{\pi} \tag{12}$$

These two equations can be combined in a single equation for the Geary real incomes:

$$\boldsymbol{z}^{G} = \boldsymbol{S}\boldsymbol{z}^{G}, \qquad \text{where:} \qquad \boldsymbol{S} \equiv \boldsymbol{Q}'\hat{\boldsymbol{q}}^{-1}\boldsymbol{W}, \tag{13}$$

The m-by-m matrix $\boldsymbol{S}$ has typical element $s_{jk}=\Sigma_i\,(q_{ij}\omega_{ik}/\Sigma_l\,q_{il})$, giving the sum over goods of country k's budget shares weighted by country j's shares in world consumption. $\boldsymbol{S}$ is strictly positive by assumption, and so, by the Perron-Frobenius theorem, its largest eigenvalue is real and positive and corresponds to a unique strictly positive eigenvector. Moreover, $\boldsymbol{S}$ is column-stochastic: $\iota'\boldsymbol{S}=\boldsymbol{q}'\hat{\boldsymbol{q}}^{-1}\boldsymbol{W}=\iota'\boldsymbol{W}=\iota'$; i.e., $\Sigma_k\, s_{jk}=1,\ \forall\ j$. Hence, its largest eigenvalue equals one, and so, from (13), $\boldsymbol{z}^G$ itself is the corresponding eigenvector. Hence, $\boldsymbol{z}^G$ is unique and strictly positive and so, from (12), the required π and ε vectors are also unique and strictly positive.

Next, repeat the preceding derivations for the GAIA system to obtain:

$$\boldsymbol{z}^{*} = \boldsymbol{S}^{*}\boldsymbol{z}^{*} \qquad \text{where:} \qquad \boldsymbol{S}^{*} \equiv \boldsymbol{Q}^{*\prime}(\hat{\boldsymbol{q}}^{*})^{-1}\boldsymbol{W}. \tag{14}$$

Unlike (13), this is a highly non-linear equation, since $\boldsymbol{S}^*$ depends on the unknown Π. Assume a particular Π^0 is given. Then, since the utility levels u_j are known, the virtual quantities q^*_{ij} are uniquely determined from Shepherd's Lemma, and so from (14) $\boldsymbol{S}^*$ is also uniquely determined. From (13), we may solve for a strictly positive $\boldsymbol{z}^*(\Pi^0)$. Finally, from the GAIA analogue of (12)(a), $\Pi=(\hat{\boldsymbol{q}}^*)^{-1}\boldsymbol{W}\boldsymbol{z}^*$, we may solve for a strictly positive $\Pi(\Pi^0)$ vector, which we may normalise such that $\Sigma_i\ \Pi_i=1$. All this defines a continuous mapping from the unit simplex into itself, and so from Brouwer's Fixed Point Theorem it must have a fixed point: $\Pi(\Pi^0)=\Pi^0$, for some Π^0. This proves the existence of a strictly positive vector of GAIA world prices, which from (9) and (12)(b) implies that strictly positive $\boldsymbol{E}$ and $\boldsymbol{z}^*$ vectors also exist. This completes the proof of

Proposition 3.

The similarity between equations (13) and (14) shows the close relationship between the Geary and GAIA real incomes, the former giving a − possible poor − approximation to the latter. These equations also suggest the following algorithm for calculating the GAIA system. First, calculate the Geary world incomes $\boldsymbol{z}^G$ using (13). Next, use (12) to solve for the Geary world prices π and then use the Hicksian demand functions to calculate first-round estimates of q_{ij}^*. Then, from the second equation in (14), calculate the implied $\boldsymbol{S}^*(\pi)$ matrix, use it in the first equation to calculate second-round estimates of $\boldsymbol{z}^*$, and continue until the algorithm converges. I have been unable to prove that this algorithm must converge.[10] However, it is clear from the proof of Proposition 3 that, if it converges, then it must converge to a solution for the GAIA prices. Moreover, from the first part of the proof, the prices at each step must be unique and positive, and so the final price vector Π must also be unique and positive. Heuristically, this suggests that the GAIA prices can be viewed as the outcome of a tâtonnement process which takes the Geary world prices as its starting point, and adjusts prices at each step to ensure worldwide virtual commodity balance.

Finally, we may state two results which are mathematically trivial but which serve the useful purpose of relating the GAIA system to the true and empirical indices introduced earlier.

Proposition 4: If preferences are homothetic, the GAIA real incomes coincide with those implied by the bilateral Allen, Konüs and Malmquist indices.

The proof is immediate by inspection. Clearly a new benchmark for multilateral comparisons is only needed if preferences are non-homothetic. Next:

Proposition 5: If prices in different countries are unrestricted (*subject only to* $p_{ij}>0$, $\forall\ i,j$), *then the GAIA real incomes coincide with (a) the Geary real incomes if and only if preferences are Leontief* (*fixed-coefficients*): $u=min_i(q_i/b_i,\ i=1, \ldots n)$*; (b) the EKS real incomes if the utility function is a homogeneous quadratic; and (c) the CCD real incomes if the utility function is a homogeneous translog.*

Proof: (a) Inspection of (13) and (14) shows they have the same solution if and only if $q^*_{ij}=q_{ij}$, $\forall\ i,j$. With no restrictions on prices (except that they are positive) this is equivalent to Leontief preferences. Parts (b) and (c) follow immediately from Propositions 1 and 2 above.

Part (a) of this proposition allows us to view the Geary real incomes as a Laspeyres-type approximation to the GAIA real incomes, just as the Laspeyres price index is a fixed-weight approximation to the Konüs true cost-of-living index.

VI. Interpretation of the GAIA World Prices

The GAIA system satisfies circularity and matrix consistency because it chooses a particular set of world prices. This raises the question: to which countries, if any, do the world prices correspond? Geary prices estimated using data from the International Comparison Project tend to come closest to the prices of middle-income countries such as Hungary or Italy. (See, for example, Nuxoll (1994).) Here, I show why this must be so for the GAIA prices, when preferences exhibit Generalized Linearity: $e(\boldsymbol{p},u)=f\,[a(\boldsymbol{p}),b(\boldsymbol{p}),u]$, where a and b are linearly homogeneous in $\boldsymbol{p}$, and f is linearly homogeneous in (a,b). This specification of preferences was introduced by John Muellbauer (1975), who showed that it is the most general which allows consistent aggregation across individuals when all face the same prices. My next result shows that, even though each country faces different prices, their demands can still be consistently aggregated at the GAIA prices:

Proposition 6: If preferences exhibit Generalized Linearity, then world demand patterns would be generated by a hypothetical country facing the GAIA world prices and with an income equal to a weighted quasi-linear mean of the individual countries' incomes.

Proof: First, we derive an implication of the GAIA system for budget shares. Let θ_j^* denote each country's share in world income, measured at world prices: $\theta_j^* \equiv z_j^*/\Sigma_k z_k^*$. Define the *world budget share* of commodity i as the average of each country's actual budget shares ω_{ij}, weighted by the θ_j^*: $\omega_i^* \equiv \Sigma_j \theta_j^* \omega_{ij}$. Using $z_j^* = E_j z_j$, and substituting from (10), this equals a weighted average of each country's budget shares in world prices, $\omega_{ij}^* \equiv \Pi_i q_{ij}^*/z_j^*$:

$$\bar{\omega}_i^* \equiv \sum_j \theta_j^* \omega_{ij} = \sum_j \theta_j^* \omega_{ij}^*, \qquad i = 1, ... n. \tag{15}$$

Equation (15) holds for any specification of preferences.[11] Under Generalised Linearity, country j's budget shares evaluated at world prices are:

$$\omega_{ij}^{*} \quad = \quad \omega_i^{GL}(\mathbf{\Pi}, z_j^{*}) \quad = \quad \phi(\mathbf{\Pi}, z_j^{*}) \,.\, A_i(\mathbf{\Pi}) \; + \; \boldsymbol{B}_i(\mathbf{\Pi}) \,. \qquad \mathbf{(16)}$$

Crucially, ϕ is independent of i, and A_i and B_i are independent of z. Weighting by country size and aggregating gives:

$$\sum_j \theta_j^{*} \omega_{ij}^{*} \quad = \quad \omega_i^{GL}(\mathbf{\Pi}, \tilde{z}^{*}) \,, \qquad \mathbf{(17)}$$

where $\tilde{z}^*$ is a weighted quasi-linear mean of individual incomes, defined implicitly by: $\phi(\Pi,\tilde{z}^*) = \Sigma_j \theta_j^* \phi(\Pi,z_j^*)$. Combining (15) and (17) yields the desired result: $\bar{\omega}_i^* = \omega_i^{GL}(\Pi,\tilde{z}^*)$, $\forall i$. In words, world expenditure patterns, in the sense of the world budget shares at world prices, would be generated by a hypothetical country which faces the same prices and whose income is an appropriate average of the individual countries' incomes.

Generalized Linearity nests many of the most widely-used demand systems, including the translog and almost ideal (AIDS) models. Proposition 6 is significantly strengthened when we specialise to these sub-cases.[12]

VII. An Empirical Application

So far, I have discussed the theoretical properties of the GAIA system, and used it as a benchmark to infer the implicit assumptions underlying the Geary and EKS indices. However, like any true index, the GAIA system can also be implemented empirically once we select a particular specification of preferences, in other words, a particular reference consumer. The approach adopted here is to choose as reference consumer that specification of preferences which comes closest to generating the observed world consumption data.[13] I estimate a number of complete systems of consumer demand equations, use the estimated parameters to calculate GAIA real income indices, and compare their performance and policy implications with those of the empirical indices.

Table 1 About Here

The data, taken from the 1980 International Comparison Project, comprise price and quantity observations of *per capita* consumer expenditure on 11 commodity groups in 60 countries.[14] Table 1 ranks the countries by total expenditure valued at current exchange rates, taking the lowest-income country (Ethiopia) as reference, and gives the values of six indices: three empirical indices, the EKS, CCD and Geary, and three "true" GAIA indices, all based on the Quadratic Almost Ideal System ("QUAIDS") of James Banks et al. (1997).[15] The QUAIDS budget shares are quadratic in $\ln y_j$ (whence the name), where $\ln y_j \equiv \ln z_j - \ln\alpha(\boldsymbol{p}^j)$; i.e., the log of nominal expenditure deflated by the "subsistence" price index $\alpha(\boldsymbol{p}^j)$:[16]

$$\omega_{ij} \;=\; \alpha_i + \sum_h \gamma_{ih} \ln p_{hj} + \beta_i \ln y_j + \frac{\lambda_i}{\beta(\boldsymbol{p}^j)} (\ln y_j)^2 . \qquad \textbf{(18)}$$

Table 1 presents GAIA indices for the full QUAIDS system and also for two special cases with restricted income responsiveness.[17] Setting λ_i=0, $\forall i$, gives the AIDS model of Deaton and Muellbauer (1980), while setting β_i=0 *and* λ_i=0, $\forall i$, gives what I call the Homothetic AIDS (HAIDS) model. Figure 1 illustrates a selection of the results, suppressing the CCD and HAIDS indices because they are visually indistinguishable from the EKS, and the AIDS index because it is indistinguishable from the QUAIDS.

Figure 1 About Here

Table 1 and Figure 1 show clearly that all six real income indices make an enormous difference to the absolute levels of real income, reducing by 50 to 70 percent their range of variation relative to the variation in expenditures at current exchange rates. They also affect the rankings across countries in similar ways: for example, the U.S., Canada and Hong Kong are ranked higher and Argentina is ranked lower by all the real income indices, reflecting substantial deviations from purchasing power parity for these countries in 1980. However, there are also significant differences between the various real income indices, and it is with these that we are primarily concerned.

Consider first the three empirical indices. As already noted, the CCD index is very close to the EKS. This is not too surprising, since they are constructed by applying the same aggregation formula to two similar superlative indices, the Fisher and Törnqvist, and it justifies our concentrating on the EKS from here on. By contrast, the EKS and Geary indices differ significantly. In particular, the Geary index compresses the distribution rather more than the EKS, lowering the coefficient of variation from 0.821 to 0.799 and the Gini coefficient from 0.458 to 0.448.

Consider next the different true indices. The simplest way of choosing between them is to compare the plausibility of their behavioral assumptions. The values of the log likelihood of each estimated demand system (given at the foot of Table 1) imply that the HAIDS specification is overwhelmingly rejected relative to the AIDS, whereas the latter can be rejected with a probability of only 22 percent relative to the QUAIDS.[18] A different perspective comes from the GAIA world prices implied by each specification, which are given in Table 2. These are somewhat implausible in the HAIDS case (with Fuel and Power costing over four times as much as, and Beverages only one seventh as much as, Miscellaneous Items). By contrast, in the AIDS and QUAIDS cases they are reasonably sensible and are highly correlated with the Geary world prices, more so in the QUAIDS case. On both econometric and economic grounds therefore, the HAIDS specification and its implied GAIA index is less persuasive than either of the other two, whereas there is little to choose between the AIDS and QUAIDS indices (which, as already noted, are visually indistinguishable in Figure 1.)

Table 2 About Here

Comparing the empirical and true indices, the first conclusion to be drawn is that homothetic tastes rationalise the EKS index. The EKS and HAIDS indices are almost identical in Table 1, which is why the latter is not shown in Figure 1. Moreover, the HAIDS index is much more correlated with the EKS than with the Geary index in Figure 2. Conversely, non-homothetic tastes are relatively more favourable to the Geary index, QUAIDS more so than AIDS, although even they are slightly more highly correlated with the EKS than with the Geary index. (Of course, no pair of correlation coefficients in Figure 2 is significantly different from each other. But the same is true of the correlations between all six real incomes indices and the undeflated expenditure index. In any case, the pattern of correlations in Figure 2 is suggestive.) Figure 1 tells a similar story: the QUAIDS index is closer to the EKS than to the Geary, but is markedly different from both.

Figure 2 About Here

A second conclusion concerns the degree of dispersion of real incomes implied by the various indices, as measured by the Gini coefficient illustrated in Figure 3. (Table 1 also gives the coefficient of variation, which is almost linearly related to the Gini.) All three GAIA indices imply a dispersion in real incomes greater than either the EKS or Geary indices, and the dispersion increases with the degree of income responsiveness.

Figure 3 About Here

Finally, the high correlations in Figure 2 might suggest that the choice between any of these indices is of little practical consequence. This ignores the fact, familiar for intertemporal comparisons from studies such as the U.S. Boskin Commission, that small differences between index numbers can have major implications for policy issues, when their effects are cumulated. Figure 3 illustrates this in the present context, taking as its starting point the U.N. target for foreign aid donations by high-income countries of 0.7 percent of GNP. To operationalise this, I calculate for each index the implied total transfer in billions of U.S. dollars (equal to 0.7 percent of real consumption expenditure grossed up by population) from countries which were members of the OECD in 1980 (denoted by a superscript in Table 1). This measure is sensitive to the choice of reference country, and with a high-income country (the U.S.) selected as reference, the implied transfer is inversely related to the dispersion of the OECD countries relative to the rest of the sample. Figure 3 shows that, despite its lower Gini coefficient, the Geary index implies a transfer which is over $0.3 billion greater than that implied by the EKS index; the HAIDS index implies considerably lower transfers; while the AIDS and QUAIDS indices imply much greater transfers, over $1 billion more than the $31.0 billion implied by the EKS index.

VIII. Conclusions

Many researchers have worked with the Penn World Table.[19] But not many have asked what exactly the numbers mean, and those who have considered the question have mostly advocated very different methods for calculating real or purchasing-power-parity-corrected incomes. In this paper I have reexamined the conceptual framework of international comparisons, proposed a new benchmark for them, and considered how well it is approximated by existing methods, including the Geary method which underlies the Penn World Table.

At a conceptual level, I have argued that there are two distinct questions which must be faced in choosing a desirable index for multilateral comparisons. First, what are we trying to measure? Second, given a particular choice of true index, which of its nature is unobservable, how can it best be approximated in practice? The importance of the first question is inadequately recognised in the literature, because it does not arise when tastes are homothetic. This is an important special case, but seems unlikely to hold for many plausible reference consumers.[20] When tastes are not homothetic, there is a threefold infinity of candidates for the "true" index. Real incomes can be measured by either the Allen, Konüs or Malmquist indices, each one of which in turn can be evaluated at any of an infinite number of reference points (a reference price vector in the Allen case or a reference utility level in the other two cases). Hence, there is no unique "true" measure of real income. For different purposes, different reference vectors may be preferred: for example, a Swiss multinational wishing to calculate local allowances for its executives might want to use a "star" system based on Swiss prices rather than the method proposed here. However, for researchers interested in explaining world growth patterns or non-economists interested in international comparisons of living standards, there seems little justification for privileging one country in this way.

The answer proposed in this paper to the first question above, the GAIA system, is a set of Allen real income indices, defined with respect to a vector of "world" prices which ensure consistent aggregation across commodities and countries. Under a wide class of demand systems, the world prices would generate actual world demand patterns. This system combines the best features of the economic approach to index numbers and of the Geary method. It meets all but one of the tests discussed in Section II (though matrix consistency is satisfied in terms of virtual rather than actual quantities); it allows for inter-commodity substitution; and it relates directly to the Gorman-Muellbauer theory of linear aggregation.

It should be stressed that this approach does not require the existence of a *representative* consumer in each country, far less in the world as a whole. None of the paper's results, except those in Sections III and VI, relies on the assumption of identical tastes worldwide. The empirical application in Section VII tries to find the parameterisation of preferences which most closely approximates the data, but does not require that a world representative consumer actually exist. By contrast, selecting a *reference* consumer seems to be a necessary requirement for making international comparisons when tastes are non-homothetic. In so far as data on real income have any meaning, it is that they provide an answer to the question: "How well-off would the same reference consumer be in different countries?" If tastes were homothetic, the answer to this question would not depend on the identity of the reference consumer. However, once we recognise the pervasive reality of non-homotheticity, we have to make a choice. From the multitude of candidate reference consumers, it seems sensible for economists to focus on the hypothetical consumer whose consumption patterns mimic world consumption behaviour as closely as possible.

As for the second question posed above, the bottom line of this paper is that the Geary method which underlies the Penn World Table is an acceptable, though not necessarily a particularly good, approximation to an appropriate ideal system. By contrast, when tastes are not homothetic, the EKS index provides a good (second-order) approximation to an *inconsistent* set of multilateral comparisons. The theoretical results of this paper suggest that the EKS index can be recommended if tastes are close to homothetic, whereas the Geary index is satisfactory if substitutability is weak. Neither assumption is attractive. However, if forced to choose, it seems more reasonable to assume that spending patterns are invariant to price changes than that they are invariant to income changes. Of course, an appreciation of the theoretical underpinnings of the Geary method draws attention to potential pitfalls in applying it. For example, since the only case where the Geary method has been shown to equal the true index is when preferences are of the fixed-coefficient type, it would not be appropriate to use data from the International Comparison Project or the Penn World Table to *test* hypotheses concerning the degree of inter-commodity substitutability.

Turning to the empirical application in Section VII, this showed how the GAIA index can be calculated, using estimates of a complete set of demand equations, based on data from the International Comparison Project which underlies the Penn World Table. As well as illustrating the pitfalls and potential of estimating true multilateral indices, this section turned up two key empirical findings. First, in accordance with the theoretical results, homothetic tastes rationalise the EKS index. However, for the specification of preferences assumed, homotheticity is overwhelmingly rejected by the data, whereas with non-homothetic tastes the EKS index is marginally preferred but there is little basis for choosing between it and the Geary index.[21] Second, both the EKS and Geary indices compress the distribution of world income (and thus underestimate the degree of inequality across countries) much more than the acceptable true indices, suggesting that conclusions about "convergence" based on either index should be treated with caution.

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ENDNOTES

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1 "EKS" refers to its originators Eltetö, Köves and Szulc. For references to the indices discussed in the paper, see Bert M. Balk (1996), Robert J. Hill (1997) and Diewert (1999). J. Peter Neary (1996) surveys the literature on multilateral comparisons and gives a non-technical introduction to the index proposed here.

2 See Alexander Gerschenkron (1951), Daniel A. Nuxoll (1994) and Neary and Bríd Gleeson

(1997). The Gerschenkron Effect is a consequence of utility maximisation if preferences are homothetic. If preferences are non-homothetic, it may not arise, even with a single utility-maximising consumer, though there is a presumption that it does. Neary and Gleeson develop and implement a test of the Effect.

3 The ICP defines true exchange rates or "purchasing power parities" as the inverse of (5), following the U.S. convention of measuring exchange rates. I follow Geary in using the U.K. convention, since it facilitates the matrix derivations below.

4 Angus Deaton (1979) gives a self-contained exposition of the distance function and its application to index numbers.

5 Strictly speaking, there is one other case where the Fisher index is known to equal the ratio of utilities, namely when the expenditure function is a homogeneous quadratic: $e(\boldsymbol{p},u) = u(\boldsymbol{p}'\boldsymbol{B}\boldsymbol{p})^{1/2}$. An analogue to Proposition 1 applies for this class of preferences too: the EKS also equals the ratio of utilities. In any case, this specification is equivalent to a homogeneous quadratic utility function when $\boldsymbol{A}$ is invertible (in which case $\boldsymbol{B} = \boldsymbol{A}^{-1}$), a condition which is necessary if the demands are to be written as single-valued functions: $\boldsymbol{q} = z(\boldsymbol{p}'\boldsymbol{A}^{-1}\boldsymbol{p})^{-1}\boldsymbol{A}^{-1}\boldsymbol{p}$.

6 The vector $\boldsymbol{b}$ is the source of non-homogeneity, which may be seen from the equations for the budget shares: $\omega = \partial \ln d(\boldsymbol{q},u)/\partial \ln \boldsymbol{q} = \boldsymbol{a}+\boldsymbol{A}\ln\boldsymbol{q}+\boldsymbol{b}\ln u$. When $\boldsymbol{b}$ is zero, (7) reduces to the homogeneous translog utility function, $\ln U(\boldsymbol{q})\mathrm{k} = a_0+\boldsymbol{a}'\ln\boldsymbol{q}+½(\ln\boldsymbol{q})'\boldsymbol{A}\ln\boldsymbol{q}$, by setting $d(\boldsymbol{q},u) = 1$, so

$u = U(\boldsymbol{q})$, and (without loss of generality, since utility is ordinal) normalizing $b_0 = -1$ and $c_0 = 0$.

7 I am grateful to a referee for pointing out that this proposition resembles one of the principal results of Caves et al. (1982a). They show that their translog multilateral index is equivalent to a set of translog bilateral comparisons between each country and a hypothetical average country. However, while instructive, the resemblance between the two results is superficial. Caves et al. are concerned with the case where individual agents differ (at least in their first-order parameters) and where tastes are homothetic. (Strictly, they explicitly consider a production context with constant returns to scale, but this is formally equivalent to a consumer context with homothetic tastes.) By contrast, I focus on the case where tastes are non-homothetic, and argue in Section VIII that for international comparisons of real income to make sense in that case requires that we pick a *unique* reference consumer. Caves et al. (1982b) extend their approach to the case of non-constant returns to scale in production (formally equivalent to non-homothetic tastes) though only in a bilateral context.

8 After the first version of this paper was written, I came across D.D. Prasada Rao and J. Salazar-Carillo (1988), who propose a system which does this. The motivation and approach of their paper is very similar to mine. However, instead of my (9) and (10), they propose a hybrid combination of (6) and (9). Summing (6) over commodities and (9) over countries shows that these two sets of equations are inconsistent in general.

9 This part of the proof is adapted from Balk (1996). The notation is as follows. All vectors

are column vectors; a prime (′) denotes a transpose; and $\hat{\boldsymbol{x}}$ denotes a diagonal matrix formed from the vector $\boldsymbol{x}$; $\boldsymbol{Z}$ denotes the n-by-m matrix of expenditures by commodity and country, with typical element $z_{ij} = p_{ij}\, q_{ij}$; $\boldsymbol{z} \equiv \boldsymbol{Z}'\iota$ (where ι is a vector of ones) denotes the m-by-1 vector of total expenditures by country, with typical element $z_j = p^{\,j}.q^{\,j}$; $\boldsymbol{Q}$ denotes the n-by-m matrix of quantities by commodity and country, with typical element q_{ij}; $\boldsymbol{q} \equiv \boldsymbol{Q}\iota$ denotes the n-by-1 vector of world consumption levels of each commodity, with typical element $q_i = \Sigma_j\, q_{ij}$; and $\boldsymbol{W} \equiv \boldsymbol{Z}\hat{z}^{-1}$ denotes the n-by-m matrix of budget shares (in domestic prices), with typical element $\omega_{ij} = z_{ij}/z_j$.

10 The algorithm is not very efficient from a computational point of view, and a different approach was used to calculate the GAIA indices in Section VII below. See footnote 17.

11 A similar relationship holds for the Geary world prices: in obvious notation, $\bar{\omega}_i^G = \Sigma_j\, \theta_j^G \omega_{ij}^G$. However, this yields no additional insights, because the Geary budget shares ω_{ij}^G have no behavioral significance: quantities are chosen facing prices $\boldsymbol{p}^j$ but aggregated using prices π.

12 For example, as shown in Neary (1996), when preferences are described by the Gorman Polar Form, due to W.M. Gorman (1961), so $e(\boldsymbol{p},u)=a(\boldsymbol{p})+ub(\boldsymbol{p})$, then world consumption patterns, not just in the sense of budget shares but in the much stronger sense of *levels* of spending on each commodity, would be generated by the GAIA prices and an income equal to the arithmetic average of world incomes. Further details are given in Appendix C to this paper, available at http://www.ucd.ie/ ~economic/staff/pneary/gaia/gaia.htm ("neary gaia" via Google) or on request.

13 An alternative approach would be to use non-parametric methods to place bounds on the true index. Steve Dowrick and John Quiggin (1997) implement this approach, though even they succumb to the demands of users for a single numerical index by privileging the mid-point of their estimated bounds, effectively treating it as a new index. See also footnote 20 below.

14 Full details of the data, taken from United Nations (1987), and the estimation methods are given in Appendix D to this paper, available at the url given in footnote 12. Note that the number of countries in the ICP is considerably less than that in the Penn World Table. It is not widely appreciated that the data on real magnitudes for non-ICP countries are derived from those for ICP countries using interpolation methods. See Kravis (1984, pp. 17-19) and Summers and Heston (1991, pp. 341-343). This is yet another reason to treat regressions which use the full Penn World Table data with caution.

15 This is a flexible functional form and it nests cleanly different hypotheses about departures from homotheticity, which is one of the key issues in the choice of real-income index. An earlier pilot project by Neary and Gleeson (1997) used the linear expenditure function. However, the wide range of variation in expenditure levels in the sample pulls the estimated subsistence parameters in the Stone-Geary utility function close to zero, which is tantamount to imposing homotheticity. I am grateful to Anton Barten for this point.

16 The log expenditure function is $\ln\alpha(\boldsymbol{p}^j)+u_j\beta(\boldsymbol{p}^j)/[1-u_j\lambda(\boldsymbol{p}^j)]$, where $\ln\alpha(\boldsymbol{p}^j)\equiv\alpha_0+\Sigma_i\,\alpha_i\ln p_{ij}$ + $\frac{1}{2}\Sigma_i\,\Sigma_h\,\gamma_{ih}\ln p_{ij}\ln p_{hj}$, $\ln\beta(\boldsymbol{p}^j)\equiv\Sigma_i\,\beta_i\ln p_{ij}$ and $\lambda(\boldsymbol{p}^j)\equiv\Sigma_i\,\lambda_i\ln p_{ij}$. Homogeneity requires $\Sigma_{i=1}\,\alpha_i = 1$ and

$\Sigma_i \beta_i = \Sigma_i \lambda_i = \Sigma_h \gamma_{ih} = 0$; while Slutsky symmetry requires $\gamma_{ih} = \gamma_{hi}$, $\forall$ *i,h*. All these restrictions, as well as negativity of the substitution matrix, were imposed in the estimation. Because of the relatively large number of parameters which appear in each equation, convergence of the maximum likelihood estimates was ensured by extending an approach developed by Lawrence J. Lau (1978) and Diewert and T.J. Wales (1988). Since this takes utility maximisation as a maintained hypothesis, it is not possible to use the estimates to *test* the restrictions. See Appendix D for further details.

17 For each set of parameter estimates, the implied GAIA world prices were calculated by solving (15) non-linearly, ensuring positive prices by iterating on the square root of Π rather than Π itself. For the HAIDS specification, the results can be cross-checked by using the fact that relative GAIA indices in that case can be derived explicitly without the need to calculate Π: $\ln z_j^* - \ln z_k^* = u_j - u_k = \{\ln z_j - \ln\alpha(\boldsymbol{p}^j)\} - \{\ln z_k - \ln\alpha(\boldsymbol{p}^k)\}$.

18 Twice the difference in log likelihoods has a chi squared distribution with degrees of freedom equal to the number of additional parameters in each bilateral comparison. The latter equals ten in both cases, so the log likelihoods imply that the HAIDS specification can be rejected relative to the AIDS at a significance level of more than 0.001 percent, whereas the AIDS specification can be rejected relative to the QUAIDS at a significance level of only 22.2 percent.

19 Heston and Summers (1996) quote an anonymous claim that over 20,000 regressions have been estimated using the Penn World Table. Xavier X. Sala-i-Martin (1997) alone exceeds this

(in combination with other data sources) by a factor of a hundred.

20 For studies of international real income comparisons which assume homotheticity see Diewert (1999) and Dowrick and Quiggin (1997). It is true that tests for homotheticity using the non-parametric approach of Sydney N. Afriat (1981) and Hal R. Varian (1983) routinely fail to reject homotheticity: see for example Dowrick and Quiggin (1997) and Dowrick, Yvonne Dunlop and Quiggin (2003). However, the power of the Afriat-Varian test is low, and other researchers have chosen to augment the non-parametric approach to allow for non-homotheticity: see Richard W. Blundell et al. (2003).

21 It is true, as a referee points out, that the HAIDS model is only one of an infinity of homothetic specifications, so its poor performance relative to the AIDS and QUAIDS models is not a conclusive rejection of homotheticity *per se*. However, the HAIDS model is a flexible functional form within the family of homothetic utility functions, so if tastes were indeed homothetic we should not expect it to perform so badly when additional spurious variables (income and income squared) are added to the budget share equations. See also the discussion of non-parametric tests of homotheticity in footnote 20.

Appendix A: Proof of Proposition 1

The proposition follows immediately from the result of Konüs and Byushgens, that the Fisher index equals the ratio of utilities, $Q^F_{jk}=u_j/u_k$, when the utility function is a homogeneous quadratic. Using this to replace the Fisher index by the ratio of utilities in the expression for the EKS index (2) yields:

$$\begin{aligned} \ln Q_{jk}^{EKS} &= \frac{1}{m}\sum_{l=1}^{m}\left\{(\ln u_j - \ln u_l) - (\ln u_k - \ln u_l)\right\}, \\ &= \ln u_j - \ln u_k . \end{aligned} \qquad \textbf{(A19)}$$

Appendix B: Proof of Proposition 2

First rewrite the expression for the Törnqvist empirical index, equation (3), in vector notation as follows:

$$\ln Q_{jk}^{T} = ½\left(\boldsymbol{\omega}^j + \boldsymbol{\omega}^k\right)'\left(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k\right). \qquad \textbf{(A20)}$$

Substituting into (4), the CCD index can be written as follows:

$$\begin{aligned} \ln Q_{jk}^{CCD} &= \frac{1}{m}\sum_{l=1}^{m}\left(\ln Q_{jl}^{T} - \ln Q_{kl}^{T}\right) \\ &= ½\left[(\boldsymbol{\omega}^j)'\ln\boldsymbol{q}^j - (\boldsymbol{\omega}^k)'\ln\boldsymbol{q}^k + (\boldsymbol{\omega}^*)'(\ln\boldsymbol{q}^j - \ln\boldsymbol{q}^k) - (\boldsymbol{\omega}^j - \boldsymbol{\omega}^k)'\ln\boldsymbol{q}^*\right], \end{aligned} \qquad \textbf{(A21)}$$

where $\boldsymbol{q}^*$ is defined in the text and ω^* is the arithmetic mean of all m countries' budget share vectors: $\omega^* \equiv (1/m)\Sigma_l\, \omega^l$. Equation (A21) holds irrespective of how demands are generated. Now suppose that they are generated by the translog distance function (7). Invoking a standard property of the distance function (see Deaton (1979)), the budget shares in this case equal:

$$\boldsymbol{\omega}^j = \frac{\partial \ln d(\boldsymbol{q}^j, u_j)}{\partial \ln \boldsymbol{q}^j} = \boldsymbol{a} + \boldsymbol{A} \ln \boldsymbol{q}^j + \boldsymbol{b} \ln u_j . \qquad \textbf{(A22)}$$

Substituting into (A21), the CCD index becomes:

$$\begin{aligned} \ln Q_{jk}^{CCD} &= a'(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k) + ½\left[(\ln \boldsymbol{q}^j)'\boldsymbol{A}\ln \boldsymbol{q}^j - (\ln \boldsymbol{q}^k)'\boldsymbol{A}\ln \boldsymbol{q}^k\right] \qquad \textbf{(A23)} \\ &+ ½\left[\left(\ln u_j\right)\boldsymbol{b}'\left(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^*\right) - \left(\ln u_k\right)\boldsymbol{b}'\left(\ln \boldsymbol{q}^k - \ln \boldsymbol{q}^*\right) + \left(\ln u^*\right)\boldsymbol{b}'\left(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k\right)\right] . \end{aligned}$$

Next, consider the Malmquist index, defined in Section II, evaluated at u^*:

$$\ln Q_{jk}^M(u^*) = \ln d(\boldsymbol{q}^j, u^*) - \ln d(\boldsymbol{q}^k, u^*) . \qquad \textbf{(A24)}$$

Substituting from the translog distance function (7), this becomes:

$$\begin{aligned} \ln Q_{jk}^M(u^*) &= a'(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k) + ½\left[(\ln \boldsymbol{q}^j)'\boldsymbol{A}\ln \boldsymbol{q}^j - (\ln \boldsymbol{q}^k)'\boldsymbol{A}\ln \boldsymbol{q}^k\right] \qquad \textbf{(A25)} \\ &+ (\ln u^*)\boldsymbol{b}'(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k) . \end{aligned}$$

Subtracting this from (A23) gives equation (8) in the text, which proves the proposition. As a corollary, the bias of the CCD index when it is evaluated not at u^* but at the geometric mean of the utilities of the two countries being compared, denoted by $\bar{u}^{jk} \equiv (u_j u_k)^{½}$, can be shown to equal:

$$\begin{aligned} \ln Q_{jk}^{CCD} - \ln Q_{jk}^M(\bar{u}^{jk}) &= ½(\ln u^*)\boldsymbol{b}'(\ln \boldsymbol{q}^j - \ln \boldsymbol{q}^k) - ½\left(\ln u_j - \ln u_k\right)\boldsymbol{b}'\ln \boldsymbol{q}^* \qquad \textbf{(A26)} \\ &+ ½\left[\left(\ln u_j\right)\boldsymbol{b}'\ln \boldsymbol{q}^k - \left(\ln u_k\right)\boldsymbol{b}'\ln \boldsymbol{q}^j\right] . \end{aligned}$$

TABLE I

ALTERNATIVE INDICES OF REAL INCOME, 1980

	Country	Expenditure	EKS	CCD	Geary	HAIDS	AIDS	QUAIDS
1	Germany[a]	79.242	30.017	29.859	25.503	30.128	40.785	41.221
2	Denmark[a]	78.833	28.995	28.645	24.699	28.894	39.968	40.533
3	Belgium[a]	73.600	29.085	28.982	24.851	29.754	40.047	40.367
4	France[a]	73.282	29.338	29.319	25.021	29.631	40.054	40.449
5	U.S.A.[a]	70.273	36.120	35.653	30.624	37.039	47.557	48.385
6	Luxembourg[a]	69.860	30.066	30.016	26.114	29.928	41.106	42.021
7	Netherlands[a]	68.652	28.265	28.013	23.993	28.875	38.096	38.222
8	Norway[a]	63.528	23.571	23.445	20.591	23.335	32.143	32.710
9	Austria[a]	62.537	26.636	26.607	22.905	26.610	37.122	37.596
10	Canada[a]	62.280	34.171	34.033	29.230	34.859	48.415	49.818
11	U.K.[a]	56.530	25.304	25.251	21.614	25.396	35.207	35.537
12	Finland[a]	52.658	22.089	22.000	19.061	22.145	31.695	32.260
13	Japan[a]	49.079	21.630	21.597	19.102	21.383	31.352	32.003
14	Italy[a]	43.309	24.784	24.755	21.596	24.791	34.565	35.011
15	Spain[a]	36.987	21.070	21.196	18.226	21.034	28.631	28.930
16	Ireland[a]	33.789	16.978	17.217	14.638	17.472	24.356	24.674
17	Argentina	31.926	11.752	11.903	10.451	11.439	15.704	15.798
18	Israel	30.784	18.791	18.685	16.525	18.444	25.388	25.706
19	Hong Kong	30.640	22.450	22.530	20.783	22.098	30.787	31.267
20	Greece[a]	27.082	16.531	16.601	14.297	16.457	22.773	23.022
21	Uruguay	24.203	14.497	14.645	13.226	14.218	19.698	19.853
22	Venezuela	20.852	14.085	14.161	12.566	13.817	19.463	19.473
23	Portugal[a]	18.026	13.741	13.887	12.340	13.495	18.665	18.805
24	Yugoslavia	16.669	11.011	10.876	9.821	11.101	15.055	15.082
25	Chile	16.536	10.755	10.811	9.396	10.664	14.464	14.498
26	Poland	13.733	12.156	12.198	10.545	12.180	16.750	16.729
27	Brazil	13.598	11.347	11.297	10.045	11.485	15.669	15.843
28	Costa Rica	13.555	10.270	10.264	9.115	10.214	13.692	13.483
29	Hungary	10.454	13.285	13.152	12.017	13.318	18.563	18.620
30	Panama	10.439	8.026	8.066	7.142	8.042	10.578	10.511
31	Paraguay	9.473	7.576	7.554	6.686	7.598	9.788	9.627
32	Korea	9.454	6.679	6.834	6.491	6.654	8.239	8.000
33	Dominican Rep.	8.698	7.635	7.561	6.820	7.665	9.811	9.683
34	Colombia	8.443	9.813	9.862	8.758	9.738	13.066	12.864
35	Ecuador	8.406	7.161	7.172	6.719	7.047	9.163	9.092
36	Tunisia	8.094	6.201	6.149	5.583	6.240	8.068	8.075
37	Guatemala	7.845	8.624	8.470	8.077	8.522	10.885	10.541
38	Côte d'Ivoire	7.114	3.410	3.396	2.924	3.419	4.218	4.228
39	Peru	6.968	7.685	7.592	7.065	7.589	9.876	9.719
40	Bolivia	6.178	4.464	4.493	4.172	4.492	5.909	5.767
41	Nigeria	5.690	2.355	2.375	2.120	2.434	2.679	2.576
42	Botswana	5.615	3.658	3.676	3.285	3.633	4.498	4.533
43	Morocco	5.599	3.896	3.870	3.520	3.885	4.712	4.693
44	Cameroon	5.313	2.803	2.804	2.509	2.777	3.014	2.966
45	El Salvador	5.178	4.804	4.824	4.413	4.728	5.239	4.890

TABLE I (cont.)

Country	Expenditure	EKS	CCD	Geary	HAIDS	AIDS	QUAIDS
46 Philippines	4.646	5.891	5.907	5.096	6.012	7.280	6.997
47 Honduras	4.398	4.079	4.051	3.642	4.054	4.509	4.376
48 Zimbabwe	4.245	2.674	2.711	2.371	2.685	3.457	3.461
49 Senegal	3.907	2.442	2.438	2.266	2.420	2.964	2.998
50 Zambia	3.369	1.620	1.629	1.421	1.587	1.876	1.927
51 Indonesia	2.753	2.958	2.946	2.802	3.027	3.706	3.466
52 Madagascar	2.652	1.974	1.963	1.862	1.942	2.120	2.082
53 Pakistan	2.619	4.175	3.983	3.624	4.361	5.007	4.798
54 Kenya	2.603	2.052	2.055	1.938	2.036	2.255	2.247
55 Sri Lanka	2.023	4.377	4.402	4.246	4.485	4.495	4.140
56 Tanzania	1.906	1.186	1.187	1.135	1.257	0.939	0.931
57 India	1.526	1.716	1.730	1.602	1.755	1.626	1.532
58 Mali	1.512	1.187	1.128	1.073	1.232	1.247	1.232
59 Malawi	1.302	1.260	1.282	1.208	1.234	1.626	1.691
60 Ethiopia	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mean	22.411	12.370	12.345	10.841	12.396	16.694	16.809
Standard Deviation	24.120	10.153	10.105	8.660	10.237	14.219	14.500
Coefficient of Variation	1.076	0.821	0.819	0.799	0.826	0.852	0.863
Correlation with EKS[b]	0.9210	1.0000	0.9999	0.9989	0.9995	0.9983	0.9977
Correlation with Geary[b]	0.9107	0.9989	0.9990	1.0000	0.9975	0.9975	0.9968
Gini Coefficient	0.5620	0.4581	0.4574	0.4479	0.4597	0.4757	0.4808
Implied Transfer[c]	33.674	30.997	31.203	31.327	30.581	32.064	32.000
Number of estimated parameters					65	75	85
Value of log likelihood					457.416	480.054	486.567

[a] Denotes an OECD member in 1980; [b] Squared correlation coefficients; [c] In billions of US$: see text for details.

TABLE II

WORLD PRICES IMPLIED BY THE GEARY AND GAIA ESTIMATES

	Commodity Group	Geary	HAIDS	AIDS	QUAIDS
1	Food	1.068	0.309	0.898	0.884
2	Beverages	0.732	0.144	0.667	0.648
3	Tobacco	0.894	0.406	1.003	0.966
4	Clothing & Footwear	1.010	0.471	1.011	0.959
5	Gross Rents	0.870	0.436	0.876	0.797
6	Fuel and Power	0.968	4.596	1.092	0.981
7	House Furnishings	1.109	0.684	1.147	1.103
8	Medical Care	1.012	0.456	1.148	1.090
9	Transport and Communications	0.940	1.241	1.086	1.055
10	Recreation and Education	1.209	0.440	1.154	1.189
11	Miscellaneous	1.000	1.000	1.000	1.000
	Correlation with Geary prices[a]		0.007	0.753	0.828

[a]Simple correlation coefficient between Geary world prices π and GAIA world prices Π

FIGURE 1 - Alternative Indices of Real Income, 1980

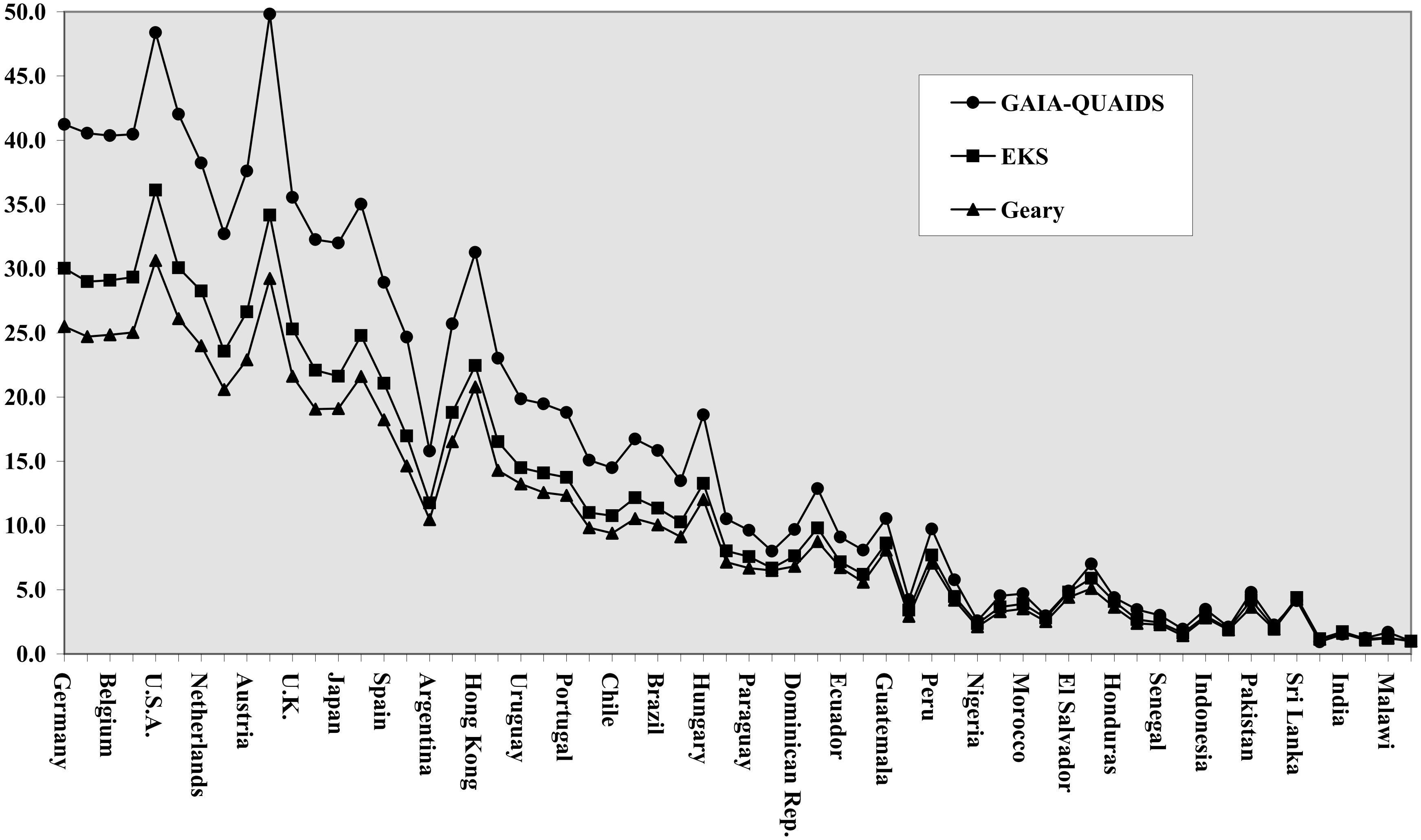

FIGURE 2 - Correlations with EKS and Geary Indices

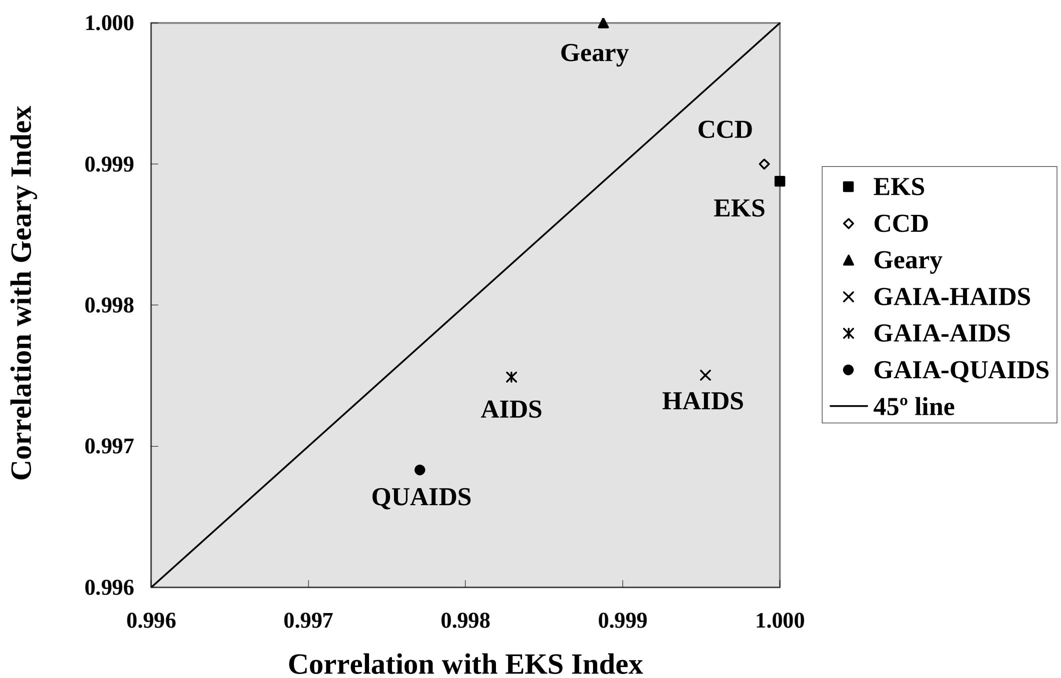

FIGURE 3 - Gini Coefficient and implied transfer