

EARNINGS FUNCTIONS, SPECIFIC HUMAN  
CAPITAL AND JOB MATCHING:  
TENURE BIAS IS NEGATIVE\*

Margaret Stevens<sup>†</sup>

---

\*I would like to thank Meg Meyer for helpful discussions of earlier versions of this paper.

<sup>†</sup>Oxford University Department of Economics, and Lincoln College, Oxford

## Abstract

This paper investigates the hypothesis that when measures of specific human capital (such as job tenure) are included in earnings functions, there may be a sample selection bias due to job-matching effects – because workers with high unobserved match quality receive, and accept, high wage offers. We develop a model for wage offers in a labour market characterized by both specific human capital and job matching. The model provides a theoretical basis for empirical earnings functions containing specific capital, and demonstrates that sample selection bias reduces the estimated return to specific human capital and tenure.

## I. Introduction

In the estimation of human capital earnings functions, the tradition established by Mincer (1974) is to represent general human capital by years of schooling and labour market experience; then, as proposed by Mincer and Jovanovic (1981), the effect on wages of the accumulation of specific human capital can be captured by including a job tenure variable. The apparent significance of job tenure in many subsequent studies involving cross-sectional estimation of earnings functions has been the subject of fierce debate. It has often been argued that the estimated return is positively biased because of a sample selection effect: workers with high unobserved match quality receive high wage offers from their existing employers and hence tend to stay in their jobs, introducing a positive correlation between wages and tenure in cross-sectional data (Abraham and Farber, 1987; Altonji and Shakotko, 1987). However, Topel (1986, 1991) and Garen (1988) have argued that with matching, the direction of the sample selection bias is ambiguous since there is, in addition, a negative effect: workers who move to new jobs, and hence have low tenure, are those who receive high alternative wage offers.<sup>1</sup>

In the existing literature, results for the direction of the bias on the return to tenure are derived on the assumption that wage offers are determined exogenously, while employment is determined endogenously by workers reacting to wage offers. I present a model in which wage offers are endogenous – firms choose their offers in anticipation of workers' reactions. Hence the model determines precisely how the wage depends on

---

<sup>1</sup>Attempts in these studies to control for sample selection effects using panel data have not resolved the argument. Abraham and Farber (1987) and Altonji and Shakotko (1987) found the effect of tenure on wages to be almost insignificant after controlling for unobserved heterogeneity. Topel (1991) found a strong positive tenure effect in his own analysis of the same data. Barth (1997), using a Norwegian dataset that enabled him to control for firm heterogeneity, also found a significant positive tenure effect.

specific capital and match quality. I show that the resulting wage offer function provides a nice interpretation of the human capital earnings function. In the estimation of this function, there is a sample selection problem exactly as described above, but the direction of the bias on the return to tenure is unambiguously negative. Workers who have high levels of specific capital stay in their jobs even when match quality is low, so in a sample of accepted wages specific human capital is negatively correlated with match quality. The model also implies that in the absence of specific human capital, matching does not introduce a positive relationship between wages and tenure.

## II. Specific Capital Models, and Job Matching

The original Mincerian earnings function, containing measures of general human capital, could be interpreted on the basis of competitive theory as a technological relationship – the effect of human capital on productivity. But the introduction of measures of specific human capital such as job tenure precludes the possibility of perfectly competitive wage determination, and raises the question: what are we estimating? The answer must be that it is some kind of wage determination function. How, then, are wages determined when workers possess specific as well as general human capital?

### A. Wage Determination with Specific Human Capital

Classical human capital theory suggests that wages respond fully to general human capital but only partially to specific human capital (Becker, 1962; Oi, 1962). The intuition that the return to specific human capital will be shared between worker and firm, in the form of a wage higher than the worker’s opportunity wage but lower than productivity, was formalised by Hashimoto (1981). In Hashimoto’s model, the value of the worker’s specific human capital is subject to shocks. Its ex-post value within the firm is observed only by the firm, and its ex-post value elsewhere (the opportunity wage) is observed only by the worker. The asymmetry of information may result in inefficient separation and loss of specific capital; to minimise this loss it is desirable for the worker and firm to predetermine the wage. Hashimoto derives the optimal predetermined wage, which depends upon the worker’s general and specific human capital. This predetermined wage function provides a potential theoretical basis for empirical earnings functions.

But if empirical earnings functions represent predetermined wage functions, then the standard matching argument for tenure bias does not apply, precisely because the wage is predetermined, before the shocks which determine employment decisions are realised.

These shocks contribute some unobserved quality to the match; they affect the separation decision and hence whether or not the wage is observed, but have no effect on the wage itself. The predetermined wage reflects only the worker's expected human capital, and observed wages are a random sample of predetermined wages.

Gibbons and Waldman (1999) argue that the predetermined wage is not a feature of many employment relationships – post-training wages are not typically specified in a contract. Hashimoto's model does not, in fact, rule out other types of employment contract for workers with specific human capital. Wages may be determined ex-post by bargaining; when asymmetry of information makes bargaining infeasible or costly, Hall and Lazear (1983) identified other simple second-best contracts, such as allowing worker or firm to set the wage unilaterally. There is no reason to suppose such contracts are less efficient than the predetermined wage; Stevens (1994) shows that it may be preferable for the firm to set the wage. Having chosen a mechanism for wage determination, a worker and firm wishing to invest in specific human capital can determine their respective shares of the expected match surplus under that contract, and bargain over the sharing of costs.

These arguments suggest that we should consider whether alternative wage determination mechanisms can be used as a basis for empirical earnings functions. As discussed in the next section, the implicit assumption behind the job matching argument for tenure bias is that wages are not predetermined.

## **B. Specific Human Capital and the Job-Matching Model**

Garen (1988) presents a simple model of job-matching with specific human capital, which neatly demonstrates the ambiguity of the tenure-bias. A worker with tenure  $T = 1$  receives a wage offer  $w_1(S)$  from his existing employer which is an exogenous function (the sum) of his general and specific human capital (or some fixed share of it) and of the realised quality of the match  $m$ . Match quality  $m$  has expected value zero, and known distribution. He also receives an offer  $w_1(A)$  from the external labour market; this is an exogenous function of his general human capital and a mean-zero random element  $u$  that can be interpreted as the quality of the alternative offer. If he accepts the alternative offer he will move to a job in which he has tenure  $T = 0$ .

The difference between the expected value of the two offers is the difference between the expected wage of two workers with the same experience, but different tenure: it is the return to tenure, or equivalently the worker's share of the return to specific human capital. However, the worker accepts the alternative offer  $w_1(A)$  if and only if it is higher than the offer of the current employer  $w_1(S)$ ; otherwise, he accepts  $w_1(S)$ . So,

in a cross-sectional dataset we observe  $w_1(A)$  and  $w_1(S)$  conditional on their having been accepted by workers. If we try to estimate the return to tenure by the difference  $E[w_1(S) \mid \text{accepted}] - E[w_1(A) \mid \text{accepted}]$ , the estimate will be biased. The bias comes from the effect of unobserved match qualities: it is equal to  $E[m \mid w_1(S) \text{ accepted}] - E[u \mid w_1(A) \text{ accepted}]$ . Since both these terms are positive (offers are accepted when the corresponding match quality is high) the overall effect is ambiguous. Furthermore, this bias may be non-zero even if the worker has no specific capital: matching can induce an apparent relationship between wages and tenure when there is none.

In this model, and similarly in Topel (1991) and many other empirical analyses, the implicit wage determination process does not involve predetermined wages. Instead, firms make wage offers after observing match quality. The bias arises because the wage offer depends on match quality as well as human capital. But this is merely an assumption: the wage offer is an exogenous function without theoretical basis, and the assumed form of this function may not be very plausible. Should we not expect, for example, that as specific human capital and match quality increase, the firm will not need to continue raising the wage offer in proportion? It will realise that it can keep the worker with high probability without doing so. This type of strategic behaviour could affect both the wage itself, and the probability of acceptance, and hence affect the conclusions for tenure bias.

### C. An Alternative Approach

The objective in this paper is to develop a theoretical model of individual wage determination with specific human capital and job matching, which can be used to analyse sample selection bias. We need a model in which wages depend on specific capital, and also (unlike the predetermined wage model) on match quality. So, the labour market is characterised by match heterogeneity. Like Hashimoto, and Hall and Lazear, we assume that information asymmetries rule out bilateral bargaining; when the worker's productivity varies across firms it is plausible to suppose that firms are not fully aware of the alternative opportunities available to their employees, and that employees are uncertain about their own value to their employer. In contrast to these two papers, the external labour market is not perfectly competitive: it is a matching market, and match productivity is the private information of the employer. We then assume that the wage and employment of an individual worker are determined according to a private-value auction mechanism: firms make simultaneous wage offers and the worker accepts the highest offer. We will show that the wage offer functions in this auction are consistent both with empirical earnings functions, and with human capital theory. In particular, the auction

implies sharing of the return to specific human capital.<sup>2</sup> We can then determine the direction of tenure bias.

### III. An Auction Model of Wage Determination

Consider a worker attached to firm 0, who has general human capital  $g \geq 1$ , and additional human capital  $k \geq 1$  specific to firm 0. His productivity if he remains in firm 0 is  $v_0$ , where:

$$\ln v_0 = \ln g + \ln k + \ln \varepsilon_0 \quad (1)$$

There are  $n \geq 1$  alternative employers, and if he moves to an alternative firm  $i$ , his productivity is  $v_i$ :

$$\ln v_i = \ln g + \ln \varepsilon_i \quad (2)$$

$\varepsilon_i > 0$  ( $i \geq 0$ ) represents the quality of his match with firm  $i$ , and is observable only by firm  $i$ ;  $g$  and  $k$  are common knowledge.

(1) and (2) capture the technological relationship between human capital and productivity. Wages are determined through a first-price private-value auction: each of the  $n + 1$  firms privately observes its own match quality, and makes a wage offer  $w_i$ ; the worker accepts the highest offer.

The parameter  $n$  can be regarded as capturing the degree of matching friction in the labour market. If search costs and information frictions in the external market are high, the worker might encounter a wage offer from only a single alternative firm ( $n = 1$ ). As frictions become smaller, the worker obtains a larger random sample of wage offers, and in the limit, as  $n$  becomes very large, he is aware of all the alternative opportunities in the market.

#### A. Distributional Assumptions

Match qualities  $\varepsilon_0, \varepsilon_1, \dots, \varepsilon_n$  are independent and identically distributed with density and distribution functions  $f(\varepsilon)$  and  $F(\varepsilon)$ . In order to apply results from auction theory we make the following distributional assumptions:

D1: Finite support:  $\varepsilon_i \in [\underline{\varepsilon}, \bar{\varepsilon}]$

---

<sup>2</sup>This is a model of wage determination for a worker already possessing some specific and some general human capital; we do not model the investment process, and the sharing of costs. However, it would be straightforward to do so: the incentives for the worker and firm to invest can be determined by calculating their expected returns.

D2:  $f(\underline{\varepsilon}) > 0$

We also require:

D3: The density of  $\ln \varepsilon_i$  is strictly log-concave.

A log-concave function is a function whose logarithm is concave. The class of log-concave densities is a wide one, and includes the uniform distribution and the truncated normal and log-normal distributions (Caplin and Nalebuff, 1991).

Strictly log-concave densities have the property that the log of the distribution function and the log of the integral of the distribution function are also strictly concave functions. So the assumption that  $\ln \varepsilon_i$  has a strictly log-concave density implies, for  $f(\varepsilon)$  and  $F(\varepsilon)$ , that the elasticity of  $F(\varepsilon)$  is strictly decreasing:

$$\frac{de}{d\varepsilon} < 0 \quad \text{where } e(\varepsilon) \equiv \frac{\varepsilon f(\varepsilon)}{F(\varepsilon)} \quad (3)$$

*A fortiori*,  $f(\varepsilon)$  and  $F(\varepsilon)$  are log-concave; so also are  $f_n(\varepsilon)$  and  $F_n(\varepsilon) \equiv F^n(\varepsilon)$ , the density and distribution of  $\max_{i=1,\dots,n} \varepsilon_i$ . For the subsequent analysis, it is useful to note that, if we define:

$$H_n(\varepsilon) \equiv \int_{\underline{\varepsilon}}^{\varepsilon} F_n(x) dx \quad (4)$$

then not only is  $F_n/H_n$  a decreasing function (since  $H_n$  is log-concave), but also:

$$\frac{d}{d\varepsilon} \left( \frac{\varepsilon F_n(\varepsilon)}{H_n(\varepsilon)} \right) < 0 \quad (5)$$

The proof of (5) is given in the appendix.

## B. Analysis of the Auction

The effect of specific human capital is to introduce an asymmetry between the players in the wage determination game. We will look for an equilibrium in which firm 0 offers  $w_0 = w_0(\varepsilon_0, g, k)$  and each of the  $n$  alternative firms uses the same strategy  $w_i = w_a(\varepsilon_i, g, k)$ . Firms choose their wage offer  $w$  to maximise payoffs:

for firm 0:

$$\Pi_0(\varepsilon_0, w) = (v_0 - w) \Pr[w_a(\varepsilon_j) \leq w \quad \forall j > 0]$$

and for firm  $i > 0$ :

$$\Pi_a(\varepsilon_i, w) = (v_i - w) \Pr[w_a(\varepsilon_j) \leq w \quad \forall j > 0, j \neq i, w_0(\varepsilon_0) \leq w]$$

Since general human capital  $g$  multiplies productivity for all firms, it is immediately clear that the elasticity of wage offers with respect to general human capital is unity:

**Lemma 1** *If  $W_0(\varepsilon, k) \equiv w_0(\varepsilon, 1, k)$  and  $W_a(\varepsilon, k) \equiv w_a(\varepsilon, 1, k)$  are equilibrium wage offers for the case  $g = 1$ , then  $w_0 = gW_0(\varepsilon, k)$  and  $w_a = gW_a(\varepsilon, k)$  are equilibrium wage offers for the general case  $g \geq 1$ .*

Hence we need only analyse the case  $g = 1$ . Maskin and Riley (2000a, 2000b) have proved the existence and uniqueness of equilibrium in auctions of this form. The equilibrium strategies are monotonic in  $\varepsilon$ , so we can define inverse wage offer functions:  $\phi_0(w, k) \equiv W_0^{-1}(w, k)$  and  $\phi_a(w, k) \equiv W_a^{-1}(w, k)$ . Following the standard analysis, the first order conditions for maximisation of the payoff functions above yield a pair of differential equations for the inverse offer functions  $\phi_0$  and  $\phi_a$ :

$$n \frac{f(\phi_a)}{F(\phi_a)} \frac{\partial \phi_a}{\partial w} = \frac{1}{k\phi_0 - w} \quad (6)$$

$$\frac{f(\phi_0)}{F(\phi_0)} \frac{\partial \phi_0}{\partial w} + (n-1) \frac{f(\phi_a)}{F(\phi_a)} \frac{\partial \phi_a}{\partial w} = \frac{1}{\phi_a - w} \quad (7)$$

Let  $\underline{w}(k)$  and  $\bar{w}(k)$  be the lowest and highest wage offers, respectively, of firm 0: that is:  $\phi_0(\underline{w}) = \underline{\varepsilon}$  and  $\phi_0(\bar{w}) = \bar{\varepsilon}$ . The following results are a direct application of the results for asymmetric first-price auctions given by Maskin and Riley (2000a, 2000b):

**Lemma 2**

- (i) *If  $k = 1$ ,  $\underline{w} = \underline{\varepsilon}$ ; otherwise  $\underline{w} = \arg \max F(w)^n (k\underline{\varepsilon} - w)$  and  $\underline{\varepsilon} < \underline{w} < k\underline{\varepsilon}$ .*
- (ii)  *$\phi_a(\underline{w}) = \underline{w}$  and  $\phi_a(\bar{w}) = \bar{\varepsilon}$ .*
- (iii) *The differential equations (6) and (7) have a unique monotonic solution  $\phi_0, \phi_a$  satisfying  $\phi_0(\underline{w}) = \underline{\varepsilon}$ ,  $\phi_a(\underline{w}) = \underline{w}$  and  $\phi_0(\bar{w}) = \phi_a(\bar{w}) = \bar{\varepsilon}$ .*

Thus all firms make the same highest offer  $\bar{w}$ , and the same *effective* lowest offer  $\underline{w}$ . The alternative firms may have realised productivity lower than  $\underline{w}$ , in which case they cannot obtain the worker. Note also that whenever the productivity of the worker in the potential match is greater than the lowest wage offer  $\underline{w}$ , firms obtain a positive expected surplus:

$$\phi_a(\varepsilon) > \varepsilon \quad \forall \varepsilon > \underline{w} \quad \text{and} \quad \phi_0(k\varepsilon) > \varepsilon \quad \forall \varepsilon > \underline{\varepsilon} \quad (8)$$

### The Symmetric Case, $k = 1$

When there is no specific capital, so that all bidders are the same, the system reduces with  $\phi_0(w, 1) = \phi_a(w, 1) \equiv \phi(w)$  to a single differential equation:

$$n \frac{f(\phi)}{F(\phi)} \frac{\partial \phi}{\partial w} = \frac{1}{\phi - w} \quad (9)$$

which can be solved directly by standard techniques to yield:

$$W_0(\varepsilon, 1) = \varepsilon - \frac{H_n(\varepsilon)}{F_n(\varepsilon)} \quad (10)$$

Using (5) it can be seen that the proportional “mark-down” of the wage offer below productivity,  $(\varepsilon - W_0)/\varepsilon$ , increases with unobserved match quality  $\varepsilon$ .

### Results for the Asymmetric Auction

In the asymmetric case, we first compare the strategy of the current employer with that of alternative employers. Examining the differential equations (6) and (7), we have:

**Lemma 3** *If  $k > 1$ :*

(i)  $W_a(\varepsilon) < W_0(\varepsilon)$  for all  $\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$

(ii)  $W_0(\varepsilon) < W_a(k\varepsilon)$  for all  $k\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$

PROOF: See Appendix. ■

The second inequality in Lemma 3 tells us that firm 0, where the worker has specific value, offers a lower wage than an alternative firm would offer with the same total productivity. Thus the current employer bids less aggressively than alternative firms (resulting, as usual in asymmetric first-price auctions, in inefficient allocation of the worker).

However, the first inequality says that firm 0 offers a higher wage than an alternative firm would offer with the same unobserved match quality. This means that, as we would expect, the presence of specific human capital tends to reduce the turnover probability. For the worker stays if and only if  $W_0(\varepsilon_0, k) > W_a(\varepsilon_{\max}, k)$ , where  $\varepsilon_{\max} = \max_{i=1, \dots, n} \varepsilon_i$ . So the probability  $P(\varepsilon_0, k)$  that the worker will stay with the current employer is given by:

$$P(\varepsilon_0, k) = F_n(\theta) \quad \text{where } \theta(\varepsilon, k) \equiv \phi_a(W_0(\varepsilon, k), k) \quad (11)$$

and Lemma 3 implies that  $\theta(\varepsilon, k) > \varepsilon = \theta(\varepsilon, 1)$ , so:

$$P(\varepsilon_0, k) > P(\varepsilon_0, 1) \quad (12)$$

Next, we examine the impact of specific human capital on the strategies of employers by comparing the asymmetric solutions  $W_0(\varepsilon, k)$  and  $W_a(\varepsilon, k)$  of (6) and (7) with the symmetric solution  $W_0(\varepsilon, 1)$  of (9):

**Lemma 4** *If  $k > 1$ , then for all  $\varepsilon \geq \underline{\varepsilon}$ :*

$$(i) \quad W_0(\varepsilon, 1) < W_a(\varepsilon, k)$$

$$(ii) \quad \text{specific human capital is shared: } W_0(\varepsilon, 1) < W_0(\varepsilon, k) < kW_0(\varepsilon, 1)$$

PROOF: See Appendix. ■

Lemma 4(i) tells us that outside firms strategically raise their wage offer when a worker has specific human capital in his current job. (From Lemma 3, however, we know that they do not raise their offers as much as the current employer.) Lemma 4(ii) states that specific human capital in the current job raises the wage, but by less than the increase in productivity. Combined with the result from Lemma 1,  $w_0 = gW_0(\varepsilon, k)$ , this implies that the wage determination process has the “classical” properties first proposed by Becker (1962): the return to general human capital accrues to the worker, but the return to specific human capital is shared between worker and firm.

## IV. Derivation of the Earnings Function

Now consider a first-order approximation to  $\ln(w_0)$ , the (log) wage offer of the current employer, valid for  $k$  close to 1 (that is, when specific human capital is small relative to general human capital):

$$\ln w_0 \approx \ln g + \alpha(\varepsilon_0) \ln k + \eta(\varepsilon_0) \quad (13)$$

where

$$\alpha(\varepsilon) \equiv \frac{1}{W_0(\varepsilon, 1)} \left. \frac{\partial W_0}{\partial k} \right|_{k=1} \quad \text{and} \quad \eta(\varepsilon) \equiv W_0(\varepsilon, 1) \quad (14)$$

From Lemma 4, we have  $0 < \frac{W_0(\varepsilon, k) - W_0(\varepsilon, 1)}{(k-1)W_0(\varepsilon, 1)} < 1$  for  $k > 1$  and  $\varepsilon \geq \underline{\varepsilon}$ . Taking the limit as  $k$  tends to 1 gives:

$$0 < \alpha(\varepsilon) < 1 \quad \forall \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}] \quad (15)$$

Equation (13) has the form of an empirical earnings function: the log-wage depends linearly on human capital, and the marginal effect of specific capital is less than one. The error term  $\eta(\varepsilon_0)$  represents the wage effect of unobserved match quality, and is independent of human capital. Thus we have established:

**Proposition 1** *A human capital earnings function can be interpreted as a log-linear approximation to the wage offer function in a labour market auction, valid when specific human capital is small relative to general human capital. The elasticity of the wage with respect to general human capital is 1, and the elasticity with respect to specific human capital lies strictly between 0 and 1.*

Note, however, that the coefficient on specific human capital is not constant, but varies with match quality. Intuitively, we might expect this coefficient to decrease as unobserved match quality  $\varepsilon_0$  increases. For we know (equation (10)) that in the absence of specific capital firms mark-down the wage more when match quality is high, so it seems plausible that high match quality obviates the need to reward observable specific human capital. Lemma 5 confirms this intuition for the normal and uniform distributions:

**Lemma 5** *When unobserved match quality,  $\ln \varepsilon$ , has a uniform distribution, or a symmetric truncated normal distribution, the elasticity of the wage with respect to observable specific human capital declines with match quality:  $\frac{d\alpha}{d\varepsilon} < 0$ .*

PROOF: See Appendix. ■

An apparent problem with Proposition 1 is that the earnings function (13) is the wage offer only for a worker previously employed in the firm. But we can also obtain the wage offer for a new worker, previously employed in a different firm, using our results for the alternative wage offer  $w_a$ . Such a worker has no specific capital in the firm making the offer, but may have human capital  $k_{-1}$  specific to his previous firm. A first-order approximation to  $\ln(w_a)$ , obtained in exactly the same way, gives:

$$\ln w_a \approx \ln g + \gamma(\varepsilon_i) \ln k_{-1} + \eta(\varepsilon_i) \quad (16)$$

where, following the same analysis as before, using Lemma 4:

$$\gamma(\varepsilon) \equiv \frac{1}{W_a(\varepsilon, 1)} \left. \frac{\partial W_a}{\partial k} \right|_{k=1} \quad \text{and} \quad 0 < \gamma(\varepsilon) < \alpha(\varepsilon) \quad (17)$$

Thus, Proposition 1 applies equally for “recent movers”, who have general, but no specific, human capital. However, (16) suggests that, since employers raise the wage offer

strategically for a worker who has observable specific capital in a different firm, we should include “specific capital in the previous job” in the earnings function for recent movers. So if we write the wage offer function as:

$$\ln w \approx \ln g + \alpha(\varepsilon)(1 - D) \ln k + \gamma(\varepsilon)D \ln k_{-1} + \eta(\varepsilon) \quad (18)$$

where  $D$  is a dummy variable indicating a recent mover, we have an earnings function encompassing both cases.

## A. Sample Selection Bias in the Return to Specific Capital

Consider the problem of estimating the wage offer function using data on the wages and human capital of a sample of workers. In this section we will assume that the econometrician has a sample of workers with varying amounts of specific and general capital, measured perfectly, and that it is a sample of “stayers”, for whom (13) is the appropriate wage offer function. (We will look at “movers”, and at the use of tenure as a proxy, in the next section.) Taking the expectation of (13) with respect to  $\varepsilon_0$  gives:

$$E[\ln w_0] \approx \ln g + \bar{\alpha} \ln k + c \quad (19)$$

where

$$c = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) dF(\varepsilon) \quad \text{and} \quad \bar{\alpha} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \alpha(\varepsilon) dF(\varepsilon) \in (0, 1) \quad (20)$$

Here  $\bar{\alpha}$  is the worker’s (ex-ante) expected share of specific capital. There is a sample selection problem in estimating the parameters of (19) because the wage offer  $w_0$  is observed only if it is accepted. The density of  $\varepsilon_0$  conditional on the wage having been accepted is given by:

$$\hat{f}(\varepsilon, k) = \frac{f(\varepsilon)F_n(\theta)}{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\theta) dF(x)} \quad (21)$$

Hence:

$$E[\ln w_0 \mid \text{acceptance}] \approx \ln g + \left( \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \alpha(\varepsilon) \hat{f}(\varepsilon, k) d\varepsilon \right) \ln k + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \hat{f}(\varepsilon, k) d\varepsilon \quad (22)$$

Continuing to exploit the assumption that specific human capital is small relative to general human capital, we can expand the expressions depending on  $k$  in (22) about  $k = 1$ , and ignore terms of order  $(\ln k)^2$ , to obtain:

$$E[\ln w_0 \mid \text{acceptance}] \approx \ln g + \hat{\alpha} \ln k + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon \quad (23)$$

where

$$\hat{\alpha} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \alpha(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \left. \frac{\partial \hat{f}}{\partial k} \right|_{k=1} d\varepsilon \quad (24)$$

A comparison of (23) and (19) shows that there are three separate biases. First, both the constant and the coefficient on specific capital are biased because of the conditional expectation of the error term  $\eta(\varepsilon_0)$ . What effect will this have? From (11) we know that, *ceteris paribus*, unobserved match quality  $\varepsilon_0$  increases the probability of acceptance  $P(\varepsilon_0, k)$ . So conditional on acceptance, match quality  $\varepsilon_0$ , and its wage effect  $\eta(\varepsilon_0)$ , will be high. This suggests that the constant term will be positively biased. But we know from (12) that specific capital,  $k$ , also increases the probability of acceptance  $P(\varepsilon_0, k)$ , suggesting a *negative* sample selection bias in estimation of the coefficient on specific human capital. Conditional on acceptance, workers with higher  $k$  will have, on average, lower unobserved match quality than those with low  $k$  – specific human capital and unobserved match quality will be negatively correlated, and this will obscure the effect of  $k$  on the wage. The third bias occurs because the average return to specific capital in a sample of accepted wages in firm 0 (the first term in (24)) is different from the ex-ante average return  $\bar{\alpha}$ . If, as for the normal and uniform distributions,  $\alpha(\varepsilon)$  decreases with  $\varepsilon$ , then the average return of “stayers” (who have higher match quality) will be lower than  $\bar{\alpha}$ .

These conjectures on the direction of the biases are verified in the proof of the following proposition:

**Proposition 2** *If the wage offer function (19) is estimated by Ordinary Least Squares on a sample of accepted wages:*

(i) *the constant is positively biased:*

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon > c$$

(ii) *there is a negative bias in the coefficient on specific capital because specific capital is negatively related to unobserved match quality:*

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \left. \frac{\partial \hat{f}}{\partial k} \right|_{k=1} d\varepsilon < 0$$

(iii) *if the distribution of  $\ln \varepsilon$  is truncated normal or uniform, there is a negative bias in the coefficient on specific capital because the average return to specific capital is*

*negatively related to unobserved match quality:*

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \alpha(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon < \bar{\alpha}$$

PROOF: See Appendix. ■

## B. Sample Selection Bias in the Return to Tenure

In practice, we rarely have a good measure of specific human capital, so use job tenure as a proxy. It is often assumed that most specific human capital is accumulated during the early months of employment, in which case we are particularly interested in the return to, say, the first year of tenure. We could try to estimate this by comparing the wages of those who have been in their current jobs for one year with the wages of workers of the same labour market experience who have recently moved. This is the case is considered by Garen (1988), described in section II. above; we now apply the wage offer model to this estimation problem.

Suppose that a wage offer game for each worker takes place when his tenure  $T = 1$  and he has accumulated specific capital  $k$ . A sample of workers with the same labour market experience will contain some workers who accept the wage offer  $w_0$  of the current employer, and have  $T = 1$ , and other workers who accept an alternative offer  $w_a$ , and have  $T = 0$ . Ex-ante, the return to tenure is the expected difference between the wage offers of the current and alternative employers (obtained as in the previous section):

$$\text{Return to tenure} = E[\ln w_0] - E[\ln w_a] \approx (\bar{\alpha} - \bar{\gamma}) \ln k \quad (25)$$

where

$$\bar{\alpha} - \bar{\gamma} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \gamma(\varepsilon)) dF(\varepsilon) \quad (26)$$

Again, there is a sample selection problem in trying to estimate (25), because we observe only accepted wages. The effect of one year of tenure on wages obtained by comparing the wages of those with  $T = 1$  and  $T = 0$  is given by:

$$\text{Estimated tenure effect} = E[\ln w_0 \mid w_0 \text{ accepted}] - E[\ln w_a \mid w_a \text{ accepted}] \quad (27)$$

We have already evaluated the first term, in (23) above. For the second term, note that a worker who leaves firm 0 receives the maximum of  $n$  alternative offers,  $w_a(\varepsilon_{\max})$ , where  $\varepsilon_{\max} = \max_{i=1, \dots, n} \varepsilon_i$ . The density function of  $\varepsilon_{\max}$ , conditional on the best alternative

wage offer having been accepted, is:

$$\hat{h}(\varepsilon_{\max}, k) = \frac{f(\varepsilon_{\max})F_{n-1}(\varepsilon_{\max})F(\theta^{-1}(\varepsilon_{\max}))}{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_{n-1}(x)F(\theta^{-1}(x)) dF(x)} \quad (28)$$

In the absence of specific capital, the two conditional densities are identical:  $\hat{f}(\varepsilon, 1) = \hat{h}(\varepsilon, 1)$ . The equivalent of (23) for the alternative wage is:

$$E[\ln w_a \mid w_a \text{ accepted}] \approx \ln g + \hat{\gamma} \ln k + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon \quad (29)$$

where:

$$\hat{\gamma} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \gamma(\varepsilon) \hat{f}(\varepsilon, 1) d\varepsilon + \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \left. \frac{\partial \hat{h}}{\partial k} \right|_{k=1} d\varepsilon \quad (30)$$

It can be verified (as in the Proof of Proposition 2(ii)) that  $\left. \frac{\partial \hat{h}}{\partial k} \right|_{k=1} = -\frac{1}{n} \left. \frac{\partial \hat{f}}{\partial k} \right|_{k=1}$ . Hence, using (23), (24), (27), (29) and (30), we obtain:

$$\text{Estimated tenure effect} = (\hat{\alpha} - \hat{\gamma}) \ln k \quad (31)$$

where

$$\hat{\alpha} - \hat{\gamma} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \gamma(\varepsilon)) \hat{f}(\varepsilon, 1) d\varepsilon + \frac{n+1}{n} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \left. \frac{\partial \hat{f}}{\partial k} \right|_{k=1} d\varepsilon \quad (32)$$

Now, comparing (32) and (26), we can see that the potential biases are similar to those in the coefficient on specific capital in the previous section. We showed there that the last term in (32) is negative. The interpretation of this term is that, when workers have specific capital, they will stay with their current firm even when match quality is low, and move only when match quality in an alternative firm is high – this biases the relative wage downwards. The other bias occurs because the coefficients on specific capital,  $\alpha$  and  $\gamma$ , depend on match quality, which is higher in accepted wages. From the previous section we know that, under distributional assumptions,  $\alpha$  decreases with quality. Intuitively we might expect  $\alpha$  to decrease faster than  $\gamma$  – when their match quality is high, alternative firms have a better chance of obtaining the worker and may therefore bid more aggressively. If so, this bias will also be negative. The following proposition confirms the downward bias:

**Proposition 3** *If the distribution of  $\ln \varepsilon$  is truncated normal or uniform, the estimated return to tenure is negatively biased:  $\hat{\alpha} - \hat{\gamma} < \bar{\alpha} - \bar{\gamma}$ .*

PROOF: See Appendix. ■

Finally, note that one obvious but important implication of the analysis above is that job-matching alone does *not* introduce an apparent tenure effect. If the worker has no specific capital ( $k = 1$ ) the wage offer functions of current and alternative employers are the same and the estimated tenure effect is zero. In a sense we have obtained this result by construction – it follows from the assumption that match quality is identically distributed in all firms. But it is surely correct to say that if a worker has no specific capital his value to all potential employers should be modelled symmetrically, as in the present model. If there were anything different ex-ante (before the realisation of match qualities) about his value to his current employer, it would, by definition, be some form of specific capital.

It might be argued that if we extended the model to allow for more periods, matches with high quality would last for more periods, inducing an apparent tenure effect. But this would only occur if match quality had a positive permanent component. And, a permanently higher expected value to the current firm is, by definition, specific human capital. Even if the firm and the worker did not intentionally invest in this capital, the problem of finding an appropriate wage to protect it from loss, once they are aware of its existence, is the same. A period of discovery of permanent match quality might, anyway, be interpreted as an investment period. Furthermore, competing employers can interpret longer tenure as a signal of the existence of permanent match quality, in which case there is no distinction at all (except the imperfection of the signal) between “permanent match quality” and “observable specific capital” as defined in the model of this paper. To model these effects precisely would clearly require a more complex dynamic approach, but this argument suggests that a positive relationship between wages and tenure should be interpreted as evidence of specific human capital.

## V. Conclusion

Without specifying the wage determination process we cannot interpret empirical earnings functions containing specific capital, nor resolve the associated problem of sample selection bias. If wages are predetermined as in the standard model of specific human capital there is no such bias, but if the earnings function represents a wage offer, sample selection bias may occur as a result of the job acceptance decisions of workers.

The interpretation of the earnings function as a wage offer is consistent with the conventional analysis of the sample selection problem. Indeed, Topel (1986) and Garen (1988) both refer to it explicitly as a wage offer. But they have a wage offer function that depends exogenously on specific human capital and match quality. The approach

here differs in that, as well as allowing for the decisions of workers to accept or reject offers, we have allowed for firm behaviour. By endogenising the wage offer, we obtain a precise understanding of the interaction between observable specific human capital and unobserved match quality, which has been at the heart of the debate on tenure bias. We have obtained two important results that directly contradict the conventional wisdom:

- (i) the OLS estimate of the return to specific human capital or tenure is negatively biased;
- (ii) job matching does not introduce an apparent return to tenure.

Why is the usual argument for a positive or ambiguous bias incorrect? The supposed positive effect is that workers with high match quality are more likely to stay, so have higher wages partly because of their unobserved match quality. We have shown (Proposition 2) that this biases (positively) the constant in the earnings function, not the coefficient on specific capital. On the contrary, the coefficient on specific capital is negatively biased because those with high specific capital stay even when their match quality is low – the error term, reflecting unobserved quality, is negatively correlated with specific capital in a sample of accepted wages.

The usual argument for an effect in the opposite direction, emphasised by Topel (1991), is that people who have high match quality in an alternative match are more likely to move, so those with low specific capital and tenure may have high wages. This is particularly important for the estimation of the return to the first period of tenure. Again, our analysis demonstrates that this effect appears in the constant term of the earnings function, and when the return to one year of tenure is estimated by comparing stayers with movers, the two matching effects cancel out. However, there is again a negative bias in the return to tenure, because when workers have specific capital, they stay even when match quality is low and move only when it is high, lowering the wage for stayers relative to movers.

# Appendix

PROOF OF EQUATION 5:

$$\begin{aligned}
\frac{d}{d\varepsilon} \left( \frac{\varepsilon F_n(\varepsilon)}{H_n(\varepsilon)} \right) &\stackrel{\text{sgn}}{=} \frac{\int F^n}{F^n} - \frac{\int G}{G} \quad \text{where } G(\varepsilon) \equiv \frac{d}{d\varepsilon} (\varepsilon F^n(\varepsilon)) = F^n(\varepsilon) (1 + n e(\varepsilon)) \\
&= \int_{\underline{\varepsilon}}^{\varepsilon} \left( \frac{F^n(x)}{F^n(\varepsilon)} - \frac{G(x)}{G(\varepsilon)} \right) dx \\
&= \int_{\underline{\varepsilon}}^{\varepsilon} \frac{n F^n(x)}{G(\varepsilon)} (e(\varepsilon) - e(x)) dx < 0 \quad \text{since by (3) } e \text{ is decreasing.}
\end{aligned}$$

■

PROOF OF LEMMA 3: Subtracting (6) from (7), and writing  $e(\varepsilon) \equiv \frac{\varepsilon f'(\varepsilon)}{F(\varepsilon)}$ :

$$\frac{e(\phi_0)}{\phi_0} \frac{\partial \phi_0}{\partial w} - \frac{e(\phi_a)}{\phi_a} \frac{\partial \phi_a}{\partial w} = \frac{1}{\phi_a - w} - \frac{1}{k\phi_0 - w} \tag{A.1}$$

First consider  $y(w) \equiv \phi_0 - \phi_a$ . If  $y = 0$  at  $w_0 \in (\underline{w}, \bar{w}]$ , (A.1)  $\Rightarrow y' > 0$ . But  $y = 0$  at  $\bar{w}$ . Hence there can be no other such  $w_0$ , and  $y(w) < 0 \forall w < \bar{w}$ .

Now let  $z(w) \equiv \phi_a - k\phi_0$ . If  $z = 0$  at  $w_0 \in (\underline{w}, \bar{w})$ , (A.1)  $\Rightarrow ke(\phi_0)\phi_0' - e(k\phi_0)\phi_a' = 0$ . And from (3),  $e(k\phi_0) < e(\phi_0)$ , so at  $w_0$ ,  $z' > 0$ . But  $z < 0$  at  $\bar{w}$ . Hence there can be no such  $w_0$ , and  $z(w) < 0 \forall w < \bar{w}$ .

Thus we have established that  $\phi_0(w) < \phi_a(w) < k\phi_0(w)$  for all  $w \in (\underline{w}, \bar{w})$ ; inverting gives the required inequalities for the wage offer functions. ■

PROOF OF LEMMA 4: Subtracting (9) from (6), and using Lemma 3:

$$\frac{e(\phi_a)}{\phi_a} \frac{\partial \phi_a}{\partial w} - \frac{e(\phi)}{\phi} \frac{\partial \phi}{\partial w} = \frac{1}{k\phi_0 - w} - \frac{1}{\phi - w} < \frac{1}{\phi_a - w} - \frac{1}{\phi - w} \tag{A.2}$$

We will prove that  $\phi_0(w) < \phi_a(w) < \phi(w) < \phi_0(kw)$  for all  $w \geq \underline{w}$ . Inverting each of these inequalities separately then gives the required results for the wage offer functions.

The lower inequality has already been proved in Lemma 3. For the middle inequality, let  $y(w) \equiv \phi_a - \phi$ . At  $\underline{w}(k) (> \underline{\varepsilon})$ , from (8),  $y < 0$ . If  $y = 0$  at  $w_0 \in (\underline{w}, \bar{w}]$ , (A.2)  $\Rightarrow y' < 0$ . Hence there is no such  $w_0$ , and  $\phi_a < \phi \forall w < \bar{w}$ .

For the upper inequality, let  $\psi \equiv \phi(w/k)$ . (9) can be written:

$$\frac{e(\psi)}{\psi} \frac{\partial \psi}{\partial w} = \frac{1}{n} \frac{1}{k\psi - w}$$

Also, from (6) and (7):

$$\frac{e(\phi_0)}{\phi_0} \frac{\partial \phi_0}{\partial w} = \frac{1}{\phi_a - w} - \frac{n-1}{n} \frac{1}{k\phi_0 - w}$$

Subtracting:

$$\frac{e(\phi_0)}{\phi_0} \frac{\partial \phi_0}{\partial w} - \frac{e(\psi)}{\psi} \frac{\partial \psi}{\partial w} = \left( \frac{1}{\phi_a - w} - \frac{1}{k\phi_0 - w} \right) + \frac{1}{n} \left( \frac{1}{k\phi_0 - w} - \frac{1}{k\psi - w} \right) \quad (\text{A.3})$$

Consider  $z(w) \equiv \phi_0 - \psi$ . When  $w = k\underline{\varepsilon}$ , from (8),  $z > 0$ . If  $z = 0$  at  $w_0 > k\underline{\varepsilon}$ , (A.3)  $\Rightarrow z' > 0$ . Hence there is no such  $w_0$ , and  $z > 0 \forall w > k\underline{\varepsilon}$ , or equivalently  $\phi(w) < \phi_0(kw) \forall w \geq \underline{\varepsilon}$ .  $\blacksquare$

PROOF OF LEMMA 5: Transform equations (6) and (7), putting  $w = W_0(\varepsilon, k)$  and  $\theta(\varepsilon, k) \equiv \phi_a(W_0(\varepsilon, k), k)$  as in (11):

$$n \frac{e(\theta)}{\theta} \frac{\partial \theta}{\partial \varepsilon} (k\varepsilon - W_0) = \frac{\partial W_0}{\partial \varepsilon} \quad (\text{A.4})$$

$$\left[ (n-1) \frac{e(\theta)}{\theta} \frac{\partial \theta}{\partial \varepsilon} + \frac{e(\varepsilon)}{\varepsilon} \right] (\theta - W_0) = \frac{\partial W_0}{\partial \varepsilon} \quad (\text{A.5})$$

Differentiating (A.4) and (A.5) with respect to  $k$ , setting  $k = 1$ , and rearranging:

$$\frac{\partial^2 W_0}{\partial \varepsilon \partial k} = n(\varepsilon - W_0) \frac{\partial}{\partial \varepsilon} \left( \frac{e(\varepsilon)}{\varepsilon} \frac{\partial \theta}{\partial k} \right) + n \frac{e(\varepsilon)}{\varepsilon} \left( \varepsilon - \frac{\partial W_0}{\partial k} \right) \quad (\text{A.6})$$

$$0 = (\varepsilon - W_0) \frac{\partial}{\partial \varepsilon} \left( \frac{e(\varepsilon)}{\varepsilon} \frac{\partial \theta}{\partial k} \right) + n \frac{e(\varepsilon)}{\varepsilon} \left( \varepsilon - \frac{\partial \theta}{\partial k} \right) \quad (\text{A.7})$$

Now, when  $k = 1$ ,  $\varepsilon - W_0 = H_n(\varepsilon)/F_n(\varepsilon)$  (equation (10)). Define  $\beta(\varepsilon)$  analogously with  $\alpha(\varepsilon)$ :

$$\alpha(\varepsilon) \equiv \frac{1}{W_0(\varepsilon, 1)} \frac{\partial W_0}{\partial k} \Big|_{k=1}, \quad \beta(\varepsilon) \equiv \frac{1}{\varepsilon} \frac{\partial \theta}{\partial k} \Big|_{k=1}$$

Then we can transform (A.6) and (A.7) to obtain a pair of differential equations in  $\alpha$  and  $\beta$ :

$$\beta' + \frac{e'}{e} \beta + \frac{nF_n}{H_n} (1 - \beta) = 0 \quad (\text{A.8})$$

$$\alpha' = \left( \frac{ne}{\varepsilon - H_n/F_n} \right) (1 - \alpha - n(1 - \beta)) \quad (\text{A.9})$$

We already know that  $0 < \alpha < 1$ . From Lemma 3  $\varepsilon < \theta < k\varepsilon$ , so  $0 < \beta < 1$ . At  $\underline{\varepsilon}$ ,  $\beta = \alpha = (1/\underline{\varepsilon}) d\underline{w}/dk$ . From Lemma 2(i)  $\underline{w}$  satisfies  $nf(\underline{w})(k\underline{\varepsilon} - \underline{w}) = F(\underline{w})$ , from which we obtain:

$$\frac{1}{\underline{\varepsilon}} \frac{d\underline{w}}{dk} = \frac{nf^2(\underline{w})}{(n+1)f^2(\underline{w}) - F(\underline{w})f'(\underline{w})}$$

When  $k = 1$ ,  $\underline{w} = \underline{\varepsilon}$ ; hence  $\alpha(\underline{\varepsilon}) = \beta(\underline{\varepsilon}) = n/(n+1)$ . Equation (A.8) can be written:

$$\beta' = \left( -\frac{1}{\varepsilon} - \frac{f'}{f} + \frac{f}{F} + \frac{nF_n}{H_n} \right) \beta - \frac{nF_n}{H_n}$$

The bracketed term is positive (by (3)); so when  $\beta \leq n/(n+1)$ :

$$\beta' \leq \frac{n}{n+1} \left( -\frac{1}{\varepsilon} - \frac{f'}{f} + \frac{f}{F} - \frac{F_n}{H_n} \right) < \frac{n}{n+1} \left( -\frac{1}{\varepsilon} - \frac{f'}{f} - (n-1) \frac{f}{F} \right)$$

(using the log-concavity of  $H_n$ ).

When  $\ln \varepsilon$  has a uniform distribution,  $\frac{1}{\varepsilon} + \frac{f'}{f} = 0$ , and  $\beta'$  is negative whenever  $\beta \leq \frac{n}{n+1}$ . Since  $\beta(\underline{\varepsilon}) = \frac{n}{n+1}$ , this means that  $\beta'$  is negative for all  $\varepsilon$ .

When  $\ln \varepsilon$  is truncated normal,<sup>3</sup>  $\frac{1}{\varepsilon} + \frac{f'}{f} = -\frac{\ln \varepsilon}{\varepsilon}$ , so by the same argument  $\beta'$  is negative for all  $\varepsilon \leq 1$ . Now suppose  $\beta' = 0$  at some  $1 < \varepsilon_0 < \bar{\varepsilon}$ . Differentiating (A.8)  $\Rightarrow \beta'' \stackrel{\text{sgn}}{=} \frac{\partial}{\partial \varepsilon} \left( -\frac{\varepsilon e'}{e} \frac{H_n(\varepsilon)}{\varepsilon F_n(\varepsilon)} \right)$  at  $\varepsilon_0$ .  $H_n/(\varepsilon F_n)$  is positive and increasing, and by Lemma 6 below, so is  $(-\varepsilon e'/e)$  for the normal distribution when  $\varepsilon > 1$ . So  $\beta'' > 0$  at  $\varepsilon_0$ . But this is impossible because at  $\bar{\varepsilon}$ ,  $\theta(\varepsilon) = \varepsilon \forall k$ , so  $\beta = 0$ , and  $\beta' < 0$ . Hence  $\beta' < 0 \forall \varepsilon$ .

Now consider equation (A.9). At  $\underline{\varepsilon}$ ,  $\alpha = \beta \Rightarrow \alpha' < 0$ . Differentiating shows that if  $\alpha' = 0$  at some  $\varepsilon_0 > \underline{\varepsilon}$ ,  $\alpha'' \stackrel{\text{sgn}}{=} \beta' < 0$ . This is impossible, so  $\alpha' < 0 \forall \varepsilon$ .  $\blacksquare$

**Lemma 6** *When  $\ln \varepsilon$  has a symmetric truncated normal distribution,  $\frac{d}{d\varepsilon} \left( -\frac{\varepsilon}{e} \frac{de}{d\varepsilon} \right) > 0$  for all  $\varepsilon > 1$ .*

PROOF:  $e(\varepsilon) \equiv \frac{\varepsilon f(\varepsilon)}{F(\varepsilon)} = \frac{g(\ln \varepsilon)}{G(\ln \varepsilon)}$  where  $g(x) = \frac{c}{\sqrt{2\pi}} \exp(-x^2/2)$  for some constant  $c$  and  $G$  is the corresponding distribution function. It is sufficient to prove that  $y(x) \equiv \frac{d^2 \ln}{dx^2} \left( \frac{g(x)}{G(x)} \right) < 0$  for  $x > 0$ . Evaluating this:  $y = \frac{g}{G} \left( x + \frac{g}{G} \right) - 1$ . When  $x = 0$ , symmetry  $\Rightarrow G = \frac{1}{2}$  and it can be verified that  $y < 0$ . Then:  $\frac{dy}{dx} = -(y+1) \left( x + \frac{g}{G} \right) - y \frac{g}{G}$  so if  $y = 0$  at any  $x_0 > 0$ ,  $dy/dx < 0$ . Hence there can be no such  $x_0$ , and  $y < 0 \forall x > 0$ .  $\blacksquare$

PROOF OF PROPOSITION 2:

(i) Since  $\eta(\varepsilon)$  and  $F_n(\varepsilon)$  are both increasing functions of  $\varepsilon$ , Lemma 7 below implies:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) F_n(\varepsilon) dF(\varepsilon) > \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) dF(\varepsilon) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\varepsilon) dF(\varepsilon)$$

Rearranging:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) \hat{f}(\varepsilon, 1) dF(\varepsilon) > \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) dF(\varepsilon) = c$$

(ii)

$$\left. \frac{\partial F_n(\theta)}{\partial k} \right|_{k=1} = f_n(\varepsilon) \left. \frac{\partial \theta}{\partial k} \right|_{k=1} = n F_n(\varepsilon) e(\varepsilon) \beta(\varepsilon)$$

where  $\beta$  is defined as in the proof of Lemma 5. Hence:

$$\left. \frac{\partial \hat{f}}{\partial k} \right|_{k=1} = n \frac{\left( \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x) dF(x) \right) f(\varepsilon) F_n(\varepsilon) e(\varepsilon) \beta(\varepsilon) - \left( \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x) e(x) \beta(x) dF(x) \right) f(\varepsilon) F_n(\varepsilon)}{\left( \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x) dF(x) \right)^2}$$

---

<sup>3</sup>This proof is for the standard normal distribution; it extends to the general case in the obvious way.

and it is required to prove that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x) dF(x) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) F_n(\varepsilon) e(\varepsilon) \beta(\varepsilon) dF(\varepsilon) < \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(x) e(x) \beta(x) dF(x) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) F_n(\varepsilon) dF(\varepsilon)$$

or equivalently, since  $F_n(\varepsilon)f(\varepsilon) = f_{n+1}(\varepsilon)/(n+1)$ :

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) e(\varepsilon) \beta(\varepsilon) dF_{n+1}(\varepsilon) < \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} e(x) \beta(x) dF_{n+1}(x) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \eta(\varepsilon) dF_{n+1}(\varepsilon)$$

But this inequality holds by Lemma 7 below, since  $\eta$  is an increasing function and from equation (A.8)  $e\beta$  is decreasing.

(iii) From Lemma 5  $\alpha$  is a decreasing function of  $\varepsilon$  so applying Lemma 7 again:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \bar{\alpha}) \hat{f}(\varepsilon, 1) d\varepsilon \stackrel{\text{sgn}}{=} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \bar{\alpha}) F_n(\varepsilon) dF(\varepsilon) < \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \bar{\alpha}) dF(\varepsilon) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F_n(\varepsilon) dF(\varepsilon)$$

and the right-hand-side is zero by definition of  $\bar{\alpha}$ . ■

**Lemma 7** *If  $K(\varepsilon)$  is a distribution function with support  $[\underline{\varepsilon}, \bar{\varepsilon}]$  and  $y(\varepsilon)$  is increasing on  $[\underline{\varepsilon}, \bar{\varepsilon}]$ , then:*

(i) *if  $z(\varepsilon)$  is increasing on  $[\underline{\varepsilon}, \bar{\varepsilon}]$*

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon) z(\varepsilon) dK(\varepsilon) > \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon) dK(\varepsilon) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} z(\varepsilon) dK(\varepsilon)$$

(ii) *if  $z(\varepsilon)$  is decreasing the reverse inequality holds.*

PROOF:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon) z(\varepsilon) dK(\varepsilon) - \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y(\varepsilon) dK(\varepsilon) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} z(\varepsilon) dK(\varepsilon) = \frac{1}{2} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (y(\varepsilon) - y(x))(z(\varepsilon) - z(x)) dK(\varepsilon) dK(x)$$

If  $z$  is increasing the integrand is everywhere positive; if  $z$  is decreasing it is negative. ■

PROOF OF PROPOSITION 3: The second term in equation (32) is negative by Proposition 2(ii). Hence it is sufficient to prove that  $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\alpha(\varepsilon) - \gamma(\varepsilon) - (\bar{\alpha} - \bar{\gamma}) \hat{f}(\varepsilon, 1)) d\varepsilon < 0$ , which follows exactly as in Proposition 2(iii), provided that we can show that  $\alpha - \gamma$  is a decreasing function of  $\varepsilon$ . For this, we have from (11)  $W_a(\theta(\varepsilon, k), k) = W_0(\varepsilon, k)$ . Differentiating with respect to  $k$ , and putting  $k = 1$ , we obtain:

$$\alpha(\varepsilon) - \gamma(\varepsilon) = \frac{\varepsilon}{W_0(\varepsilon)} \frac{\partial W_0}{\partial \varepsilon} \beta(\varepsilon) = \frac{ne\beta}{\varepsilon F_n / H_n - 1}$$

where the second equality follows from (10). Differentiating and using (A.8) gives:

$$(\alpha - \gamma)' \stackrel{\text{sgn}}{\equiv} -ne\beta + \left( \frac{\varepsilon F_n}{H_n} - 1 \right) (\beta - n(1 - \beta))$$

From the proof of Lemma 5 we know that (under the distributional assumptions)  $\beta \leq \frac{n}{n+1}$  for all  $\varepsilon$ , which implies that  $(\alpha - \gamma)' < 0$ . ■

## References

- Abraham, Katharine G., and Farber, Henry S. "Job Duration, Seniority and Earnings." *American Economic Review* 77 (June 1987): 278–97.
- Altonji, Joseph G., and Shakotko, Robert A. "Do Wages Rise with Job Seniority?" *Review of Economic Studies* 54 (July 1987): 437–59.
- Barth, Erling. "Firm-Specific Seniority and Wages." *Journal of Labor Economics* 15 (July 1997): 495–506.
- Becker, Gary S. "Investment in Human Capital: A Theoretical Analysis." *Journal of Political Economy* 70, suppl. (October 1962): 9–49.
- Caplin, Andrew, and Nalebuff, Barry. "Aggregation and Social Choice: A Mean Voter Theorem." *Econometrica* 59 (January 1991): 1–23.
- Carmichael, Lorne. "Firm Specific Human Capital and Promotion Ladders." *Bell Journal* 14 (Spring 1983): 251–8.
- Garen, John E. "Empirical Studies of the Job Matching Hypothesis." *Research in Labor Economics* 9 (1988): 187–244.
- Gibbons, Robert, and Waldman, Michael. "Careers in Organizations: Theory and Evidence." In *Handbook of Labour Economics*, vol. 3B, edited by Orley Ashenfelter and David Card. Amsterdam: North Holland, 1999.
- Hall, Robert E., and Lazear, Edward P. "The Excess Sensitivity of Quits and Layoffs to Demand." *Journal of Labor Economics* 2 (April 1984): 233–57.
- Hashimoto, Masanori. "Firm-Specific Human Capital as a Shared Investment." *American Economic Review* 71 (June 1981): 475–82.
- Lazear, Edward P. "Agency, Earnings Profiles, and Hours Restrictions." *American Economic Review* 71 (September 1981): 606–20.
- Maskin, Eric, and Riley, John. "Asymmetric Auctions." *Review of Economic Studies* 67 (July 2000): 423–38. (a)
- . "Equilibrium in Sealed High Bid Auctions." *Review of Economic Studies* 67 (July 2000): 439–54. (b)
- Mincer, Jacob. *Schooling, Experience and Earnings*. New York: Columbia University Press, 1974.
- Mincer, Jacob, and Jovanovic, Boyan. "Labor Mobility and Wages." In *Studies in Labor Markets*, edited by S. Rosen. Chicago: University of Chicago Press, 1981.
- Oi, Walter Y. "Labour as a Quasi-Fixed Factor." *Journal of Political Economy* 70 (December 1962): 538–55.
- Stevens, Margaret. "Labour Contracts and Efficiency in On-the-Job Training." *Economic*

*Journal* 104 (March 1994): 408-20.

Topel, Robert. "Job Mobility, Search and Earnings Growth: A Reinterpretation of Human Capital Earnings Functions." *Research in Labor Economics* 8A (1986): 199–233.

—. "Specific Capital, Mobility and Wages: Wages Rise with Job Seniority." *Journal of Political Economy* 99 (February 1991): 145-76.